MTH5118 Probability II. Problem Sheet 3.

Please staple your coursework and post it in the Orange Box on the ground floor of the Maths building by **15:00** *on Wednesday 15th October 2006.*

1. Gonzo has £100 to gamble on roulette at a casino with no zero, so that $P(red) = \frac{1}{2}$. His stake at each game is £1, which he bets on red. If red comes up he wins £1 and gets his stake back. If red does not come up he loses his £1 stake. He decides to play until he loses all his money or has doubled his money to £200. Find $E[X]$, where X counts the number of games he plays.

By changing units, obtain $E[X]$ if his stake at each game is £10. Nothing else is altered.

2. Jack plays a series of games. At each game Jack wins, loses or draws with equal probabilities of a third. He bets one unit each time and receives an additional unit, gets the unit back, or loses the unit depending upon whether he wins, draws or loses the game. He starts with k units and stops playing when he reaches 0 or N units ($0 \leq k \leq N$). Find $E_k = E[X_k]$, where X_k counts the number of games he plays.

3. A mouse is put at the centre of a maze. The time X until he escapes from the maze is measured in minutes. There are 3 routes he may select, two of which lead to dead ends so that the mouse has to retrace his path back to the centre of the maze. The time taken for route 1 is 10 minutes and for route 2 is 15 minutes. The third path leads out of the maze in 8 minutes. Find $E[X]$ for each of the following cases:

(a) Each time he is at the centre of the maze he selects one of the 3 routes at random.

(b) Initially he chooses one of the 3 routes at random, but subsequently he does not choose the route he has just returned along but chooses one of the remaining two routes at random. Hint: Let X_i be the time to get out of the maze if his choice initially is restricted so that he cannot choose route i but chooses one of the other two routes at random.

4. Consider a randomly selected female from a population. Let Y count the number of children she will have during her lifetime. Assume that each child is equally likely to be male or female and the sex is independent of the sex of previous children. Let X count the number of her daughters. State the distribution of $X|Y = y$ i.e. the distribution of the number who are female out of the fixed number x of children. Hence give $E[X|Y = y]$, $Var(X|Y = y)$ and $E[t^X|Y = y]$.

(a) If $E[Y] = 4$ and $Var(Y) = 1$ use the results above to obtain $E[X]$ and $Var(X)$.

(b) If $Y \sim Bernoulli(\theta)$ state $G_Y(t)$ and use the appropriate result above to obtain $G_X(t)$. Hence state the distribution of the number of daughters, X.