MTH Probability II. Problem Sheet 2.

Please staple your coursework and post it in the Orange Box on the ground floor of the Maths building by **15:00** *on Wednesday 8th October 2008.*

1. A pair of fair six-sided dice are thrown repeatedly until their sum is either 12 or 7. The sum on the final throw is recorded. Let E be the event that the recorded sum is 12. By conditioning on the outcome of the first throw of the two dice, use the theorem of total probability to obtain $P(E)$.

2 The roulette wheel at a casino has integers from 1 to 36, together with 0. Half of the non-zero numbers are red, the other half are black, and 0 is green. Any of the numbers between 0 and 36 is equally likely to occur each time the wheel is spun.

Gonzo has £100 to gamble on roulette at the casino. His stake at each game is £1, which he bets on red. If red comes up he wins £1 and gets his stake back. If red does not come up he loses his £1 stake. He decides to play until he loses all his money or has doubled his money to £200. Find the probability that Gonzo doubles his money.

State the probability that Gonzo doubles his money if he has £1000 to gamble and bets £10 each time. Again he stops if he either loses all his money or doubles it.

3. Repeat the first part of question 2 if the roulette wheel in the casino has no zero.

4. Jack plays a series of games. At each game Jack wins, loses or draws with equal probabilities of a third. He bets one unit each time and receives an additional unit, gets the unit back, or loses the unit depending upon whether he wins, draws or loses the game. He starts with k units and stops playing when he reaches 0 or N units ($0 \leq k \leq N$). Find the probability that he loses all his money. Note that this is a very minor adaptation of the standard gambler's ruin problem from lectures.

5. A frog performs a random walk on the integers. At each stage he jumps either one integer forwards or one integer back with equal probability. If he starts at the integer 1, find the probability that he reaches 0 before he reaches 12. If he starts at the integer 11 find the probability that he reaches 12 before he reaches 0.

Now suppose that he performs his random walk on a circle with 12 points on the circumference labelled 0 to 11 which are equidistant apart (and the length of a frog jump!). He starts at the point marked 0. Find the probability that he returns to 0 without making a complete circuit of the circle (in either direction). (Hint: You need to think about which way he goes on the first jump and split into two separate random walks).

What would your answers be if the probability of a clockwise jump is p where $p \neq 1/2$?