MTH5118 Probability II. Problem Sheet 10.

Please staple your coursework and post it in the Orange Box on the ground floor of the Maths building by **15:00** *on Wednesday 10th December 2008.*

1. X takes non-negative values and has mean μ and variance $\sigma^2 > 0$.

(a) Use Markov's inequality to obtain an upper bound for $P(X \geq \mu + 2\sigma)$.

(b) Use Chebyshev's inequality to obtain an upper bound for $P(|X - \mu| \geq 2\sigma)$.

If $X \sim Exp(\theta)$, for each of cases (a) and (b), (i) state the upper bound by writing μ and σ in terms of θ; (ii) integrate the exponential p.d.f. to obtain the actual probability specified.

2. Let $X_1, X_2, X_3, ...$ be a sequence of independent random variables each with $Bernoulli(p)$ distribution (where $0 < p < 1$) and let $\overline{X}_n = \frac{1}{n}\sum_{j=1}^{n} X_j$. State $E[\overline{X}_n]$ and $Var(\overline{X}_n)$.

(a) Use Chebyshev's inequality to obtain an upper bound for $P(|\overline{X}_n - p| \geq 0.1p)$. Hence show that $P(|\overline{X}_n - p| \geq 0.1p) \leq 0.05$ for $n \geq \frac{2000(1-p)}{p}$.

(b) Use the Central Limit Theorem to find the value of n (in terms of p) so that

$$P(|\overline{X}_n - p| \geq 0.1p) \simeq 0.05$$

Assume that p is not close to zero or one. Note that $\Phi(1.96) = 0.975$, i.e. the upper 2.5% point of the $N(0,1)$ distribution is 1.96.

3. Let $\mathbf{X}$ be a vector of random variables with entries X_1 and X_2, where $E[X_1] = 5$, $E[X_2] = 3$, $Var(X_1) = 9$, $Var(X_2) = 4$ and $Cov(X_1, X_2) = -2$. Write down the vector of means and the variance-covariance matrix for $\mathbf{X}$. Let $Y_1 = 2 + X_1 + X_2$ and $Y_2 = 2X_1 - 7X_2$ and let $\mathbf{Y}$ be the vector with entries Y_1 and Y_2. Calculate the vector of means and the variance-covariance matrix for $\mathbf{Y}$.

Are Y_1 and Y_2 independent?

If X_1 and X_2 have bivariate normal distribution, state the joint distribution of Y_1 and Y_2.