

Probability 2 - REQUIREMENTS FOR TEST.

1. P.g.f.:

- (a) Know p.g.f. for standard discrete distributions.
- (b) Know how to find $P(X = x)$ by expanding or differentiating the p.g.f.
- (c) Know how to find $E[X]$ and $Var(X)$ by differentiating the p.g.f.
- (d) Know that the p.g.f. of the sum of independent r.v.'s is the product of the individual p.g.f.'s.

2. Know the Theorem of Total Probability and be able to do simple examples.

Know the gambler's ruin result (either $R_k(M, N)$ or $L_k(M, N)$ since they sum to one) for cases: (i) $p \neq q$; (ii) $p = q = \frac{1}{2}$. Be able to use the results in simple examples.

3. Know the Theorem of Total Probability for Expectations. Be able to do simple examples (NOT the expected duration of game for gambler's ruin)

4. Know results on conditional distributions and conditional expectations and be able to use this for simple examples.

In particular know:

$$E[Y] = E[E[Y|X]] \text{ and } Var(Y) = E[Var(Y|X)] + Var(E[Y|X])$$

Know equivalent result for random sums. If $Y = \sum_{j=1}^N X_j$ where the X_j 's are i.i.d. with common mean μ and common variance σ^2 and N is a random variable which is independent of the X_j 's, then:

$$E[Y] = E[N]\mu \text{ and } Var(Y) = E[N]\sigma^2 + Var(N)\mu^2$$

5. Branching processes.

- (a) Be able to find the p.g.f. for Y_n for small n in simple examples using $G_{Y_{n+1}}(t) = G_{Y_n}(G_X(t))$
- (b) Be able to find $\theta_n = P(Y_n = 0)$ for small n in simple examples using $\theta_{n+1} = G_X(\theta_n)$.
- (c) Be able to find the probability of eventual extinction θ by solving $G_X(t) = t$.

6. M.g.f.:

- (a) Be able to find the m.g.f. for simple cases only.
- (b) Be able to find $E[X]$ and $Var(X)$ by expanding or differentiating the m.g.f.