

# A CLASS OF GROUPS UNIVERSAL FOR FREE $\mathbb{R}$ -TREE ACTIONS

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ABSTRACT. I report on a new construction in group theory giving rise to a kind of continuous analogue of free groups. More explicitly, given any (discrete) group  $G$ , we construct a group  $\mathcal{RF}(G)$  equipped with a natural (real-valued) Lyndon length function, and thus with a canonical action on an associated  $\mathbb{R}$ -tree  $\mathbf{X}_G$ , which turns out to be transitive. Analysis of these groups  $\mathcal{RF}(G)$  is difficult. However, conjugacy of hyperbolic elements is understood, as are the centralizers and normalizers of hyperbolic elements; moreover, we show that  $\mathcal{RF}$ -groups and their associated  $\mathbb{R}$ -trees are *universal* (with respect to inclusion) for free  $\mathbb{R}$ -tree actions. Furthermore, we prove that

$$|\mathcal{RF}(G)| = |G|^{2^{\aleph_0}},$$

and that non-trivial normal subgroups of  $\mathcal{RF}(G)$  contain a free subgroup of rank  $|\mathcal{RF}(G)|$ , as well as a number of further structural properties of  $\mathcal{RF}(G)$  and its quotient by the span of the elliptic elements.