MAS400: Exercises 8

Let $v_1, \ldots, v_n$ be linearly independent vectors in $\mathbb{R}^n$, such that $(v_1, \ldots, v_n)$ has Gram–Schmidt orthogonalisation $((v_1^*, \ldots, v_n^*), (\mu_{st}))$. Determine the Gram–Schmidt orthogonalisations $((w_1^*, \ldots, w_n^*), (\xi_{st}))$ of $(w_1, \ldots, w_n)$ in the following cases.

1. $w_i = \lambda v_i$, $w_j = v_j$ if $j \neq i$, where $\lambda \neq 0$ ($\lambda \in \mathbb{R}$).
2. $w_i = v_i - \lambda v_k$ with $k < i$, $w_j = v_j$ if $j \neq i$.
3. $w_i = v_{i+1}$, $w_{i+1} = v_i$, $w_j = v_j$ if $j \neq i, i+1$.

You may like to think about what happens in these two cases too. (It is not really possible to write down exact answers for these.)

4. $w_i = v_i - \lambda v_k$ with $k > i$, $w_j = v_j$ if $j \neq i$.
5. $w_i = v_k$, $w_k = v_i$ (wlog $i < k$), $w_j = v_j$ if $j \neq i, k$.