MAS400: Exercises 2

Throughout this sheet F will denote an arbitrary field unless otherwise stated.

1. Show that $\mathbb{Z}[x]$ has ideals that are not generated by a single element. Thus $\mathbb{Z}[x]$ is not a principal ideal domain, even though it is a unique factorisation domain.

2. Write, using pseudo-code, the univariate division algorithm (that is the algorithm that given $f, g \in F[x]$ with $g \neq 0$ will return $q, r \in F[x]$ such that $f = qg + r$ and $\deg r < \deg g$).

3. Rewrite the Euclidean Algorithm so that only a bounded number of variables is used. (These variables should be elements of $F[x]$, and not such things as an arbitrarily long sequence of elements of $F[x]$.)

4. Show that the only monomial order on $\mathbb{N}$ is $x^0 < x^1 < x^2 < \cdots < x^i < x^{i+1} < \cdots$.

5. Show that a totally ordered set is well ordered if and only if it has no infinite descending chains.

6. Any exercises that may be embedded in my lecture notes.