# CW9 Solutions

1. (a) Following Euclid's algorithm:

$$\begin{aligned} 285 &= 2\cdot 117+51\\ 117 &= 2\cdot 51+15\\ 51 &= 3\cdot 15+6\\ 15 &= 2\cdot 6+3\\ 6 &= 2\cdot 3+0 \end{aligned}$$

so $\gcd(285,117)=\gcd(117,51)=\gcd(51,15)=\gcd(15,6)=\gcd(6,3)=\gcd(3,0)=3$.

From the above equations,

$$\begin{aligned} 3 &= 15-2\cdot 6\\ &= 15-2\cdot(51-3\cdot 15)\\ &= (-2)\cdot 51+7\cdot 15\\ &= (-2)\cdot 51+7(117-2\cdot 51)\\ &= 7\cdot 117-16\cdot 51\\ &= 7\cdot 117-16(285-2\cdot 117)\\ &= (-16)\cdot 285+39\cdot 117 \end{aligned}$$

so the equation holds with $x=(-16)$, $y=39$.

CHECK: $39\cdot 117-16\cdot 285=4563-4560=3$ so we didn't make a mistake!

(b) In this case, we have

$$\begin{aligned} 4199 &= 2\cdot 1771+657\\ 1771 &= 2\cdot 657+457\\ 657 &= 1\cdot 457+200\\ 457 &= 2\cdot 200+57\\ 200 &= 3\cdot 57+29\\ 57 &= 1\cdot 29+28\\ 29 &= 1\cdot 28+1\\ 28 &= 28\cdot 1 \end{aligned}$$

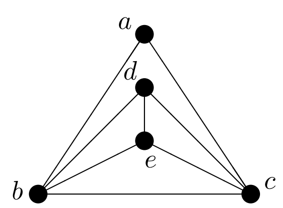
so $\gcd(4199,1771)=1$.

Going backwards,

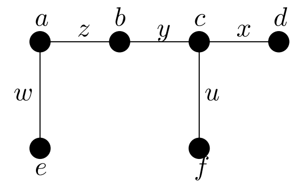
$$\begin{aligned} 1 &= 1\cdot 29-1\cdot 28=1\cdot 29-1\cdot(57-1\cdot 29)\\ &= (-1)\cdot 57+2\cdot 29=(-1)\cdot 57+2\cdot(200-3\cdot 57)\\ &= 2\cdot 200-7\cdot 57=2\cdot 200-7\cdot(457-2\cdot 200)\\ &= (-7)\cdot 457+16\cdot 200=(-7)\cdot 457+16\cdot(657-1\cdot 457)\\ &= 16\cdot 657-23\cdot 457=16\cdot 657-23\cdot(1771-2\cdot 657)\\ &= (-23)\cdot 1771+62\cdot 657=(-23)\cdot 1771+62\cdot(4199-2\cdot 1771)\\ &= 62\cdot 4199-147\cdot 1771. \end{aligned}$$

CHECK: $62\cdot 4199-147\cdot 1771=260338-260337=1$ so we didn't make a mistake!

2. $G$ is as follows:



3. (a) $\{a, z, b, y, c, x, d\}$ (among many others)
   (b) $\{a, w, e, t, f, u, c, y, b, z, a\}$ (or the same thing backwards),
   (c) Plenty of examples, one such is




4. (a) $\{a, d, f, i, b, h, j\}$ [the algorithm presented in lectures should be used and described clearly].
   (b) No - the connected component containing $b$ is not the whole graph.

5. (a) $a$ is a source and $h$ is a sink.
   (b) Which (if any) of the following functions $f$ are *flows* on $N$?
      (i) Not a flow - symmetry broken at vertex $e$.
      (ii) Not a flow - $gh$ over capacity.
      (iii) Not a flow - symmetry broken at vertex $e$.
   (c) The min cut is 12, formed by $ch$, $gh$, $ep$ and $ap$. Therefore the max flow is 12 [we are using the Max-Flow Min-Cut Theorem]. One solution is the following: