CW8 - SOLUTIONS

① (a) $C(7,5) = \frac{7!}{5!\,(7-5)!} = \frac{7!}{5!\,2!} = \frac{7\cdot 6\cdot \cancel{5!}}{\cancel{5!}\cdot 2} = 7\cdot 3 = 21$

(b) $P(6,4) = \frac{6!}{(6-4)!} = \frac{6!}{2!} = 6\cdot 5\cdot 4\cdot 3 = 360$

(c) $C(100,2) = \frac{100!}{2!\,98!} = \frac{100\cdot 99\cdot \cancel{98!}}{2\cdot \cancel{98!}} = 50\cdot 99 = 4950$

(d) $P(8,5) = \frac{8!}{(8-5)!} = \frac{8!}{3!} = 8\cdot 7\cdot 6\cdot 5\cdot 4 = 6720$

② We'll need Pascal's triangle up to the 7'th level:

0: 1
1: 1 1
2: 1 2 1
3: 1 3 3 1
4: 1 4 6 4 1
5: 1 5 10 10 5 1
6: 1 6 15 20 15 6 1
7: 1 7 21 35 35 21 7 1

Binomial formula in general:

$$(x+y)^n = C(n,n)x^n + C(n,n-1)x^{n-1}y + C(n,n-2)x^{n-2}y^2 + \dots + C(n,2)x^2y^{n-2} + C(n,1)xy^{n-1} + C(n,0)y^n$$

(a) $(x+y)^6 = C(6,6)x^6 + C(6,5)x^5y + C(6,4)x^4y^2 + C(6,3)x^3y^3 + C(6,2)x^2y^4 + C(6,1)xy^5 + C(6,0)y^6 =$

$= \underline{x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6}$

(b) $(a+b)^7 = C(7,7)a^7 + C(7,6)a^6b + C(7,5)a^5b^2 + C(7,4)a^4b^3 +$

$+ C(7,3)a^3b^4 + C(7,2)a^2b^5 + C(7,1)ab^6 + C(7,0)b^7 =$

$= \underline{a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7}$

③ (a) $\frac{5!}{2!}$ (divided by 2! since there are two P's)

(b) There are $\frac{4!}{2!}$ rearrangements of XPPL, and each "X" can be replaced either by "AE" or "EA", so get:

$\frac{4!}{2!}\cdot 2! = 4!$

(c) we're in fact looking for rearrangements of

$\underbrace{AA}_{2}BEP\underbrace{RRRR}_{4}\underbrace{II}_{2}Y$ There are $\dfrac{12!}{2!\,4!\,2!}$ such words.

(d) Once we fix R at the start and at the end R_____R, we are looking at rearrangements of $\underbrace{AA}BEP\underbrace{RR}\underbrace{II}Y$, and there are $\dfrac{10!}{2!2!2!}$ such words.

4 (a) It is the same as the number of rearrangements of the word $xxx\,yyyy\,z$: $\dfrac{8!}{3!\,4!\,1!} = \dfrac{8!}{3!\,4!}$

(b) If we write $u = y^3$, $v = z^2$, we are in fact looking for the coefficient of x^2uv^2 in $(x+u+v)^5$, which is $\dfrac{5!}{2!\,1!\,2!} = \dfrac{5!}{2!\,2!}$.

(c) Writing $a = x^2$, $b = y^2$, $c = z^2$, this coefficient is equal to the coefficient of $w^7a^4b^5c^7$ in $(w+a+b+c)^n$, so n must equal the degree of $w^7a^4b^5c^7$, which means $n = 7+4+5+7 = 23$.

5 Choose 5 out of 12 children who get vanilla in $C(12,5)$ ways

— " — 3 — — — 12-5 — — — — — — — chocolate in $C(7,3)$ ways

remaining children get strawberry.

Thus, # of ways is : $C(12,5)\cdot C(7,3) = \dfrac{12!}{5!\,\cancel{7!}}\cdot\dfrac{\cancel{7!}}{3!\,4!} = \dfrac{12!}{5!\,3!\,4!}$