MAE113 DISCRETE TECHNIQUES FOR COMPUTING

Coursework 7 - Solutions.

1. $f : \mathbb{R} \to \mathbb{R}$, $f(y) = y^2(y+2)$.

2. (a) The following is the bare minimum information:
   $n_1 = 4$, $k_1 = 5$
   $u_2 = n_1 = 4$, $n_2 = n_1 = 4$, $k_2 = k_1 = 5$
   $n_{3,1} = n_2 * u_2 = 4 * 4 = 4^2$, $k_{3,1} = k_2 = 5$, $u_{3,1} = u_2 = 4$
   $n_{4,1} = n_{3,1} = 4^2$, $k_{4,1} = k_{3,1} - 1 = 4$, $u_{4,1} = u_{3,1} = 4$
   $n_{3,2} = n_{4,1} * u_{4,1} = 4^2 * 4 = 4^3$, $k_{3,2} = k_{4,1} = 4$, $u_{3,2} = u_{4,1} = 4$
   $n_{4,2} = n_{3,2} = 4^3$, $k_{4,2} = k_{4,1} - 1 = 3$, $u_{4,2} = u_{4,2} = 4$
   $n_{3,3} = n_{4,2} * u_{4,2} = 4^3 * 4 = 4^4$, $k_{3,3} = k_{4,2} = 3$, $u_{3,3} = u_{4,2} = 4$
   $n_{4,3} = n_{3,3} = 4^4$, $k_{4,3} = k_{3,3} - 1 = 2$, $u_{4,3} = u_{3,3} = 4$
   $n_{3,4} = n_{4,3} * u_{4,3} = 4^4 * 4 = 4^5$, $k_{3,4} = k_{4,3} = 2$, $u_{3,4} = u_{4,3} = 4$
   $n_{4,4} = n_{3,4} = 4^5$, $k_{4,4} = k_{3,4} - 1 = 1$, $u_{4,4} = u_{3,4} = 4$
   return $n_{4,4} = 4^5 = 1024$.
   $f(4,5) = 4^5 = 1024$

   (b) $f : {\mathbb{Z}_+}^2 \to \mathbb{Z}_+$, $f(n,k) = n^k$. Not polynomial, because there is a loop.

3. (a) $(3x^3+5x^2+x+3)(x^2+3x+2) = (3x^5+5x^4+x^3+3x^2)+(9x^4+15x^3+3x^2+9x)+(6x^3+10x^2+2x+6) = 3x^5+14x^4+22x^3+16x^2+11x+6$.

   (b) $(x^3+x^2+x+1)(x-1) = x^4+x^3+x^2+x-x^3-x^2-x-1 = x^4-1$.

4. (a)
$$\frac{1001!}{1000!} = \frac{1001 \cdot 1000!}{1000!} = 1001.$$

   (b)
$$\frac{7!}{4!} = \frac{7 \cdot 6 \cdot 5 \cdot 4!}{4!} = 7 \cdot 6 \cdot 5.$$

   (c)
$$\frac{(n+3)!}{(n+2)(n+1)!} = \frac{(n+3) \cdot (n+2)!}{(n+2)!} = n+3.$$

5. $6! \times 7! = 7! \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 7! \times (4 \times 2) \times (6 \times 5 \times 3 \times 1) = 7! \times 8 \times 90 = 7! \times 8 \times 9 \times 10 = 10!$.