CW10 Solutions

1. First of all, note that the sample space S consists of $2^4 = 16$ elements (two possibilities for each of the four children).

   (a) The event A' is 'all children are the same sex', so $A' = \{BBBB, GGGG\}$ and $P(A') = \frac{|A'|}{|S|} = \frac{2}{16} = \frac{1}{8}$. Thus $P(A) = 1 - P(A') = \frac{7}{8}$.

   (b) The event $B = \{GGGG, BGGG, GBGG, GGBG, GGGB\}$ so $P(B) = \frac{5}{16}$.

   (c) We have $A \cap B = \{BGGG, GBGG, GGBG, GGGB\}$ so $P(A \cap B) = \frac{4}{16} = \frac{1}{4}$.

   (d) We have seen in (c) that $P(A \cap B) = \frac{1}{4}$ while $P(A)P(B) = \frac{35}{128}$, so these events are **not** independent.

2. We use the fact that throws are independent from each other.

   (a) $P(A) = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$.

   (b) $P(B) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$.

   (c) $P(C) = P(\{HHH, HHT, THH, HTH\}) = P(HHH)+P(HHT)+P(THH)+P(HTH) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{1}{2}$.

   (d) $P(D) = P\{HHH, HHT, THH\} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$

   (e) $P(A \cap D) = P(\{HHH, HHT\} = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$. On the other hand, $P(A) \cdot P(D) = \frac{1}{2} \cdot \frac{3}{8} = \frac{3}{8} \neq \frac{1}{4}$ so the events are not independent.

3. We use the fact that throws are independent from each other.

   (a) $P(A) = \frac{2}{3} \cdot 1 \cdot 1 = \frac{1}{2}$.

   (b) $P(B) = P(HHT) = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} = \frac{4}{27}$.

   (c) $P(C) = P(\{HHH, HHT, THH, HTH\}) = P(HHH)+P(HHT)+P(THH)+P(HTH) = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} + \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} + \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} = \frac{8}{27} + \frac{4}{27} + \frac{4}{27} + \frac{4}{27} = \frac{20}{27}$.

4. (a)

	E	F	G	probability
1.	T	T	T	$0.1 \cdot 0.2 \cdot 0.3 = 0.006$
2.	T	T	F	$0.1 \cdot 0.2 \cdot 0.7 = 0.014$
3.	T	F	T	$0.1 \cdot 0.8 \cdot 0.3 = 0.024$
4.	T	F	F	$0.1 \cdot 0.8 \cdot 0.7 = 0.056$
5.	F	T	T	$0.9 \cdot 0.2 \cdot 0.3 = 0.054$
6.	F	T	F	$0.9 \cdot 0.2 \cdot 0.7 = 0.126$
7.	F	F	T	$0.9 \cdot 0.8 \cdot 0.3 = 0.216$
8.	F	F	F	$0.9 \cdot 0.8 \cdot 0.7 = 0.504$

   (b) Good rows are those where E is false and at least one of F and G are false, i.e. rows 6, 7 and 8.

   (c) The probability that a message will go through is the probability of the event consisting of rows 6, 7 and 8, which is $0.126 + 0.216 + 0.504 = 0.846$.