MAE113 DISCRETE TECHNIQUES FOR COMPUTING

Coursework 9–to be handed in by 11am, Thursday 09/12/2010.

Write your name and student number at the top of your assignment before handing it in. You should attempt all questions because as little as one question might be marked.

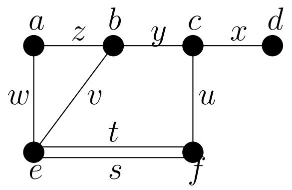
1. (a) Using Euclid's algorithm, find the greatest common divisor d of 285 and 117, and express d in the form $d = x \cdot 285 + y \cdot 117$ for some integers x and y.

   (b) Do the same for 4199 and 1771.

2. Draw the graph G whose adjacency matrix is as follows:

	a	b	c	d	e
a	0	1	1	0	0
b	1	0	1	1	1
c	1	1	0	1	1
d	0	1	1	0	1
e	0	1	1	1	0

3. Let G the following graph:



   Give an example of

   (a) A path in G of length three,

   (b) A cycle in G of length five,

   (c) A spanning tree of G.

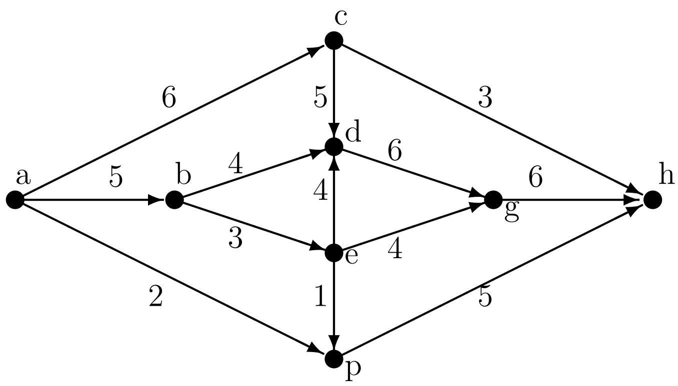
4. The following array is the adjacency matrix of a graph G.

	a	b	c	d	e	f	g	h	i	j	k
a	1	0	0	1	0	1	0	0	0	0	0
b	0	0	0	0	0	0	0	0	1	0	0
c	0	0	1	0	1	0	1	0	0	0	2
d	1	0	0	1	0	5	0	0	1	0	0
e	0	0	1	0	0	0	0	0	0	0	0
f	1	0	0	5	0	0	0	0	0	0	0
g	0	0	1	0	0	0	1	0	0	0	0
h	0	0	0	0	0	0	0	1	2	1	0
i	0	1	0	1	0	0	0	2	0	0	0
j	0	0	0	0	0	0	0	1	0	1	0
k	0	0	2	0	0	0	0	0	0	0	0

(a) Find the connected component of G containing the vertex b (you are not expected to draw G),

(b) Is G connected? Explain your answer.

5. Answer the following questions for the network N given below. In this network, the edges are labelled by their endpoints, so ab is the edge joining vertices a and b, and so on. The numbers on edges indicate capacities.



(a) Which vertices in this network are sources? Which are sinks?

(b) Which (if any) of the following functions f are *flows* on N?

(i) $f(ab) = 5$, $f(bd) = 4$, $f(be) = 1$, $f(ed) = 1$, $f(dg) = 5$, $f(eg) = 1$, $f(gh) = 6$, f takes the value 0 on all other edges.

(ii) $f(ab) = 4$, $f(ac) = 6$, $f(bd) = 4$, $f(cd) = 3$, $f(dg) = 7$, $f(gh) = 7$, $f(ch) = 3$, f takes the value 0 on all other edges.

(iii) $f(ab) = 3$, $f(ap) = 2$, $f(be) = 2$, $f(bd) = 1$, $f(de) = 1$, $f(ep) = 1$, $f(eg) = 1$, $f(gh) = 2$, $f(dg) = 1$, $f(ph) = 3$, $f(eg) = 1$, f takes the value 0 on all other edges.

(c) Find a maximum flow in the above network. State the value of your flow and show that it is a maximum by finding a cut of the same value.