MTH5121 Probability Models. Problem Sheet 3.

You are required to submit solutions to Problems 1 and 3 only. You are strongly encouraged to solve all problems. *Please staple your coursework and post it in the Green Box on the ground floor of the Maths building by* **16:30** *on Wednesday, 26th October 2011.*

1. Gonzo has £100 to gamble on roulette at a casino with no zero, so that $P(red) = \frac{1}{2}$. If 10 red comes up he wins £1. If red does not come up he loses £1. He decides to play until he loses all his money or has doubled his money to £200. Find $E[X]$, where X counts the number of games he plays.

By changing units, obtain $E[X]$ if his stake at each game is £10. Nothing else is altered. 15

Consider the case of roulette with a zero which means that $P(red) = \frac{18}{37}$ and he will 30 eventually lose £100. The stake is £10. What is $E[X]$?

2. Jack plays a series of games. At each game Jack wins, loses or draws with equal probabilities of a third. He bets one unit each time and receives an additional unit, gets the unit back, or loses the unit depending upon whether he wins, draws or loses the game. He starts with k units and stops playing when he reaches 0 or N units ($0 \leq k \leq N$). Find $E_k = E[X_k]$, where X_k counts the number of games he plays.

3. A mouse is put at the centre of a maze. The time X until he escapes from the maze is measured in minutes. There are 3 routes he may select, two of which lead to dead ends so that the mouse has to retrace his path back to the centre of the maze. The time taken for route 1 is 10 minutes and for route 2 is 15 minutes. The third path leads out of the maze in 8 minutes. Find $E[X]$ for each of the following cases:

(a) Each time he is at the centre of the maze he selects one of the 3 routes at random. 20

(b) Initially he chooses one of the 3 routes at random, but subsequently he does not 25 choose the route he has just returned along but chooses one of the remaining two routes at random. Hint: Let X_i be the time to get out of the maze if his choice initially is restricted so that he cannot choose route i but chooses one of the other two routes at random.