MTH5121 Probability Models. Problem Sheet 2.

You are required to submit solutions to Problem 5 only and you are strongly encouraged to solve all problems on this problem sheet. *Marks awarded are shown next to the question. Please staple your coursework and post it in the Green Box on the ground floor of the Maths building by* **16:30** *on Wednesday 19th October 2011.*

1. A pair of fair six-sided dice are thrown repeatedly until their sum is either 12 or 7. The sum on the final throw is recorded. Let E be the event that the recorded sum is 12. By conditioning on the outcome of the first throw of the two dice, use the theorem of total probability to obtain $P(E)$.

2 Jack plays a series of games. At each game Jack wins, loses or draws with equal probabilities of a third. He bets one unit each time and receives an additional unit, gets the unit back, or loses the unit depending upon whether he wins, draws or loses the game. He starts with k units and stops playing when he reaches 0 or N units ($0 \leq k \leq N$). Find the probability that he loses all his money.

3. (*From one of past exams!*) Galileo releases a feather from a window 80 metres above the ground in a building 100 metres high. Each second after the feather is let go either the feather drops 20 metres with probability 2/3 or the wind blows the feather up 20 metres with probability 1/3. What is the probability that the feather reaches the top of the building before it hits the ground?

4. Repeat the first part of question 5.(a) if the roulette wheel in the casino has no zero.

5.(a) The roulette wheel at a casino has integers from 1 to 36, together with 0. Half of the non-zero numbers are red, the other half are black, and 0 is green. Any of the numbers between 0 and 36 is equally likely to occur each time the wheel is spun. 40

Gonzo has £100 to gamble on roulette at the casino. His stake at each game is £1, which he bets on red. If red comes up he wins £1 and gets his stake back. If red does not come up he loses his £1 stake. He decides to play until he loses all his money or has doubled his money to £200. Find the probability that Gonzo doubles his money.

State the probability that Gonzo doubles his money if he has £1000 to gamble and bets £10 each time. Again he stops if he either loses all his money or doubles it.

(b) A frog performs a random walk on the integers. At each stage he jumps either one integer forwards or one integer back with equal probability. He starts at zero. Find the probability that he reaches 10 before he reaches -20. 25

(c) Now suppose that he performs his random walk on a circle with points on the circumference labeled 0 to 11 which are equidistant apart (and the length of a frog jump!). He 35

starts at the point marked 0. Find the probability that he returns to 0 without making a complete circuit of the circle (in either direction). (Hint: You need to think about which way he goes on the first jump and split into two random walks).