MTH5121 Probability Models. Problem Sheet 1.

You are required to submit solutions to Problem 4 only. However, you are strongly encouraged to solve all problems on this problem sheet. *Marks awarded are shown next to the question. Please staple your coursework and post it in the Green Box on the ground floor of the Maths building by* **16:30** *on Wednesday 12th October 2011.*

1. For each of the following functions $G_X(t)$ state (with reasons) if it can be the probability generating function of a discrete random variable X which takes non-negative integer values and (if appropriate) determine the probability mass function of X:

(a) $G_X(t) = \frac{1}{2} + \frac{1}{3}t^2 + \frac{1}{4}t^3$; (b) $G_X(t) = \frac{1}{4}t(1+t)^2$; (c) $G_X(t) = \frac{2t}{(1+t)}$; (d) $G_X(t) = \frac{1}{(2-t)}$.

2. X is a random variable with probability generating function $G_X(t)$. In each of the following cases state the probability distribution of X, i.e. name the distribution and specify its parameters: (a) $G_X(t) = e^{7t-7}$; (b) $G_X(t) = \left(\frac{4}{5} + \frac{1}{5}t\right)^2$; (c) $G_X(t) = \frac{1}{25}(4+t)^2$.

3. Let X be a random variable with probability generating function $G_X(t) = \frac{t}{4-3t}$. Name the distribution of X, including any parameter values. Using $G_X(t)$, find $P(X = 1)$, $P(X = 2)$, $E(X)$ and $\mathrm{Var}(X)$.

4 (a). Let X and Y be two independent random variables, $X \sim \mathrm{Binomial}(2, 1/2)$ and $Y \sim \mathrm{Bernoulli}(1/3)$. Write down the probability generating functions $G_X(t)$ and $G_Y(t)$. Hence obtain the probability generating function of $Z = X + Y$ and use this to derive the probability mass function of $Z = X + Y$ (i.e. find $P(Z = z)$ for the values of z for which this probability is positive). 30

(b). Let X and Y be independent random variables each with Poisson distribution, with parameters λ and μ respectively. Show that $Z = X + Y$ has Poisson distribution and state its parameter. If $X_1, X_2, ..., X_n$ are independent identically distributed random variables, with common distribution which is Poisson with parameter λ, find the probability generating function of $W = \sum_{j=1}^{n} X_j$ and hence state the distribution of W. 30

(c). Let X be a random variable with probability generating function $G_X(t) = \frac{(1+2t)t}{3(2-t)}$. Using $G_X(t)$, find $E(X)$ and $\mathrm{Var}(X)$. 40

Factor $G_X(t)$ into the product of two probability generating functions $G_X(t) = G_Y(t)G_Z(t)$, hence proving that X may be expressed in the form $X = Y + Z$ where Y and Z are independent random variables. Name the distributions of Y and Z and their parameters. Use results for the mean and variance of the sum of two independent random variables Y and Z to obtain $E(X)$ and $\mathrm{Var}(X)$ (which should be identical to the results obtained above).