MTH5121 Probability Models. Problem Sheet 9.

You are supposed to submit problems 2(b)-(iii), 2(b)-(iv), and 3. *Please staple your coursework and post it in the Green Box on the ground floor of the Maths building by* **16:30** *by Wednesday 14th December 2011.*

1. Sally has a bus journey then a walk in order to get to work each morning. Her bus journey takes time $a + X$, where X measures the excess over the minimum journey time a. Her walk takes time $b + Y$ where Y measures the excess over the minimum walk time b. X and Y are independent $U(0, 1)$. Then $Z = X + Y$ measures the excess total journey time above the minimum time of $a + b$. Let $U = X$. Find the joint p.d.f. of U and Z (which gives the joint p.d.f. of X and Z since $U = X$). Hence find the conditional p.d.f. for $X|Z = z$.

2. This problem is a continuation of Problem 3, CW8. Last time, You were supposed to solve (a), (b)-(i), (b)-(ii). The answers to these questions can be found in the solutions to CW8 (on the web) and used while solving (b)-(iii) an (b)-(iv) which is what you are supposed to do here.

(a) If $f_U(u) = \theta e^{-\theta(u-\alpha)}$ for $\alpha < u < \infty$, show that $V = U - \alpha \sim Exp(\theta)$.to Hence state $E[U]$ and $Var(U)$.
(b) A device contains two components working in parallel, so that the device continues working whilst at least one of the components is still working. Let X be the time until one of the components fails and Y be the time until both fail (so that the device stops working). It is known that the joint p.d.f. of X and Y is

$$f_{X,Y}(x, y) = 2\theta^2 e^{-\theta(x+y)} \text{ if } 0 < x < y < \infty$$

and $f_{X,Y}(x, y) = 0$ otherwise.
(i) Find the marginal p.d.f. for X and hence state the distribution, mean and variance of X.
(ii) Find the conditional distribution of $Y|X = x$.
(iii) Obtain $E[Y|X]$ and $Var(Y|X)$. Hence find $E[Y]$ and $Var(Y)$. 15
(iv) Use the result that $E[XY] = E[XE[Y|X]]$ to obtain $Cov(X, Y)$ and $\rho(X, Y)$. 15

3. X takes non-negative values and has mean μ and variance $\sigma^2 > 0$.

(a) Use Markov's inequality to obtain an upper bound for $P(X \geq \mu + 2\sigma)$. 15

(b) Use Chebyshev's inequality to obtain an upper bound for $P(|X - \mu| \geq 2\sigma)$. 15

If $X \sim Exp(\theta)$, for each of cases (a) and (b),

(i) state the upper bound by writing μ and σ in terms of θ; 15

(ii) integrate the exponential p.d.f. to obtain the actual probability specified. 25