MTH5121 Probability Models. Problem Sheet 8.

You are supposed to submit problems 1, 2. *Please staple your coursework and post it in the Green Box on the ground floor of the Maths building by* **16:30** *by Wednesday 7th December 2011.*

You will need formula (1) and (2) from Notes 8 in order to solve the problems below. Read Notes 8 before attempting the problems.

1. X and Y have joint p.d.f. $f_{X,Y}(x, y) = 2$ for $0 < x < y < 1$ and $f_{X,Y}(x, y) = 0$ elsewhere.

(a) Find the joint p.d.f. for $U = Y - X$ and $V = X$.   15
(b) Hence find the marginal p.d.f. for U.   10
(c) Find the covariance $Cov(U, V)$.   25

2. X and Y are independent with $X \sim Exp(\theta)$ and $Y \sim Exp(\theta)$. Find the joint p.d.f. of   50
$U = \frac{X}{Y}$ and $V = X + Y$. State whether U and V are independent and find their marginal p.d.f.'s.

3. (a) If $f_U(u) = \theta e^{-\theta(u-\alpha)}$ for $\alpha < u < \infty$, show that $V = U - \alpha \sim Exp(\theta)$. Hence state $E[U]$ and $Var(U)$.
(b) A device contains two components working in parallel, so that the device continues working whilst at least one of the components is still working. Let X be the time until one of the components fails and Y be the time until both fail (so that the device stops working). It is known that the joint p.d.f. of X and Y is

$$f_{X,Y}(x, y) = 2\theta^2 e^{-\theta(x+y)} \quad \text{if } 0 < x < y < \infty$$

and $f_{X,Y}(x, y) = 0$ otherwise.
(i) Find the marginal p.d.f. for X and hence state the distribution, mean and variance of X.
(ii) Find the conditional p.d.f. for $Y|X = x$.