## MTH5121 Probability Models. Problem Sheet 7.

You are supposed to submit problems 1, 2, 5. Please staple your coursework and post it in the Green Box on the ground floor of the Maths building by 16:30 on Wednesday 30 November 2011.

To solve the problems below remind yourself the definition of the joint p.d.f.  $f_{X,Y}(x,y)$  of two random variables X, Y and that  $f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y) dy$ ,  $f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y) dx$ .

**1.** X has p.d.f.  $f_X(x) = 2\theta x e^{-\theta x^2}$  for x > 0 and  $f_X(x)$  is zero elsewhere. Let  $Y = X^2$ . Note that this is a one to one transformation for the range of X for which the p.d.f. is non-zero. Use the standard transformation of variables result to obtain the p.d.f. for Y.

**2.** Let  $X \sim Exp(\theta)$ . Use the standard transformation of variables result to obtain the p.d.f. for  $Y = 1 - e^{-\theta X}$ . State the distribution of Y.

**3.** Let X and Y have joint p.d.f.  $f_{X,Y}(x,y) = C$  for 0 < x < 2y < 2 and  $f_{X,Y}(x,y) = 0$  elsewhere. Find the marginal p.d.f.'s for X and Y,  $f_X(x)$  and  $f_Y(y)$  and obtain C.

**4.** Random variables X and Y have joint p.d.f.  $f_{X,Y}(x,y) = C(x^2 + xy)$  for 0 < x < 1, 0 < y < 1 and  $f_{X,Y}(x,y) = 0$  elsewhere.

Find the marginal p.d.f.'s  $f_X(x)$  and  $f_Y(y)$  in terms of C. Find C.

**5.** Random variables X and Y have joint p.d.f.  $f_{X,Y}(x,y) = Ce^{-(x+y)}$  for  $0 < x < y < \infty$  and  $f_{X,Y}(x,y) = 0$  elsewhere.

Obtain the marginal p.d.f.'s  $f_X(x)$  and  $f_Y(y)$  in terms of C. Find C.