# MTH5121 Probability Models. Problem Sheet 6.

***You are supposed to submit problems 1, 3, 5.*** *Please staple your coursework and post it in the Green Box on the ground floor of the Maths building by* **16:30** *on Wednesday 23 November 2011.*

**1.** Let $X$ be a continuous random variable with p.d.f. $f_X(x) = e^{-(x-\alpha)}$ for $x > \alpha$ and $f_X(x) = 0$ elsewhere.

(a) Show that, for $t < 1$, the m.g.f. for $X$ is $M_X(t) = \frac{e^{\alpha t}}{(1-t)}$. 10

(b) Differentiate the m.g.f. to find $E[X]$ and $Var(X)$. 10

(c) Let $Y = (X - \alpha)$. Show that the m.g.f. $M_Y(t) = e^{-\alpha t} M_X(t)$ and hence obtain $M_Y(t)$ 10
and state the distribution of $Y$.

**2.** Let $X \sim N(\mu, \sigma^2)$. State the m.g.f. of $X$, $M_X(t)$. Write the m.g.f. of $Y = \frac{(X-\mu)}{\sigma}$, $M_Y(t)$, in terms of the m.g.f. for $X$. Hence find $M_Y(t)$ and state the distribution of $Y$.

Expand the m.g.f. for $Y$ in a power series and hence show that $E[Y^{2r+1}] = 0$ for all $r = 0, 1, ...$ and find $E[Y^{2r}]$ for $r = 1, 2, ....$

**3.** $X$ has p.d.f. $f_X(x) = \frac{\theta}{2} e^{-\theta|x|}$ for all $-\infty < x < \infty$ (i.e. $f_X(x) = \frac{\theta}{2} e^{-\theta x}$ for $x \geq 0$ and $f_X(x) = \frac{\theta}{2} e^{\theta x}$ for $x < 0$).

(a) Show that the m.g.f. is $M_X(t) = \left(1 - \frac{t^2}{\theta^2}\right)^{-1}$, for $-\theta < t < \theta$. You will need to split 10
the integral into two ranges corresponding to $x \geq 0$ and $x < 0$.

(b) Use the m.g.f. to obtain $E[X]$ and $Var(X)$. 10

(c) Let $Y = |X|$, so that $Y$ takes values on $[0, \infty)$. Find the m.g.f. for $Y$, for $t < \theta$, (you 10
will again need to split the range) and hence state the distribution of $Y$.

**4.** Integrate by parts to show that $\Gamma(\alpha + 1) = \alpha\Gamma(\alpha)$ for $\alpha > 0$, where $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$.

**5.** (a) Let $X \sim Exp(\theta)$. Use the standard transformation of variables result to obtain 15
the p.d.f. for $Y = 1 - e^{-\theta X}$. State the distribution of $Y$.

(b) $X \sim N(0, 1)$. Let $Y = |X|$, so that $Y$ takes values on $[0, \infty)$. For $y > 0$, write the 25
event $Y \leq y$ as an equivalent event for $X$. Hence find the c.d.f for $Y$, $F_Y(y) = P(Y \leq y)$, in terms of the c.d.f. for $X$. Differentiate $F_Y(y)$ to obtain the density function for $Y$.