MTH5121 Probability Models. Problem Sheet 4.

This time you are required to submit solutions to all problems. *Please staple your coursework and post it in the Green Box on the ground floor of the Maths building by* **16:30** *on Wednesday, 2nd November 2011.*

1. (i) Let $S = \sum_{j=1}^{N} X_j$, where X_j are i.i.d.r.v.'s with $E(X_j) = a$, $Var(X_j) = \sigma^2$ and N is an integer-valued non-negative r.v. independent of the sequence X_j. Write down the formulae for $E(S)$ and $Var(S)$ (derived in the lecture). 5

(ii) The number of customers arriving at a shop during one day is a r.v. $N \sim Poisson(80)$. The customers act independently of each other spending a random amount of X_j pounds in the shop. Given that $E(X_j) = 20$, $Var(X_j) = 20$, find the average value and the variance of the daily cash flow S in this shop. 15

2. The number of spam messages Y in a day has Poisson distribution with parameter μ. Each spam message (independently) has probability p of not being detected by the spam filter. Let X be the number of messages getting through the filter.

(i) Find $E(X)$ and $Var(X)$. (Hint: note that $X = \sum_{j=1}^{Y} Z_j$, where $Z_j \sim Bernoulli(p)$ i.i.d.r.v.'s.) 10

(ii) Compute $E(t^X)$ and hence find the p.g.f. of X. To this end: 1. Compute $E(t^X|Y = y)$; 2. Use the fact that $E(t^X) = E[E(t^X|Y)]$. 20

(iii) Hence find the p.m.f. of X and identify the name of this distribution. 10

(iv) Repeat (i), (ii), and (ii) if $Y \sim Geometric(\bar{p})$, that is $P(Y = k) = \bar{p}\bar{q}^k,\ k = 0, 1, 2, ...$ 40