MTH5121 Probability Models. Problem Sheet 10.

You don't have to submit solutions to any of these problems but are strongly encouraged to solve them.

1. Let $X_1, X_2, X_3, \ldots$ be a sequence of independent random variables each with $Bernoulli(p)$ distribution (where $0 < p < 1$) and let $\overline{X}_n = \frac{1}{n}\sum_{j=1}^{n} X_j$. State $E[\overline{X}_n]$ and $Var(\overline{X}_n)$.

(a) Use Chebyshev's inequality to obtain an upper bound for $P(|\overline{X}_n - p| \geq 0.1p)$. Hence show that $P(|\overline{X}_n - p| \geq 0.1p) \leq 0.05$ for $n \geq \frac{2000(1-p)}{p}$.

(b) Use the Central Limit Theorem to find the value of n (in terms of p) so that

$$P(|\overline{X}_n - p| \geq 0.1p) \simeq 0.05$$

Assume that p is not close to zero or one. Note that $\Phi(1.96) = 0.975$, i.e. the upper 2.5% point of the $N(0,1)$ distribution is 1.96.