Probability III – 2008/09

Exercise Sheet 4

Write your name and student number at the top of your assignment before handing it in. Staple all pages together. Return the assignment by 16:00 on Thursday, 26 February

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1. Show that a Markov chain whose transition probability matrix is [25]

$$\mathbb{P} = \left\| \begin{array}{ccccc} 0 & 1/2 & 1/2 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 1/3 & 1/3 & 0 & 1/3 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 & 0 \end{array} \right\|$$

   is regular and determine the limiting distribution.

2. A Markov chain has state space $S = \{0, 1, 2\}$ and transition probability matrix [25]

$$\mathbb{P} = \left\| \begin{array}{ccc} 0.3 & 0.5 & 0.2 \\ 0.2 & 0.7 & 0.1 \\ 0.4 & 0.2 & 0.4 \end{array} \right\|$$

   After a long period of time, you observe the chain and see that it is in state 2. What is the conditional probability that the previous state was 1? That is, find

$$\lim_{n\to\infty} \mathbb{P}(X_{n-1} = 1 | X_n = 2).$$

3. Suppose that a production process changes state according to a Markov chain on state space $S = \{0, 1, 2, 3\}$ whose transition probability matrix is given by [25]

$$\mathbb{P} = \left\| \begin{array}{cccc} 0.2 & 0.2 & 0.4 & 0.2 \\ 0.5 & 0.2 & 0.2 & 0.1 \\ 0.2 & 0.3 & 0.4 & 0.1 \\ 0.1 & 0.2 & 0.4 & 0.3 \end{array} \right\|$$

   a) Determine the limiting distribution for the process.

   b) Suppose that states 0 and 1 are "in-control," while states 2 and 3 are deemed "out-of-control." In the long run, what fraction of time is the process out-of-control?

4. Five balls are distributed between two urns, labelled A and B. Each period, an urn is selected at random, and if it is not empty, a ball from that urn is removed and placed into the other urn. In the long run, what fraction of time, on the average, is urn A empty. [25]