Probability III – 2008/09

Exercise Sheet 2

Write your name and student number at the top of your assignment before handing it in. Staple all pages together. Return the assignment by 16:00 on Thursday, 29 January

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1. An insurance company offers a "no claims discount" system of 10% for each claim free year up to a maximum of 50%. A customer initially has no discount. For each year the customer does not make a claim 10% is added to the discount, unless the discount is already 50% in which case there is no change. If the customer does make a claim then the discount is reduced to 0%. Suppose that the probability of a given customer making a claim in any year is p, and is independent of whether a claim is made in any other year.

a) Explain how to model the level of discount as a Markov chain.

b) Write down the transition matrix of this Markov chain.

c) What is the probability that the customer's discount in year 4 is 20%?

d) What is the probability that the customer's discount in year 16 is 0%?

2. A Markov chain on state space $\{0,1,2\}$ has a transition matrix

$$\mathbb{P} = \begin{pmatrix} 0 & 0.2 & 0.8 \\ 0.25 & 0.4 & 0.35 \\ 0 & 0 & 1 \end{pmatrix}.$$

The process starts in state 0.

a) Calculate the expected time to absorbtion.

b) Calculate the expected number of visits to state 1 before absorbtion.

c) Suppose that you lose £3 for each visit to state 0 and gain £5 for each visit to state 1. Calculate your expected gain.

3. A fair 6-sided die is rolled repeatedly until the sum of two consecutive results is 11. What is the expected number of rolls in this game? What is the probability that the game will end at 5,6 (rather than 6,5)?

To solve this problem set up a Markov chain X_n with the following states: 0, 5, 6, (56), (65), where
0 represents the starting point;
5 represents a single observed 5;

6 represents a single observed 6;
(56) represents the sequence 5,6;
(65) represents the sequence 6,5.

By definition the process starts at 0, that is $X_0 = 0$. If at the n-th roll the die shows up with a number which is not 5 or 6, then $X_{n+1} = 0$. The other states are reached in a natural way, e. g. $X_{n+1} = (56)$ if $X_n = 5$ and the die shows up 6 at the next roll.
Write down the corresponding transition matrix and use the first step analysis.