

Probability III – 2007/08

Please hand in solutions by 12:00, December 13

Exercise Sheet 7

1. Customers arrive at a single server according to a Poisson process of rate 1 per minute. The server can be operated in one of two modes “fast” or “slow”. The service times are exponentially distributed, with mean 30 seconds in “fast” mode and 50 seconds in “slow” mode.

The system is set to either “fast” or “slow” mode and is run for a long time in this mode. Compare the mean queue lengths for each mode of service.

2. Consider an $M/M/2$ queueing system with traffic intensity $\frac{1}{4}$.

- i) In the long run what is the probability that both servers are idle?
- ii) In the long run what is the probability that the system is busy?
- iii) In the long run what is the probability that both servers are busy?
- iv) In the long run what is the probability that there are exactly 2 customers waiting to be served?

3. Find the equilibrium distribution for a $M(\lambda)/M(\mu)/3$ queueing system.

4. Let $Q(t)$ be the number of customers in an $M(\lambda)/D(d)/\infty$ queueing system. What is the distribution of $Q(t)$ for fixed t ?

Recall that $D(d)$ means service times are deterministic taking value d with probability 1.

5. Consider an $M(\lambda)/M(\mu)/1$ queueing system with $\lambda < \mu$. What is the expected time that an arriving customer must wait to be served conditioned on the fact that there are n customers in the system when he arrives?

Find the expected time an arriving customer has to wait if the queue is in equilibrium.