Probability III – 2007/08

Please hand in solutions by 12:00, November 22

Exercise Sheet 4

1. Let $\{X_n\}$ be the Markov chain with transition matrix

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 1/2 & 0 \\ 1/4 & 0 & 0 & 1/2 & 1/4 & 0 \\ 0 & 0 & 1/2 & 1/4 & 0 & 1/4 \\ 0 & 1/2 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Find the equivalence classes of states of the chain. Classify the states as recurrent or transient

2. Customers arrive at a server according to a Poisson process of rate 4 per hour. Let $X(t)$ be the number of customers who have arrived up to time t hours. Determine the following:

a) $\mathbb{P}(X(4) = 2)$

b) $\mathbb{P}(X(4) = 2, X(1) = 1)$

c) $\mathbb{P}(X(4) = 2 | X(1) = 1)$

d) $\mathbb{P}(X(1) = 1 | X(4) = 2)$

e) $\mathbb{P}(X(4) = 1 | X(1) = 2)$.

3. A long pipe is subject to two sorts of fault, cracks and punctures. Suppose that cracks occur according to a Poisson process of intensity λ per mile, and punctures occur according to a Poisson process of intensity μ per mile. Show that the number of faults in the first t miles forms a Poisson process of intensity $\lambda + \mu$.

Suppose that punctures can always be repaired but each crack has probability p of not being repairable, independent of all other events. Show that the number of un-repairable faults in the first t miles forms a Poisson process of intensity $p\lambda$.