Probability III – 2007/08

Exercise Sheet 3

Write your name and student number at the top of your assignment before handing it in. Staple all pages together. Return the assignment to me by 11:00 on Tuesday, 30 October

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1. An urn contains 5 red balls and 3 green balls. A ball is chosen at random with each ball in the urn equally likely to be chosen. If the chosen ball is red then it is removed. If the chosen ball is green then it is replaced. The process is repeated until all the red balls have been removed. By considering a suitable Markov chain find the expected duration of the process.

2. Explain why the following chains, given by their transition matrices, are regular. In each case find the limiting distribution. What proportion of time do you expect the chain to spend in each state in the long run?

a)

$$\begin{pmatrix} 1/10 & 1/2 & 0 & 2/5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

b)

$$\begin{pmatrix} 0 & 1/3 & 0 & 2/3 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

3. For each of the following transition matrices, state whether the corresponding Markov chain is regular, irreucible or neither. Determine the equilibrium distribution if it exists.

a)

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

b)

$$\begin{pmatrix} 0 & 1/4 & 3/4 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

c)

$$\begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

4. A *graph* is a set of points called *vertices*, some pairs of which are joined by (undirected) lines called *edges* (note, we do not allow loops or multiple edges). The *degree* of a vertex is the number of edges which are incident with it. A particle is placed at a vertex v of a graph G, and at each time step moves along some edge chosen randomly with all edges incident with v equally likely to be chosen. This is called a *random walk* on G. Check that the position of the particle defines a Markov chain with state space the set of vertices of G. Show that, provided G is finite, there is an equilibrium distribution for the random walk and specify it in terms of the degrees of vertices.