Probability III – 2007/08

Exercise Sheet 2

Write your name and student number at the top of your assignment before handing it in. Staple all pages together. Return the assignment to me by 11:00 on Tuesday, 23 October

---

1. A Markov chain on state space $\{1,2,3,4\}$ has transition matrix

$$\begin{pmatrix} 0 & 1/2 & 1/2 & 0 \\ 1/3 & 0 & 1/3 & 1/3 \\ 1/3 & 1/3 & 0 & 1/3 \\ 0 & 1/2 & 1/2 & 0 \end{pmatrix}.$$

Draw the transition graph of the chain.

Calculate the following probabilities:

i) $P(X_1 = 2 | X_0 = 1)$,

ii) $P(X_2 = 3 | X_1 = 2)$,

iii) $P(X_2 = 3 | X_1 = 2, X_0 = 1)$,

iv) $P(X_2 = 3, X_1 = 2 | X_0 = 1)$.

2. A simplified model for weather is defined as follows. Each day is either wet or dry. The probability that each day is wet/dry depends only on whether the previous day is wet/dry. The probability that the day following a wet day is wet is 3/4. A dry day is equally likely to be followed by a wet day or a dry day. Calculate the following probabilities:

i) $P(17 \text{ October is dry} | 14 \text{ October is dry})$,

ii) $P(17 \text{ October is wet} | 14 \text{ October is wet})$,

iii) $P(17 \text{ and } 18 \text{ October are both wet} | 14 \text{ October is wet})$.

[Hint: You may find it useful to calculate the 3-step transition matrix.]

3. Five points are marked on the circumference of a circle. A particle is placed on one of the points and moves according to the throw of a standard fair die as follows.

- If 1,2 or 3 is rolled then the particle moves clockwise round the circle to the next point and the die is thrown again.
- If 4 or 5 is rolled then the particle moves anti-clockwise round the circle to the next point and the die is thrown again.
- If 6 is rolled then the process ends.

Draw the transition graph for this Markov chain and determine the transition matrix.

What is the probability that after two throws of the die the particle is back at its starting position?

What is the probability that the process lasts exactly 7 throws?

4. A Markov chain on state space $\{0,1,2,3\}$ has transition matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/6 & 1/6 & 1/6 & 1/2 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Which states are absorbing?

What is the probability that the process ends up in state 0 given that it starts in state 1?

5. A Markov chain on state space $\{0,1,2\}$ has transition matrix

$$\begin{pmatrix} 0 & 1/7 & 6/7 \\ 1/4 & 1/2 & 1/4 \\ 0 & 0 & 1 \end{pmatrix}.$$

The process starts in state 1.

i) Calculate the expected time to absorbtion.

ii) Calculate the expected number of visits to state 0 before absorbtion.

iii) Suppose that you lose £5 for each visit to state 0 and gain £10 for each visit to state 1. Calculate your expected gain.