Probability III – 2007/08

Exercise Sheet 1

Write your name and student number at the top of your assignment before handing it in. Staple all pages together. Return the assignment to me by 11:00 on Tuesday, 9 October

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1. Let X and Y be independent random variables. Show that:

   a) if X and Y both have distribution $\text{Bin}(n, p)$ then $X+Y$ has distribution $\text{Bin}(2n, p)$,

   b) if X and Y both have distribution $\text{Po}(\lambda)$ then $X+Y$ has distribution $\text{Po}(2\lambda)$,

   c) if X and Y both have distribution $\text{Po}(\lambda)$ then the conditional distribution of X given than $X+Y=n$ is $\text{Bin}(n, \frac{1}{2})$.

2.

   a) Show that if X is a random variable taking positive integer values then

$$\mathbb{E}(X) = \sum_{k\geq 1} \mathbb{P}(X \geq k).$$

   b) A fair coin is tossed repeatedly. Let N be the number of the toss at which the first head appears. Determine the distribution of N and its expectation.

3. Let N be a Poisson random variable with parameter λ. A coin which has a probability p of showing heads is tossed N times. What is the distribution of the number of heads observed?

4. An urn contains n balls labeled $1, 2, \ldots, n$. We select k balls at random (without replacement) and add up the numbers on them obtaining a value X. Show that

$$\mathbb{E}(X) = \frac{k(n+1)}{2}.$$

(Hint: Let ξ_j be the random number chosen at j-th selection. Write X in terms of the random variables ξ_j.)

5. A standard die is rolled repeatedly. Let

   - X_n be the largest value seen in the first n rolls,
   - S_n be the number of 6s seen in the first n rolls,
   - T_n be the number of 6s seen on rolls $n-1$ and n.

(so $T_n = S_n - S_{n-2}$).
Which of X_n, S_n, T_n are Markov chains?
For those that are draw their transition graphs.