MAS111 Convergence and Continuity

Key Learning Objectives

1. **Real numbers.** Understand the axioms for the set of real numbers; be able to prove basic inequalities; know what is the least upper bound/the largest lower bound (supremum/infimum) of a set of real numbers; be able to state the theorem about the existence of the least upper (largest lower) bound of a set of real numbers.

2. **Sequences and limits.** Know the definition of a limit of a sequence of numbers; be able to prove the theorems about properties of limits: arithmetic properties (such as properties of sums, products, ratios), the Bolzano-Weierstrass Theorem; be able to compute limits of sequences.

3. **Functions.** Functions; definition of a continuous function; simple examples of discontinuous functions. Derivation of basic properties of continuous functions on closed intervals. The Intermediate Value Theorem and its applications (such as existence of $\sqrt[k]{x}$).

4. **Series.** Definition of convergence of a series; alternating series and the condition for their convergence; geometric series; absolutely convergent series; comparison test and ratio test for convergence; conditions for convergence of $\sum_{n=1}^{\infty} \frac{1}{n^{\alpha}}$. Power series: definitions of radius of convergence, domain of convergence; finding radius and domain of convergence.