**BSc examination by course unit**

# MAS111 Convergence and Continuity

**?? May 2008 ??:?0** **Duration 2 hours**

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*You should attempt ALL questions. Marks awarded are shown next to the question.*

*Calculators are not permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.*

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**You are not allowed to start reading the question paper until instructed to do so by the invigilator**

**You must not remove the examination paper from the examination room**

**THIS IS A SAMPLE EXAM PAPER**

1. Basics
   4 (a) prove that if $0 \leq x \leq z$ then $z^2 \geq x^2$.
   6 (b) Prove that $|x - a| < \varepsilon$ if and only if $a - \varepsilon < x < a + \varepsilon$.
   8 (c) Prove that if $0 < q < 1$ then $lim_{n \to \infty} nq^n = 0$

      Remak. You can use (without proving it) the following fact: if $c \geq 0$ then $(1 + c)^n \geq 1 + 0.1n^2c^2$, where $n \geq 2$ is an integer number.

      Hint: present $q = \frac{1}{1+c}$.
   5 (d) Give the definition of a limit of a sequence. State (do not prove) the Bolzano-Weierstrass theorem.

2. Properties of continuous functions and some of their applications.
   13 (a) Prove that if $f : X \mapsto \mathbb{R}$ and $g : , X \mapsto \mathbb{R}$ are continuous functions on $X$ then $f(x)g(x)$ is a continuous function.
   14 (b) Let $f : [a, b] \mapsto \mathbb{R}$ be a continuous function. What is the definition of the maximal value and the minimal value of $f$.

      Let $M$ and $m$ be, respectively, the maximal and the minimal value of the function $f$.

      Using the Intermediate Value Theorem, prove that for every $y$ such that $m \leq y \leq M$ there is a number $c \in [a, b]$ such that $f(c) = y$.

5 3. State (do not prove) the Cauchy criterion for convergence of a sequence.

4. Series.
   10 (a) State (do not prove) the comparison test for convergence of a series.

      Using the comparison test prove that if $a_n \geq 0$ and $\sum_{n=1}^{\infty} a_n$ converges then also $\sum_{n=1}^{\infty} a_n^2$ converges.
   10 (b) Does the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt[3]{n}}$ converge? Explain your answer.

      Is this series absolutely convergent? Explain your answer.
   15 (c) What is the radius of convergence and the domain of convergence of the following series:

$$\sum_{n=1}^{\infty} \frac{x^n}{n^2 + \sqrt{n}} \ ?$$

10 (d) State (do not prove) the condensation test for convergence of a series. Use this test to prove that the series

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

diverges.