Book of Abstracts for the
30th British Combinatorial Conference

Last updated 2024/06/25 16:11:40 +01'00'

Welcome

The British Combinatorial Committee and the local organisers are delighted to welcome you to the 30th British Combinatorial Conference and hope that you have a productive and enjoyable week.

Code of Conduct

Open exchange of ideas and freedom of thought and expression are central to research meetings. We expect cooperation from conference participants to ensure a safe environment for everyone in all conference venues, including ancillary events and unofficial social gatherings.

Expected behaviour

- Exercise consideration and respect in your speech and actions.
- Refrain from demeaning, discriminatory, or harassing behaviour and speech.
- Be mindful of your surroundings and of your fellow participants.
- Alert the conference organisers if you notice a dangerous situation, someone in distress, or violations of this code, even if they seem inconsequential.

Unacceptable behaviour

- Any action directed at an individual that (a) interferes substantially with that person's participation; or (b) causes that person to fear for their safety. This includes threats, intimidation, bullying, stalking, and other types of abuse.
- Any conduct that discriminates or denigrates an individual based on age, belief, disability, ethnicity, gender identity, race, religion, sexual identity, or any other characteristic protected by law.
- Unwelcome sexual advances, requests for sexual favours, or other verbal or physical conduct of a sexual nature.

Harassment can occur when there is no deliberate intention to offend. Be careful in the words that you choose. Unacceptable behaviour committed in a joking manner or disguised as a compliment still constitutes unacceptable behaviour.

Contacts

If you feel that you have experienced or witnessed an instance of unacceptable behaviour, you are encouraged to approach the following contacts for support and advice:

- the conference organisers at `bcc-2024@qmul.ac.uk`;
- QM Report + Support at `https://reportandsupport.qmul.ac.uk/`, where you can make a report anonymously;
- the S&E EDI Lead, Professor Claudia Garetto, at `c.garetto@qmul.ac.uk`.

Consequences of unacceptable behaviour

If a conference participant engages in unacceptable behaviour, the conference organisers reserve the right to take appropriate action to ensure the physical and emotional safety of all attendees. This can include exclusion from the conference without warning or refund.

Emergency Contacts

Campus security	020 7882 3333 (3333 from internal phones)
Emergency services	999

Campus Map

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(i) Information

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The smoking of cigarettes or tobacco products is **only** permitted at designated smoking areas / shelters indicated on this map.

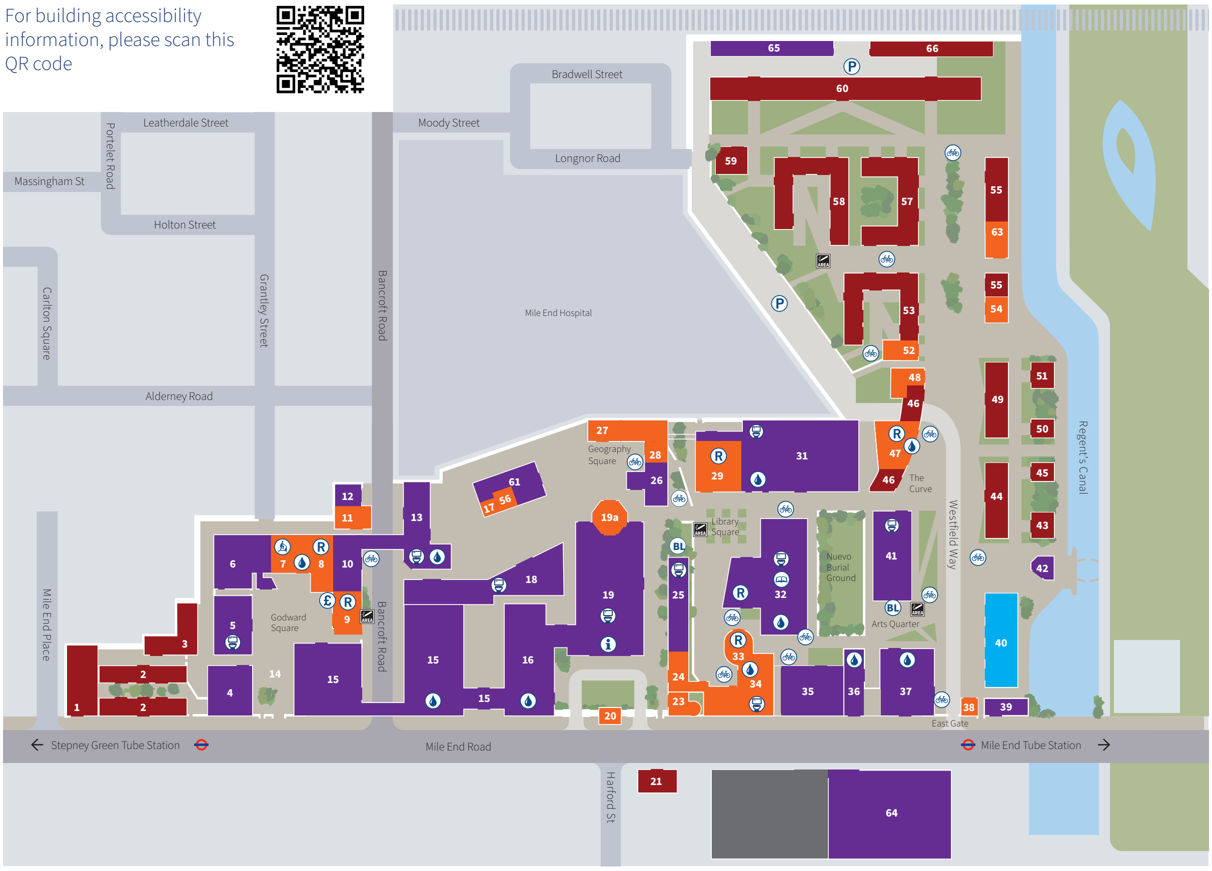
Electronic cigarettes permitted on outside spaces **only**.

These premises are alarmed and monitored by CCTV; please call Security on +44 (0)20 7882 5000 for more information.

Key

- Library/bookshop
- Fitness centre
- (R) Refreshments: Bar/Eatery/Coffee place
- (P) Staff car park
- Bicycle parking
- (BL) Bicycle lockers
- (£) Cash machine
- Water fountain
- Smoking area/shelter
- Gender-neutral toilet

For building accessibility information, please scan this QR code



Schedule

In the event of unplanned changes to the schedule, a revised version will be posted to the conference web page at the start of each day.

Monday 1st July

08:15 – 09:00	**Registration** in Arts Two Foyer
09:00 – 09:30	**Welcome** in Arts Two LT
09:30 – 10:30	Plenary talk in Arts Two LT **Martin Skutella** *An Introduction to Transshipments Over Time*
10:30 – 11:00	**Refreshments** in Arts Two Foyer

Contributed Talks

	Arts Two LT	Sizer LT	Mason LT	Bancroft 2.40	Bancroft 4.08
Chair	*Hyde*	*Jedwab*	*Jackson*	*Blackburn*	*Jerrum*
11:00 – 11:20	Kontogeorgiou	Selig	Hilton	Bean	Lauri
11:25 – 11:45	Munha Correia	Yan	Perry	Chalermsook	Mifsud
11:50 – 12:10	Pehova	Zhu	Soicher	Ollson	Zarb

12:15 – 13:45	**Lunch** in Arts Two Foyer
13:45 – 14:45	Plenary talk in Arts Two LT **Maya Stein** *Oriented Trees and Paths in Digraphs*
14:45 – 15:15	**Refreshments** in Arts Two Foyer

Contributed Talks

	Arts Two LT	Sizer LT	Mason LT	Bancroft 2.40	Bancroft 4.08
Chair	*Hng*	*Penman*	*Tiba*	*Clinch*	*Hancock*
15:15 – 15:35	Bowtell	Moffatt	Adeshola	Jackson	Diskin
15:40 – 16:00	Lo	Browne	Mattheus	Lundqvist	Jerrum
16:05 – 16:25	Smith	Somlai	Milojević	Michaeli	Limbach
16:30 – 16:50	Wang	Flower	Nagy	Nixon	Zakharov

17:00 – 17:40	Steven Noble: **A Tribute to Dominic Welsh** in Arts Two LT
17:45	**Reception** Canalside

Tuesday 2nd July

09:00 – 10:00 Plenary talk in Arts Two LT
Richard Montgomery *Transversals in Latin Squares*

10:00 – 10:30 **Refreshments** in Arts Two Foyer

10:30 – 10:40 **BCC PhD Prize announcement**

10:40 – 11:20 **BCC PhD Prize lecture**

Contributed Talks

	Arts Two LT	Sizer LT	Mason LT	Bancroft 2.40	Bancroft 4.08
Chair	*Behague*	*Illingworth*	*Walters*	*van Hintum*	*Ward*
11:25 – 11:45	Calbet	Hollom	Katsamaktsis	Costa	Gagarin
11:50 – 12:10	Zhukovskii	Zarate Gueren	Messinger	Ivan	Shiri

12:15 – 13:45 **Lunch** in Arts Two Foyer

13:45 – 14:45 Plenary talk in Arts Two LT
Shoham Letzter *Sublinear Expanders and Their Applications*

14:45 – 15:15 **Refreshments** in Arts Two Foyer

Mini-Symposia

	Arts Two LT	Sizer LT	Mason LT
	Probabilistic Combinatorics	Combinatorial Optimisation	Designs / Algebraic Combinatorics
Chair	*Kronenberg / Shapira*	*Vegh*	*Huczynska*
15:15 – 15:45	Krivelevich	Ward	Davis
15:50 – 16:20	Staden	Zambelli	Paterson
16:25 – 16:55	Helmuth	Svensson	Li
17:00 – 17:30	Mond	Yuditsky	Webb
17:35 – 18:05			Wanless

18:10 – 19:00 **Business Meeting** in Arts Two LT

19:00 – **BBQ dinner** Canalside

Wednesday 3rd July

09:00 – 10:00 Plenary talk in Arts Two LT
Lisa Sauermann *The Slice Rank Polynomial Method – A Survey a Few Years Later*

10:00 – 10:30 **Refreshments** in Arts Two Foyer

Contributed Talks

	Arts Two LT	Sizer LT	Mason LT	Bancroft 2.40	Bancroft 4.08
Chair	*Lo*	*Fink*	*RF Bailey*	*Cameron*	*Prellberg*
10:30 – 10:50	Bandyopadhyay	Dankelmann	Mella	Blackburn	Deb
10:55 – 11:15	Im	Mafunda	Huang	Ironmonger	Gutiérrez
11:20 – 11:40	Pfenninger	Mallu	Gordeev	Johnson	Jedwab
11:45 – 12:05	Lang	Tuite	Erskine	Rattan	Tahmi

12:15 – 13:45 **Lunch** in Arts Two Foyer

13:45 – **Self-Guided Stroll** from Blackheath to Canary Wharf

Thursday 4th July

09:00 – 10:00 Plenary talk in Arts Two LT
Sarah Peluse *Finite Field Models in Arithmetic Combinatorics – Twenty Years On*

10:00 – 10:30 **Refreshments** in Arts Two Foyer

Contributed Talks

	Arts Two LT	Sizer LT	Mason LT	Bancroft 2.40	Bancroft 4.08
Chair	*Tyomkyn*	*Draganic*	*Noble*	*Paterson*	*Gagarin*
10:30 – 10:50	Christoph	Collins	Farr	RA Bailey	Dalirrooyfard
10:55 – 11:15	Freschi	Illingworth	Howell	O'Brien	Ellingham
11:20 – 11:40	Hancock	Volec	Chapman	Huczynska	Salia
11:45 – 12:05	Hattingh	Zhou	Park	Kemp	Xie

12:15 – 13:45 **Lunch** in Arts Two Foyer

13:45 – 14:45 Plenary talk in Arts Two LT
Federico Ardila *Intersection Theory of Matroids: Variations on a Theme*

14:45 – 15:15 **Refreshments** in Arts Two Foyer

Mini-Symposia

	Arts Two LT	Sizer LT	Mason LT
	Extremal Combinatorics	Additive Combinatorics	Combinatorial Geometry / Matroids
Chair	*Bowtell / Long*	*Bloom*	*Rincón*
15:15 – 15:45	Kamcev	Bowen	Ferroni
15:50 – 16:20	Kwan	Hunter	Smith
16:25 – 16:55	Müyesser	Kousek	Pendavingh
17:00 – 17:30	Shapira	Souza	Maclagan

17:40 – 18:40 **Open Problem Session** in Arts Two LT

Friday 5th July

09:00 – 10:00 Plenary talk in Arts Two LT
Paul Balister *Erdős Covering Systems*

10:00 – 10:30 **Refreshments** in Arts Two Foyer

Contributed Talks

	Arts Two LT	Sizer LT	Mason LT	Bancroft 2.40	Bancroft 4.08
Chair	*Volec*	*Pehova*	*Soicher*	*Talbot*	*Stark*
10:30 – 10:50	Anastos	Boyadzhiyska	RF Bailey	Behague	Araujo
10:55 – 11:15	Kucheriya	Bradac	Cameron	Tyrell	Draganic
11:20 – 11:40	Treglown	Chen	Ó Catháin	Ghosal	Hng
11:45 – 12:05	Tyomkyn	Mycroft	Safarji	Malekshahian	Portier

12:15 – 13:45 **Lunch** in Arts Two Foyer

Contributed Talks

	Arts Two LT	Sizer LT	Mason LT	Bancroft 2.40	Bancroft 4.08
Chair	*B Janzer*	*Mycroft*	*O Janzer*	*RA Bailey*	*Fischer*
13:45 – 14:05	Chau	Schoot Uiterkamp	Tiba	Noble	Basak
14:10 – 14:30	Williams	Di Braccio	van Hintum	Pirzada	Chishti

14:30 – 15:00 **Refreshments** in Arts Two Foyer

15:00 – 16:00 Plenary talk in Arts Two LT
Matthew Jenssen *The Cluster Expansion in Combinatorics*

16:00 Close

An Introduction to Transshipments Over Time

Martin Skutella

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Technische Universität Berlin

Network flows over time are a fascinating generalization of classical (static) network flows, introducing an element of time. They naturally model problems where travel and transmission are not instantaneous and flow may vary over time. Not surprisingly, flow over time problems turn out to be more challenging to solve than their static counterparts. In this survey, we mainly focus on the efficient computation of transshipments over time in networks with several source and sink nodes with given supplies and demands, which is arguably the most difficult flow over time problem that can still be solved in polynomial time.

Oriented Trees and Paths in Digraphs

Maya Stein

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University of Chile

Which conditions ensure that a digraph contains all oriented paths of some given length, or even a all oriented trees of some given size, as a subgraph? One possible condition could be that the host digraph is a tournament of a certain order. In arbitrary digraphs and oriented graphs, conditions on the chromatic number, on the edge density, on the minimum outdegree and on the minimum semidegree have been proposed. We review the known results, and highlight some open questions in the area.

Transversals in Latin Squares

Richard Montgomery

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University of Warwick

A Latin square is an n by n grid filled with n symbols so that each symbol appears exactly once in each row and each column. A transversal in a Latin square is a collection of cells which do not share any row, column, or symbol. This survey will focus on results from the last decade which have continued the long history of the study of transversals in Latin squares.

Sublinear expanders

Shoham Letzter

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University College London (UCL)

(This talk is based on joint work with António Girão, Tao Jiang, Abhishek Methuku, and Liana Yepremyan.)

In this talk I will give an overview of results using sublinear expanders. The term *sublinear expanders* refers to a variety of definitions of expanders, which typically are defined to be graphs G such that every not-too-small and not-too-large set of vertices U has neighbourhood of size at least $\alpha|U|$, where α is a function of n and $|U|$. This is in contrast with *linear expanders*, where α is typically a constant. I will describe proof ideas of some of the results mentioned here, and will state some related open problems.

The Slice Rank Polynomial Method

Lisa Sauermann

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University of Bonn

The slice rank polynomial method was introduced by Tao in 2016 following the breakthrough of Ellenberg and Gijswijt on the famous Cap-Set Problem, which in turn was building on work of Croot, Lev and Pach. This talk gives an introduction to the slice rank polynomial method, shows some of its early applications, and discusses the developments since then.

Finite field models in arithmetic combinatorics – twenty years on

Sarah Peluse

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University of Michigan

About twenty years ago, Green wrote a survey article on the utility of looking at toy versions over finite fields of problems in additive combinatorics. This article was extremely influential, and the rapid development of additive combinatorics necessitated a follow-up survey ten years later, which was written by Wolf. Since the publication of Wolf's article, an immense amount of progress has been made on several central open problems in additive combinatorics in both the finite field model and integer settings. This talk covers some of the most significant results of the past ten years and suggests future directions.

Intersection Theory of Matroids

Federico Ardila–Mantilla

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San Francisco State University

Chow rings of toric varieties, which originate in intersection theory, feature a rich combinatorial structure of independent interest. We survey four different ways of computing in these rings, due to Billera, Brion, Fulton–Sturmfels, and Allermann–Rau. We illustrate the beauty and power of these methods by giving four proofs of Huh and Huh–Katz's formula $\mu^k(\mathsf{M}) = \deg_{\mathsf{M}}(\alpha^{r-k}\beta^k)$ for the coefficients of the reduced characteristic polynomial of a matroid M as the mixed intersection numbers of the hyperplane and reciprocal hyperplane classes α and β in the Chow ring of M. Each of these proofs sheds light on a different aspect of matroid combinatorics, and provides a framework for further developments in the intersection theory of matroids.

Our presentation is combinatorial, and does not assume previous knowledge of toric varieties, Chow rings, or intersection theory.

Erdős Covering Systems

Paul Balister

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University of Oxford

Introduced by Erdős in 1950, a covering system of the integers is a finite collection of infinite arithmetic progressions whose union is the set of all integers. Many beautiful questions and conjectures about covering systems have been posed over the past several decades, but until the last decade little was known about their properties. Most famously, the so-called minimum modulus problem of Erdős was resolved in 2015 by Hough, who proved that in every covering system with distinct moduli, the minimum modulus is at most a fixed constant. The ideas of Hough were simplified and extended in 2018 by Balister, Bollobás, Morris, Sahasrabudhe and Tiba, to give solutions (or progress towards solutions) to a number of related questions. We give a summary of this and other progress that has been made since.

The cluster expansion in combinatorics

Matthew Jenssen

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King's College London

(This talk is based on joint work with Ewan Davies, Tyler Helmuth, Peter Keevash, Alexandru Malekshahian, Jinyoung Park, Will Perkins, Aditya Potukuchi.)

The cluster expansion is a classical tool from statistical physics used to study the phase diagram of interacting particle systems. Recently, the cluster expansion has seen a number of applications in combinatorics and the field of approximate counting/sampling. In this talk, I will give an introduction to the cluster expansion and survey some of these recent developments.

Colouring graphs from random lists

Michael Krivelevich

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Tel Aviv University, Israel

(This talk is based on joint work with Dan Hefetz.)

Given an n-vertex graph G of maximum degree d, palette size m and list length k, generate a family $\mathcal{L} = \{L(v) : v \in G\}$ of colour lists for the vertices of G by choosing a uniformly random k-subset $L(v)$ of $[m]$ for every vertex v of G independently. How large should be the palette m so as to guarantee that with high probability G is $\mathcal{L}$-choosable (namely, one can choose a colour $c(v)$ from the list $L(v)$ for every vertex v, so that the chosen colours form a proper colouring of G)? This fairly natural setting has been researched for the last twenty years, and gained more visibility recently due to palette sparsification results.

We provide nearly optimal bounds for this problem for the case of constant k. We also address the case of G being H-free, for a fixed graph H.

Probabilistic aspects of transversal embedding

Katherine Staden

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The Open University

A classical question in graph theory is to find optimal sufficient conditions which guarantee that a graph G contains a given subgraph H. A colourful variant of this problem has graphs $G_1, \ldots, G_s$ on the same vertex set, where $s = e(H)$ and we think of each graph as having a different colour, and the goal is to find a *transversal* (or *rainbow*) copy of H that contains exactly one edge from each graph G_i. I will present an overview of some probabilistic aspects of this problem, including results in random graphs and how probabilistic methods can be used in this area.

The Arboreal Gas

Tyler Helmuth

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Durham University

(This talk is based on joint work with Roland Bauerschmidt, Nick Crawford, and Andrew Swan.)

In Bernoulli bond percolation each edge of a graph is declared open with probability p, and closed otherwise. Typically one asks questions about the geometry of the random subgraph of open edges. The arboreal gas is the probability measure obtained by conditioning on the event that the percolation subgraph is a forest, i.e., contains no cycles. Physically, this is a model for studying the gelation of branched polymers. What are the percolative properties of these random forests? Do they contain giant trees? I will discuss what is known and conjectured.

Maker-Breaker percolation games on a random board

Adva Mond

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University of Cambridge and King's College London

(This talk is based on joint work with Vojtěch Dvořák and Victor Souza.)

The (m, b) Maker-Breaker percolation game on an infinite graph G is played in the following way. Maker starts by choosing a vertex and naming it the origin. Maker and Breaker then alternately claim m and b unclaimed edges of G, respectively. Breaker wins if the component containing the origin becomes finite when his edges are deleted from G. Maker wins if she can indefinitely avoid a win of Breaker.

We will discuss the state of the art for this game, with special attention to the case where G is the result of bond percolation on the square lattice. In particular, we will show how this game can be analysed using tools from bootstrap percolation.

Improved algorithms for weighted matchings in k-uniform hypergraphs

Justin Ward

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Queen Mary University of London

(This talk is based on joint work with Theophile Thiery.)

In the Weighted k-Uniform Hypergraph Matching Problem—also known as the Weighted k-Set Packing Problem—we are given an (hyper)edge weighted hypergraph in which each hyperedge contains exactly k vertices. The goal is to find a maximum weight set of hyperedges that is pairwise disjoint. For $k = 2$, the problem is equivalent to the classical weighted matching problem in graphs, which is solvable in polynomial time. For $k > 2$, however, the problem is NP-hard to solve exactly, and so we consider instead polynomial-time algorithms guaranteed to find a matching of total weight within some constant factor of the optimum. In the case of $k = 3$, the best-known approximation result guaranteed a solution within a factor 2 of the optimal, until a recent breakthrough by Neuwohner, which obtain a guarantee of roughly $2 - 6 \cdot 10^{-7}$. In this talk, I will present recent joint work with Theophile Thiery, which gives a polynomial-time $\sqrt{3}$ approximation in the case $k = 3$, as well new results for $k > 3$.

Arc connectivity and submodular flows in digraphs

Giacomo Zambelli

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LSE

(This talk is based on joint work with Ahmad Abdi and Gérard Cornuéjols.)

The paper introduces a sufficient condition for the existence of capacitated integral solutions to the intersection of two submodular flow systems. This sufficient condition has the consequence that the intersection of two submodular flow systems is totally dual integral in weakly connected digraphs, and it implies the classic result of Edmonds and Giles on the box-total dual integrality of a submodular flow system. The result has several applications to Graph Orientations. In particular, it strengthens Nash-Williams' "weak orientation theorem" and proves a weaker variant of Woodall's conjecture on digraphs.

Advancements in Online Edge Coloring Algorithms

Ola Svensson

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EPFL

(This talk is based on joint work with Joakim Blikstad, Radu Vintan, and David Wajc.)

In this presentation, we explore recent advancements in online edge coloring, highlighting the use of martingales to get close to optimal solutions. Our focus will be on two recent results:

Firstly, we address the longstanding conjecture by Bar-Noy, Motwani, and Naor, showing that a $(1+o(1))\Delta$-edge-coloring can be achieved online for graphs with maximum degree $\Delta = \omega(\log n)$. This result holds even in the challenging setting of adversarial edge arrivals, and the techniques also allow us to extend known results in list edge coloring and local edge coloring.

Secondly, we consider the task of designing deterministic algorithms for online bipartite edge coloring. Contrary to the common belief that randomization is necessary to outperform the greedy algorithm, we give a deterministic approach in the vertex arrival model that achieves a competitive ratio of $\mathrm{e}/(\mathrm{e}-1)+o(1)$ for sufficiently large $\Delta = \omega(\log n)$.

Finally, we discuss several intriguing open questions related to these problems.

Integer programs with nearly totally unimodular matrices: the cographic case

Yelena Yuditsky

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Université libre de Bruxelles

(This talk is based on joint work with M. Aprile, S. Fiorini, G. Joret, S. Kober, M.T. Seweryn, and S. Weltge.)

It is a notorious open question whether integer programs (IPs), with an integer coefficient matrix M whose subdeterminants are all bounded by a constant in absolute value, can be solved in polynomial time. We answer this question in the affirmative if we further require that, by removing a constant number of rows and columns from M, one obtains the transpose of a network matrix. We achieve our result in two main steps, the first related to the theory of IPs and the second related to graph minor theory. In this talk, I will present the ideas behind some of the results we obtain in either of those steps.

New Spence Difference Sets

James Davis

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(This talk is based on joint work with John Polhill, Ken Smith, Eric Swartz, Jordan Webster.)

There are five families of (v, k, λ)-difference sets with the property that $\gcd(v, k-\lambda) > 1$: Hadamard, McFarland, Spence, Davis-Jedwab, and Chen. The Hadamard and McFarland families have been studied extensively, the other three families less. The Spence family was discovered in 1977 [2], and Drisko generalized this construction in 1998 [1]. Other than these constructions, we are not aware of any other constructions. We provide new examples in nonabelian groups, and we point to a new approach that will likely generalize to other families of difference sets.

[1] A. Drisko. Transversals in Row-Latin Rectangles, J. Comb. Th. (A), 84 (2):181–195, 1998.

[2] E. Spence. A family of difference sets in non-cyclic groups, J Comb. Th. (A), 22:103–106, 1977.

Circular External Difference Families

Maura B. Paterson

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Birkbeck, University of London

(This talk is based on joint work with Douglas R. Stinson.)

In 2023, Veitch and Stinson proposed the notion of a *circular external difference family (CEDF)* as a tool for constructing unconditionally secure non-malleable secret sharing schemes. Let G be an abelian group of order n, and let $\mathcal{A} = (A_0, A_1, \ldots, A_{m-1})$ be a set of m disjoint subsets of G, each of size ℓ. For $i \in \mathbb{Z}_m$, let $\mathcal{D}_i$ be the multiset $\{a_i - a_{i-1} : a_i \in A_i, a_{i-1} \in A_{i-1}\}$. Then $\mathcal{A}$ is an $(n, m, \ell; \lambda)$-CEDF if the multiset $\cup_{i \in \mathbb{Z}_m} \mathcal{D}_i$ contains each nonzero element of G precisely λ times.

In this talk we motivate the definition of CEDFs, before presenting a variety of recent approaches to constructing CEDFs. This will include a discussion of connections with graph labellings, as well as some open problems.

On the Nonexistence of Generalized Bent Functions

Shuxing Li

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University of Delaware

(This talk is based on joint work with Ka Hin Leung (National University of Singapore) and Songtao Mao (Johns Hopkins University).)

An (m, n)-generalized bent function is a function from $\mathbb{Z}_2^n$ to $\mathbb{Z}_m$ so that its associated Fourier transformations have constant absolute value. It is known that an (m, n)-generalized bent function exists whenever one of the following holds:

(1) both m and n are even.

(2) $4 \mid m$.

On the other hand, all known results suggest that for (m, n) pair that fails to satisfy both of the above conditions, (m, n)-generalized bent function does not exist. In this talk, we will discuss the recent nonexistence result of $(m, 4)$ generalized bent functions with m being odd. This result crucially relies on analyzing vanishing sums of complex roots of unity.

Homogeneous and omega-categorical Steiner triple systems

Bridget S. Webb

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Open University

A mathematical structure is *homogeneous* if every isomorphism between two of its finitely generated substructures can be extended to an automorphism of the whole. Fraïssé's theorem says that if a countably infinite class of finitely generated structures obeys certain properties, and so is an *amalgamation class*, then there is a homogeneous countably infinite structure, its *Fraïssé limit*, whose finitely generated substructures are precisely the elements of the amalgamation class. For example, the Fraïssé limit of the class of all finite graphs is the well-known Rado graph.

We consider a Steiner triple system as a Steiner quasigroup so the substructures in this context are the subsystems; that is, we consider a Steiner triple system as a functional structure. Graphs are relational structures, so, although there are some analogies between homogeneous graphs and homogeneous Steiner triple systems, there are also significant differences. For example, a mathematical structure M is *omega-categorical* if and only if its automorphism group has finitely many orbits in its action on M^n (where automorphisms act on n-tuples from M^n coordinatewise). A homogenous relational structure is necessarily omega-categorical, but this is not the case for homogeneous functional structures.

Unlike the case for graphs, it is unknown whether it is possible to completely classify all homogeneous Steiner triple systems, or even the omega-categorical homogeneous Steiner triple systems. In this talk we will look at homogeneous, omega-catagorical and some other related countably infinite Steiner triple systesms and will consider future avenues of research that may help shed light on these difficult classification problems.

Subsquares of Latin squares

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(This talk is based on joint work with Jack Allsop.)

A *Latin square* is a matrix in which each row and column is a permutation of the same set of symbols. A *subsquare* is any submatrix which is itself a Latin square. Every Latin square of order n trivially has n^2 subsquares of order 1 and one subsquare of order n. Any subsquare between these two extremes is *proper*. Subsquares of order 2 are called *intercalates*. A Latin square without intercalates is said to be N_2 and a Latin square without proper subsquares is said to be N_∞.

In this talk I will survey results and open questions relating to the number of subsquares in a Latin square. We might be trying to minimise or maximise this number, or to understand its distribution among all Latin squares of a given order. The existence question for N_2 Latin squares was settled a long time ago, but the corresponding question for N_∞ Latin squares has only just been settled. There has also been exciting recent progress on understanding the distribution of intercalates among Latin squares of order n. But many questions remain.

Clique factors in randomly augmented graphs

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(This talk is based on joint work with Sylwia Antoniuk and Christian Reiher.)

A randomly augmented graph $G^p = G_\alpha \cup G_{n,p}$ is obtained by taking a deterministic n-vertex graph G_α with minimum degree $\delta(G) \geq \alpha n$ and adding the edges of the binomial random graph $G_{n,p}$ defined on the same vertex set. For which value p does the graph G^p contain a K_r-factor with high probability?

The order of magnitude of the minimal value of p has been determined whenever $\alpha \neq 1-\frac{s}{r}$ for an integer s (see Han, Morris, and Treglown [RSA, 2021] and Balogh, Treglown, and Wagner [CPC, 2019]).

We establish the minimal probability p_s (up to a constant factor) for all values of $\alpha = 1 - \frac{s}{r} \leq \frac{1}{2}$, and show that the threshold exhibits a polynomial jump at $\alpha = 1 - \frac{s}{r}$ compared to the surrounding intervals. An extremal example G_α which shows that p_s is optimal up to a constant factor differs from the previous (usually multipartite) examples in containing a pseudorandom subgraph.

Edge statistics and the quadratic Littlewood-Offord problem

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(This talk is based on joint work with Lisa Sauermann.)

For positive integers ℓ, k, n, what is the maximum possible number of k-vertex subsets which span exactly ℓ edges, in an n-vertex graph? Extremal "edge statistics" questions of this type are closely related to the so-called *quadratic Littlewood-Offord problem*, on concentration properties of quadratic polynomials of independent random variables. With Lisa Sauermann, we developed new *decoupling* techniques to prove an optimal bound for the quadratic Littlewood-Offord problem (as conjectured by Nguyen and Vu); one application is the resolution of a conjecture due to Alon, Hefetz, Krivelevich and Tyomkyn on edge statistics in graphs. I'll briefly introduce these topics and say a few words about our new techniques.

Approximate path decompositions of regular graphs

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(This talk is based on joint work with Richard Montgomery, Alexey Pokrovskiy, Benny Sudakov.)

We show that the edges of any d-regular graph can be almost decomposed into paths of length roughly d, giving an approximate solution to a problem of Kotzig from 1957. Along the way, we show that almost all of the vertices of a d-regular graph can be partitioned into $n/(d+1)$ paths, asymptotically confirming a conjecture of Magnant and Martin from 2009.

Trimming Forests is Hard

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(This talk is based on joint work with Lior Gishboliner and Yevgeny Levanzov.)

Graph modification problems ask for the minimal number of vertex/edge additions/deletions needed to make a graph satisfy some predetermined property. A (meta) problem of this type, which was raised by Yannakakis in 1981, asks to determine for which properties $\mathcal{P}$, it is NP-hard to compute the smallest number of edge deletions needed to make a graph satisfy $\mathcal{P}$. Despite being extensively studied in the past 40 years, this problem is still wide open. In fact, it is open even when $\mathcal{P}$ is the property of being H-free, for some fixed graph H. In this case we use $\mathrm{Rem}_H(G)$ to denote the smallest number of edge deletions needed to turn G into an H-free graph.

Alon, Sudakov and Shapira proved that if H is not bipartite, then computing $\mathrm{Rem}_H(G)$ is NP-hard. They left open the problem of classifying the bipartite graphs H for which computing $\mathrm{Rem}_H(G)$ is NP-hard. In this paper we resolve this problem when H is a forest, showing that computing $\mathrm{Rem}_H(G)$ is polynomial-time solvable if H is a star forest and NP-hard otherwise. Our main innovation in this work lies in introducing a new graph theoretic approach for Yannakakis's problem, which differs significantly from all prior works on this subject. In particular, we prove new results concerning an old and famous conjecture of Erdős and Sós, which are of independent interest.

Monochromatic non-commuting products

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We show that any finite coloring of an amenable group (in particular, any finite group) contains 'many' monochromatic sets of the form $\{x, y, xy, yx\}$, as well as generalizations to more variables. This gives the first combinatorial proof and refinements of Bergelson and McCutcheon's non-commutative Schur theorem. Our main new tool is the introduction of what we call 'quasi-random colorings,' a property which is satisfied by all colorings of quasi-random groups, and a reduction to this case.

About improved lower bounds for Szemerédi's theorem

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(This talk is based on joint work with Elsholtz, Proske, and Sauermann.)

Let $r_k(N)$ be the maximum cardinality of a subset of $\{1, \ldots, N\}$ lacking k-term arithmetic progressions. Szemerédi proved $r_k(N) = o(N)$, and it has been a central problem in combinatorics to better understand how this function grows.

The best known lower bounds for $r_3(N)$ came from (a slight modification of) Behrend's construction, relying upon geometric insights and Freiman homomorphisms. For $k \geq 5$, an improved lower bound was given by Rankin, building upon this geometric perspective.

I will be discussing the ideas behind two works of mine (one of these works is joint with Elsholtz, Proske, and Sauermann), which establish the first quasipolynomial improvements to the classical bounds of Behrend and Rankin.

Infinite unrestricted $B + B$ in sets with large density

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(This talk is based on joint work with Radic.)

For a set $A \subset \mathbb{N}$ we characterize in terms of its density when there exists an infinite set $B \subset \mathbb{N}$ and $t \in \{0, 1\}$ such that $B + B \subset A - t$, where $B + B := \{b_1 + b_2 \colon b_1, b_2 \in B\}$. Specifically, when the lower density $\underline{d}(A) > 1/2$ or the upper density $\overline{d}(A) > 3/4$, the existence of such a set $B \subset \mathbb{N}$ and $t \in \{0, 1\}$ is assured. Furthermore, whenever $\underline{d}(A) > 3/4$ or $\overline{d}(A) > 5/6$, we show that the shift t is unnecessary and we also provide examples to show that these bounds are sharp. Finally, we construct a syndetic three-coloring of the natural numbers that does not contain a monochromatic $B + B + t$ for any infinite set $B \subset \mathbb{N}$ and number $t \in \mathbb{N}$. This is joint work with T. Radic.

On the number of monochromatic solutions to multiplicative equations

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(This talk is based on joint work with Lucas Aragão, Jonathan Chapman and Miquel Ortega.)

The following question was asked by Prendiville in the British Combinatorial Conference in 2022: given an r-colouring of the interval $\{2, \ldots, N\}$, what is the minimum number of monochromatic solutions the equation $xy = z$?

We give an asymptotically sharp answer for this question when $r = 2$. Indeed, we show the minimum number of monochromatic solutions to $xy = z$ that a 2-colouring of $\{2, \ldots, N\}$ can have is

$$\Big(\frac{1}{2\sqrt{2}} + o(1)\Big) N^{1/2} \log N$$

as $N \to \infty$. The proof involves a blend of elementary techniques, the sum-product phenomena and the divisor bound. We also establish a stability version of this result.

For general r, we show that there are at least $C_r N^{1/S(r-1)}$ monochromatic solutions, where $S(r)$ is the Schur number for r colours and C_r is a constant. This bound is sharp up to logarithmic factors when $r \leq 4$.

We also obtain results for more general multiplicative equations of the form $x_1^{a_1} \cdots x_k^{a_k} = y$, where $a_1, \ldots, a_k$ are positive integers, at least one of which equals 1. Our corresponding upper bounds are given in terms of certain 'interval Rado numbers' for additive equations.

These results highlight a distinction on quantititative aspects Ramsey theory for additive equations versus multiplicative equations. Indeed, it is well known that that for a partition regular linear equation, a positive proportion of all solutions are guaranteed to be monochromatic in any r-colouring of $\{1, \ldots, N\}$. For multiplicative equations like $xy = z$, the number of available solutions in $\{2, \ldots, N\}$ is $N \log N$. Therefore, only a $o(1)$ proportion of the solutions are guaranteed to be monochromatic in an r-colouring.

The Chow polynomial of a poset

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(This talk is based on joint work with Jacob P. Matherne and Lorenzo Vecchi.)

We develop a theory of poset functions that parallels the theory of Kazhdan–Lusztig–Stanley polynomials. To each kernel in a locally finite, weakly ranked poset, we can associate a special function that we call "Chow function". These are palindromic polynomials that in general may fail to be non-negative, but in most interesting cases actually can be described via adequate cohomologies.

When the poset is the lattice of flats of a matroid, and when the kernel is the characteristic polynomial, the Chow function coincides with the Hilbert series of the associated Chow ring. Also, we show that in the setting of Eulerian posets (or face posets of polytopes) and using the Eulerian P-kernel, the Chow function is related to the h-vector of the barycentric subdivision of the poset. We describe how ideas in one framework can be translated into the other.

One important main result is that the non-negativity of Kazhdan–Lusztig–Stanley functions implies the non-negativity of Chow functions. A few open problems and conjectures will be mentioned and discussed.

Quotients in matroid theory and discrete convex analysis

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(This talk is based on joint work with Marie-Charlotte Brandenburg and Georg Loho.)

A *matroid* can be seen as a combinatorial model of a configuration of vectors that records independence. In the same spirit, *matroid quotients* (or strong maps) are an abstraction of linear maps between configurations of vectors. Matroids quotients are fundamental in matroid theory: they are the building blocks for flag matroids, and are morphisms in the category of matroids.

Matroids can be generalised in several orthogonal directions. By considering configurations of linear spaces rather than vectors, we obtain *polymatroids*, combinatorial objects whose bases are lattice points rather than sets. Alternatively, we can consider matroids with real coefficients to obtain *valuated matroids*, a fundamental object within tropical geometry. In this talk, we unify the study of quotients of matroids, polymatroids and valuated matroids in the framework of Murota's discrete convex analysis. We compile a list of equivalent characterisations of quotients of discrete convex sets, generalising existing formulations for (poly)matroids. We also construct a hierarchy of characterisations of quotients of discrete convex functions, including valuated matroids.

Bounds on the number of cells of the Dressian

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A *valuation* of a matroid M with set of bases $\mathcal{B}$ is a function $\nu : \mathcal{B} \to \mathbb{R}$ so that

(V) for all $B, B' \in \mathcal{B}$ and all $e \in B \setminus B'$ there exists an $f \in B' \setminus B$ such that

$$\nu(B) + \nu(B') \geq \nu(B - e + f) + \nu(B' + e - f).$$

The *Dressian* $\mathcal{D}(M)$ is defined as the set of all valuations of M. The Dressian of M is the support of a polyhedral complex in $\mathbb{R}^{\mathcal{B}}$ whose cells are polyhedral cones. We investigate $\#\mathcal{D}(M)$, the number of cells, and $\dim \mathcal{D}(M)$, the maximum dimension of any cell.

We show that if M is a matroid on an n-element groundset of rank $r \geq t \geq 3$, then

$$\frac{\dim \mathcal{D}(M)}{\binom{n}{r}} \leq \frac{\max\{\dim \mathcal{D}(M/S),\ S \in \binom{E}{r-t} \text{ independent}\}}{\binom{n-r+t}{t}} \leq \frac{3}{n-r+3}$$

and

$$\frac{\log_2 \#\mathcal{D}(M)}{\binom{n}{r}} \leq \frac{\max\{\log_2 \#\mathcal{D}(M/S) :\ S \in \binom{E}{r-t} \text{ independent }\}}{\binom{n-r+t}{t}} O(\ln(n)) \leq \frac{O\left(\ln(n)^2\right)}{n}.$$

For uniform matroids $M = U(r, n)$ these upper bounds may be compared to the lower bounds

$$\frac{1}{n} \leq \frac{\dim \mathcal{D}(M)}{\binom{n}{r}}, \qquad \frac{1}{n} \leq \frac{\log_2 \#\mathcal{D}(M)}{\binom{n}{r}}$$

that follow a previously known construction of valuations of $U(r, n)$ from matroids of rank r on n elements. For non-uniform matroids, we also obtain somewhat tighter upper bounds in terms of the number of free generators of the Tutte group of M.

Our methods include an analogue of Shearers' Entropy Lemma for bounding the dimension of a linear subspace, and a container method for collections of linear subspaces.

Tropical schemes

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(This talk is based on joint work with Felipe Rincón.)

In this talk I will highlight the role of matroids in developing the foundations of tropical geometry. The tropicalization of a homogeneous ideal in a polynomial ring is a sequence of (valuated) matroids $\{M_d\}_{d\geq 1}$ where M_d is a matroid on the monomials of degree d satisfying certain compatibility conditions. Such sequences of matroids, regardless of whether they arise as tropicalizations, can be used to define tropical subschemes of tropical affine space. I will explain some of the properties of these objects, with an emphasis on the matroidal aspects and remaining challenges.

Small weakly separating path systems for complete graphs

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(This talk is based on joint work with Maya Stein.)

Let G be a graph and let $\mathcal{P}$ be a set of paths of G. We say that $\mathcal{P}$ weakly separates G if for every pair of edges of G there exists a path in $\mathcal{P}$ that contains exactly one of them. It is a well-known problem to determine the size of the smallest weakly separating path system of a given graph on n vertices. Around a decade ago, Falgas-Ravry, Kittipassorn, Korandi, Letzter and Narayanan conjectured an upper bound of $O(n)$ paths. This was proved last year by Bonamy, Botler, Dross, Naia and Skokan, who further conjectured an upper bound of $(1 + o(1))n$ paths.

Some authors have considered the restriction of this problem to complete graphs. It is known that a weakly separating path system for a complete graph on n vertices must contain at least $n-1$ paths. Recently, Fernandes, Oliveira Mota and Sanhueza-Matamala proved that $(1 + o(1))n$ paths suffice.

In recent work with Maya Stein, we proved that every complete graph on n vertices has a weakly separating path system of $n + 2$ paths. I will provide a brief view of the history of the problem and a proof sketch.

On friendship and cyclic parking functions

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(This talk is based on joint work with Yujia Kang, Guanyi Yang, Yanting Zhang and Haoyue Zhu.)

In parking problems, a given number of cars enter a one-way street sequentially, and try to park according to a specified preferred spot in the street. Various models are possible depending on the chosen rule for collisions, when two cars have the same preferred spot. Parking functions were initially introduced by Konheim and Weiss [1] in their study of hashing functions. In classical parking functions, if a car's preferred spot is already occupied by a previous car, it drives forward and looks for the first unoccupied spot to park.

In this work [2], we introduce a variant called *friendship parking functions*, which imposes additional restrictions on where cars can park. Namely, a car can only end up parking next to cars which are its friends (friendship will correspond to adjacency in an underlying graph). We characterise and enumerate friendship parking functions according to their outcome permutation, which describes the final configuration when all cars have parked. Inspired by this, we then return to the classical parking function case, where the outcome is a (cyclically) increasing permutation. We show that parking functions with such outcomes are in bijection with the set of all permutation components.

This talk is based on a summer research project conducted with Undergraduate students. As such, a theme of the talk will be on how to get Undergraduate students involved in academic research.

[1] A. G. Konheim and B. Weiss, *An occupancy discipline and applications*, SIAM J. Appl. Math. 14(6) (1966), 1266–1274.

[2] Y. Kang, T. Selig, G. Yang, Y. Zhang, and H. Zhu, *On friendship and cyclic parking functions*, Enumer. Comb. Appl. 4(3): Article#S2R21, 2024.

A Solution to the Total Chromatic Number Conjecture

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(This talk is based on joint work with with M. J. Henderson and R. Mary Jeya Jothi.)

The *total chromatic number* $\chi_T(G)$ of a graph G is the smallest number of colours needed to colour the vertices of G so that

1. no colour is used on two adjacent vertices,
2. no colour is used on two edges which have a common vertex,
3. no colour is used on a vertex v and an edge incident with v.

In 1964 and 1965 Behzad [1] and Vizing [?] independently conjectured that, for a simple graph G,

$$\chi_T \leq \Delta(G) + 2.$$

In this paper we prove this conjecture.

We actually prove a generalisation of this conjecture. Suppose that to each vertex and each edge there is assigned a paintbox (or list) so that if each paintbox contains $c_T(G)$ coloured paints, then G has a total colouring (i.e. each edge and each vertex gets a colour which is selected from the paintbox for that edge or for that vertex). If all the paintboxes have the same size then the least size for which this is always possible whatever the choice of paintboxes is called the *total choice number* $c_T(G)$ of G. We show that, for a simple graph G, $c_T(G) \leq \Delta(G) + 2$. Clearly $\chi_T(G) \leq c_T(G)$.

[1] M. Behzad. Graphs and their chromatic numbers. In *Ph.D. Thesis, Michigan State University*, 1965.

Combinatorial exploration of permutation classes

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(This talk is based on joint work with Michael H. Albert, Anders Claesson, Émile Nadeau, Jay Pantone, and Henning Ulfarsson.)

Permutations, words, tableaux, and other such families of objects often play a role in diverse subfields of mathematics, physics and computer science. When the structure of the object under investigation is known there are well-established tools, such as symbolic and analytic combinatorics, that derive an enumeration, asymptotics, and the ability to randomly generate instances of the objects. However, the initial step from a definition of the object to a structural description is often ad-hoc, human-staring-at-a-blackboard type of work. This is the gap *combinatorial exploration* attempts to fill.

Combinatorial exploration is a domain-agnostic algorithmic framework to automatically and rigorously study the structure of combinatorial objects and derive their counting sequences and generating functions. We describe how it works and provide an open-source Python implementation. As a prerequisite, we build up a new theoretical foundation for combinatorial decomposition strategies and combinatorial specifications.

Combinatorial exploration has been most extensively applied to permutation classes, rederiving hundreds of results in the literature as well as discovering many novel results (which can be found on permpal.com). As well as unifying earlier methods, one key advantage of our approach is its ability to utilise a growing library of strategies in a simultaneous manner to build a greater understanding of the structure of the permutation classes.

Michael H. Albert, Christian Bean, Anders Claesson, Émile Nadeau, Jay Pantone, and Henning Ulfarsson. *Combinatorial Exploration: An algorithmic framework for enumeration.* 2022. arXiv:2202.07715 [math.CO]

Flip colouring of graphs

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(This talk is based on joint work with Yair Caro, Xandru Mifsud, Raphael Yuster, and Christina Zarb.)

Suppose that G is a graph whose edges are coloured red/blue such that every vertex is incident to r red edges and b blue edges with $r > b$ but such that in the closed neighbourhood of every vertex there are more blue edges than red edges. Such a colouring will be called a (b, r)-flip colouring of G. Examples of such flip colourings are shown, constructed using Cayley graphs or strong products of graphs. This type of colouring can be extended such that for every vertex v, its closed t-neighbourhood consisting of vertices at distance at most t from v contains more blue than red edges. Another extension considers k instead of 2 colours whose majorities are flipped at every vertex. Some general results are given, for example, that for integers b, r such that $3 \leq b < r \leq \binom{b+1}{2} - 1$, there always exist graphs with a (b, r)-flip colouring, that the upper bound $r \leq \binom{b+1}{2} - 1$ is best possible, but that if $k > 3$ colours are used, then there are $(b_1, b_2, \ldots, b_k)$-flip colourings of graphs where where b_1 is fixed and b_k can be arbitrary large.

A useful result required in some of the proofs about flip colourings, which is of independent interest, is that for integers r, c such that $0 \leq c \leq \frac{r^2}{2} - 5r^{3/2}$, there exists an r-regular graph in which each open neighbourhood induces precisely c edges. Co-author Xandru Mifsud will be talking about such graphs in his presentation.

Hamilton cycles in expanders

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(This talk is based on joint work with Nemanja Draganić, Richard Montgomery, Alexey Pokrovskiy and Benny Sudakov.)

Deciding whether a graph is Hamiltonian or not is an NP-complete problem, and thus it is an important area of research to find simple conditions which imply Hamiltonicity. There are numerous such examples for dense graphs, the classic example being Dirac's theorem that any n-vertex graph with minimum degree at least $n/2$ is Hamiltonian. For sparse graphs, the paradigm is distinct.

The Hamiltonicity of sparse random graphs is very well-studied. The threshold of p for which binomial random graphs $G(n,p)$ are likely Hamiltonian is known and it is also known that random regular graphs $G_{n,d}$ are likely to be Hamiltonian for all $d \geq 3$. This points to considering natural 'pseudorandom' conditions which are required by a deterministic graph to resemble a random graph, and understand if these imply Hamiltonicity. Indeed, Krivelevich and Sudakov conjectured that (n,d,λ)-graphs, which resemble random graphs in their edge distribution, should be Hamiltonian if there is only a slight resemblence, that is if $d/\lambda > C$ for some large constant C.

An even more bold problem has been put forward in the last twenty years, with the aim of singling out the key properties of (n,d,λ)-graphs and random graphs thought to give potential for proving Hamiltonicity. An n-vertex graph G with $n \geq 3$ is a C-*expander* if: $|N(X)| \geq C|X|$ for all vertex sets $X \subseteq V(G)$ with $|X| < n/2C$; and there is an edge between any disjoint vertex sets $X, Y \subseteq V(G)$ with $|X|, |Y| \geq n/2C$. Both random graphs and (n,d,λ)-graphs are expanders and moreover, it is well-known that if C is a large constant, every C-expander contains a cycle covering almost all the vertices.

The central question is whether C-expanders are Hamiltonian. In this talk I will present our solution to this problem, thus completing an extensive line of research on Hamiltonicity problems and in particular, solving the Krivelevich-Sudakov conjecture.

Sums over Catalan paths in connection with some parking function enumeration problems

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Let $\mathcal{C}_n$ be the set of Catalan paths of length $2n$. For each $C \in \mathcal{C}_n$, let $\mathbf{u}(C)$ be the vector recording the length of each block of consecutive up-steps in C. For example, for $C = UUDUDUUUDDDD \in \mathcal{C}_6$, $\mathbf{u}(C) = (2, 1, 3)$. A function $f : [n] \to [n]$ is a parking function of size n if for all $i \in [n]$, $|\{j \mid f(j) \leq i\}| \geq i$.

In this talk, we investigate several sums over all Catalan paths $C \in \mathcal{C}_n$, whose summands are all products involving entries of $\mathbf{u}(C)$, and show their connections with several enumeration problems on parking functions. An example evaluation of such a sum is

$$\sum_{C \in \mathcal{C}_n} \prod_{i=1}^{|\mathbf{u}(C)|-1} (\mathbf{u}(C)_i + 1) = \frac{\binom{3n+1}{n}}{n+1} - \sum_{k=0}^{n-1} \frac{\binom{3n-3k+1}{n-k}}{2^{k+1}(n-k+1)},$$

and it turns out that this is equal to the number of parking functions of size n that, using a certain notion of pattern avoidance, avoid the patterns 312 and 321.

[1] J. Yan. Results on pattern avoidance in parking functions. *arXiv:2404.07958*, 2024.

Breaking Symmetry in Graphs

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(This talk is based on joint work with Sonwabile Mafunda, Jonathan Merzel, Anna Varvak.)

Given a graph G, a natural question to ask is, *"How symmetric is G?"* In this talk, we investigate this question under the umbrella of two symmetry breaking parameters: distinguishing number and fixing number. A coloring of the vertices of a graph G with d colors is said to be a *d-distinguishing coloring* of G if only the trivial automorphism preserves the color classes of G. A graph is *d-distinguishable* if it has a d-distinguishing coloring. The smallest d for which G is d-distinguishable is its *distinguishing number*. A subset $S \subseteq V(G)$, is said to be a *fixing set* for a graph G if the only automorphism that fixes the elements of S pointwise is the trivial automorphism. The size of a smallest fixing set in G is its *fixing number*. Though the two parameters were introduced separately and with different motivations, relationships exist. In this talk, we look at such relationships with special emphasis on tree graphs.

Sorting Pattern-Avoiding Permutations via 0-1 Matrices Forbidding Product Patterns

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(This talk is based on joint work with Seth Pettie and Sorrachai Yingchareonthawornchai.)

Pattern-avoidance is a fundamental topic in combinatorics that arises routinely in algorithms and data structures. In this work, we consider the problem of comparison-sorting an length-n permutation S that *avoids* some length-k permutation π. The main open question is whether sorting π-free sequences can be performed in $O(n)$ time. When $k \leq 3$, a classical result of Knutch (1973) shows a linear time sorting algorithm. For $k \geq 4$, the problem has remained open. Our main result is an $O(n2^{(1+o(1))\alpha(n)})$ time sorting based on binary search trees, where $\alpha(n)$ is an inverse Ackermann function. This result is tight up to the factor of $2^{(1+o(1))\alpha(n)}$. The key technical ingredient is an analysis of the density of 0-1 matrices forbidding a certain "product pattern".

On (r,c)-constant circulants

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(This talk is based on joint work with Yair Caro.)

This talk concerns (r,c)-constant graphs, which are r-regular graphs in which the subgraph induced by the open neighbourhood of every vertex has precisely c edges.

The family of (r,c)-constant graphs encapsulates within it many important and familiar families of graphs, such as vertex-transitive graphs (and in particular Cayley graphs), graphs with constant link, (r,b)-regular graphs, strongly regular graphs, and much more.

This family was recently introduced to demonstrate the existence of k-flip graphs, which Josef Lauri will talk about in his presentation. A number of results on (r,c)-constant graphs have been established, such as that for $r \geq 1$ and $0 \leq c \leq \frac{r^2}{2} - 5r^{\frac{3}{2}}$, there exists an (r,c)-graph. The proof is constructive and follows from the feasibility of line graphs problem, which Christina Zarb will talk about in her presentation.

In this talk we consider the existence and non-existence of (r,c)-constant circulants. We outline how (r,c)-constant circulants can be constructed when $c \equiv 0,1 \pmod 3$, $c > 0$ and $r \geq 6 + \sqrt{\frac{8c-5}{3}}$. Surprisingly, however, we shall see that for $c \equiv 2 \pmod 3$ no (r,c)-constant circulant exists.

Universality for transversal Hamilton cycles

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(This talk is based on joint work with Candida Bowtell, Patrick Morris and Katherine Staden.)

Let $\mathbf{G} = \{G_1, \ldots, G_m\}$ be a graph collection on a common vertex set V of size n such that $\delta(G_i) \geq (1 + o(1))n/2$ for every $i \in [m]$. We show that $\mathbf{G}$ contains every Hamilton cycle pattern. That is, for every map $\chi : [n] \to [m]$ there is a Hamilton cycle whose i-th edge lies in $G_{\chi(i)}$.

New combinatorial perspectives on MVP parking functions and their outcome map

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(This talk is based on joint work with Thomas Selig.)

In parking problems, a given number of cars enter a one-way street sequentially, and try to park according to a specified preferred spot in the street. Various models are possible depending on the chosen rule for collisions, when a car's preferred spot is already occupied by a previous car in the sequence.

In this work [2], we study the so-called MVP parking model, originally introduced by Harris *et al.* [1]. In this model, priority is given to the cars arriving later in the sequence. When a car finds its preferred spot occupied by a previous car, it "bumps" that car out of the spot and parks there. The bumped car drives forward and parks in the first unoccupied spot. If all cars manage to park through this process, we say that the list of preferences is an MVP parking function.

We study MVP parking functions according to their outcome permutation, which describes the final order in which the cars end up parking. In particular, we interpret parking functions with a fixed permutation outcome as certain subgraphs of the corresponding permutation's inversion graph. This allows us to give improved upper and lower bounds on the number of such parking functions.

[1] P. E. Harris, B. M. Kamau, J. C. M. Mori, and R. Tian, *On the outcome map of MVP parking functions: permutations avoiding 321 and 3412, and Motzkin paths*, Enumer. Comb. Appl. 3(2) (2023) Article #S2R11.

[2] T. Selig and H. Zhu, *New combinatorial perspectives on MVP parking functions and their outcome map*, ArXiv preprint (2023), arXiv:2309.11788.

Proper vertex-colouring exploiting graph symmetry and parallel computing

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This talk will focus on the functionality for proper vertex-colouring in the GRAPE package [3] for GAP [1]. This functionality includes the calculation of a minimum vertex-colouring of a graph, and hence its chromatic number. The software exploits graph symmetry and has been adapted to make use of large-scale parallel computing on the QMUL Apocrita cluster [2].

[1] The GAP Group, GAP — Groups, Algorithms, and Programming, Version 4.13.0, 2024. https://www.gap-system.org

[2] T. King, S. Butcher and L. Zalewski, Apocrita - High Performance Computing Cluster for Queen Mary University of London, Zenodo, 2017. https://doi.org/10.5281/zenodo.438045

[3] L.H. Soicher, The GRAPE package for GAP, Version 4.9.0, 2022. https://gap-packages.github.io/grape/

The Insertion Encoding of Cayley Permutations

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(This talk is based on joint work with Christian Bean, Paul C. Bell.)

A Cayley permutation is a sequence of numbers 1 to n such that every number appears at least once; for example 112 is a Cayley permutation of size three. If every number appears exactly once in a Cayley permutation, then it is precisely a permutation. Cayley permutations are counted by the Bell numbers as they are in bijection with ordered set partitions; the value i in the j^{th} block of the ordered set partition corresponds to the i^{th} index of the Cayley permutation having value j. A Cayley permutation π contains another Cayley permutation σ if there is a subsequence of π that is order isomorphic to σ, not necessarily at consecutive indices. For example, in 123413 the subsequence 3413 implies an occurrence of 2312. Sets of Cayley permutations that are closed downwards with respect to pattern containment are called Cayley permutation classes and can be defined by the set of minimal Cayley permutations not in the class, called the basis. For a basis B, we denote the class of Cayley permutations avoiding B as $Av(B)$.

The insertion encoding is a language that encodes how to construct permutations by adding new maximums that was introduced by Albert, Linton and Ruskuc [**?**]. We extend this to Cayley permutations. As it is possible to have multiple maximum elements in a Cayley permutation, we specify when the value being added is a new maximum or a repeated element, and prioritise the rightmost maximum.

When the insertion encoding for a Cayley permutation class is a regular language, we call the class regular. We give a condition on the basis of a class for the class to be regular, and by using the insertion encoding, we determine an algorithm to compute this regular language. This includes $Av(12)$, $Av(231, 312, 121)$ and $Av(1211, 1234, 3421)$, for example. We also compute the regular language for $Av(231, 312, 1212)$, a class studied by Cerbai [**?**] called the hare pop-stack sortable Cayley permutations.

The feasibility of the family $\mathcal{F}(G)$ of all induced G-free graphs

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(This talk is based on joint work with Yair Caro, Matthew Cassar and Josef Lauri.)

The *Feasibility Problem* is an umbrella for various specific problems in extremal combinatorics: Let $\mathcal{F}$ be an infinite family of graphs. Then $\mathcal{F}$ is called feasible if for every $n \geq 1$, $0 \leq m \leq \binom{n}{2}$, there is as graph $G \in \mathcal{F}$ having exactly n vertices and m edges. If $\mathcal{F}$ is not feasible, it is of interest to find the set of all feasible pairs

$$FP(\mathcal{F}) = \{(n, m) : \text{ there is a graph } G \in \mathcal{F} \text{ having exactly } n \text{ vertices and } m \text{ edges}\},$$

as well as the complementary set

$$\overline{FP}(\mathcal{F}) = \{(n, m) : \text{ no member of } \mathcal{F} \text{ has precisely } n \text{ vertices and } m \text{ edges}\}.$$

In the paper [1], the feasibility problem for line graphs was considered and the sets $\overline{FP}(\mathcal{F})$ and $FP(\mathcal{F})$ when $\mathcal{F}$ is the family of all line graphs were determined.

Inspired by the fact that line graphs are characterised by the nine forbidden Bieneke graphs [2], we here consider the family $\mathcal{F}(G)$ of all induced G-free graphs and show that $\mathcal{F}(G)$ is feasible if and only if G is not K_k, $K_k \backslash K_2$, $\overline{K_k}$, $\overline{K_k \backslash K_2}$, for $k \geq 2$, continuing the work initiated in [1].

We then consider the following natural question : Suppose G and H are graphs such that $\mathcal{F}(G)$ and $\mathcal{F}(H)$ are both feasible families. Is $\mathcal{F}(G, H)$, the family of all graphs which are simultaneously induced G-free and induced H-free, necessarily feasible? We give examples which result in non-feasibility, and examples which result in feasible families, and discuss some conditions which explain these results.

[1] Y. Caro, J. Lauri, and C. Zarb. The feasibility problem for line graphs. *Discrete Applied Mathematics*, 324:167–180, 2023.

[2] L.W. Beineke. Characterizations of derived graphs. *J. of Combin. Theory*, 9(2):129–135, 1970.

Tiling thresholds in 3-uniform hypergraphs

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(This talk is based on joint work with Amarja Kathapurkar, Natasha Morrison, Richard Mycroft.)

A classical question in extremal (hyper)graph theory asks for tight minimum degree conditions which force the existence of certain spanning structures in large graphs, generalising Dirac's theorem from 1952. One aspect of this concerns tiling graphs with identical vertex disjoint copies of a small subgraph. For example, asking for tight minimum codegree conditions in a k-uniform hypergraph which force a perfect matching (under the obvious additional necessary condition that the number of vertices is divisible by k). Whilst there has been a lot of interest in these types of tiling problem, still very few results are known. We share some recent progress in this area.

Why are graph polynomials knot theory weight systems?

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In knot theory *weight systems* are linear functionals on chord diagrams. Their significance arises from the Vassiliev–Kontsevich Theorem which gives that every Vassiliev knot invariant determines and is determined by a weight system. There is a method for constructing weight systems from Lie algebras and many important and familiar knot invariants arise in this way. For example, the Jones polynomial is determined by a weight system coming from sl(2).

Turning to combinatorics, a *graph polynomial* is a polynomial-valued invariant of graphs. Over the last few years there has been interest in constructing weight systems from graph polynomials. For example, Chmutov showed that Gross, Mansour, and Tucker's *partial dual genus polynomial* gives a weight system, and Kodaneva and Netrusova showed that Arratia, Bollobás and Sorkin's *interlace polynomial* is a weight system. In both cases, the proofs rely on direct combinatorial analysis.

In this talk I will explain how it follows from a quick application of some results of Bar Natan that various common graph polynomials are Lie algebra weight systems. In particular I will resolve a question of Chmutov by showing that the partial dual genus polynomial is a Lie algebra weight system.

I will keep the exposition gentle and won't assume prior knowledge of graph polynomials, knot theory, weight systems or Lie algebras.

Combinatorics Factorization of Finite Geometry and Weak Mutually Unbiased Bases in Finite Quantum Systems

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(This talk is based on joint work with S.O. Oladejo, A.O., Abdulkareem, and G.R. Ibrahim.)

A combinatoric phase-space factorization of lines in finite geometry $G(m)$ with variables in Z_m and its correspondence in finite Hilbert space $H(m)$ for m a non-prime was discussed. Using a counting techniques, lines in $G(m)$ were factorized as products of lines $G(m_i)$ where m_i is a prime divisor of m. A lattice was formed between the non trivial sublines of $G(m)$ and lines of $G(m_i)$ and between a subspace of $H(m)$ and bases of $H(m_i)$ and existence of a link between lines in phase space finite geometry and bases in Hilbert space of finite quantum systems were counted and the results follows.

Rigidity of Triangulated Surfaces

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(This talk is based on joint work with James Cruickshank and Shin-ichi Tanigawa.)

A *d-dimensional framework* is a pair (G, p) where $G = (V, E)$ is a graph and $p : V \to \mathbb{R}^d$. The *lemgth* of each edge of (G, p) is the Euclidean distance between its endpoints. The framework is *rigid* if every continuous motion of the vertices of (G, p) which preserves the edge lengths, preserves the distances between all pairs of vertices. It is *globally rigid* if every realisation of G in $\mathbb{R}^d$ which has the same edge lengths as (G, p), has the same distances between all pairs of vertices. A graph G is *rigid*, respectively *globally rigid*, in $\mathbb{R}^d$ if every generic realisation of G in $\mathbb{R}^d$ is rigid, respectively globally rigid. Both properties have been characterised for graphs when $d = 1, 2$. Extending these characterizations to $\mathbb{R}^3$ is an important open problem in discrete geometry.

Triangulations of surfaces provide a large family of graphs for which we can determine both rigidity and global rigidity in $\mathbb{R}^3$. A celebrated result of Cauchy implies that every plane triangulation is rigid in $\mathbb{R}^3$. This result was extended to all surfaces by Fogelsanger in his PhD thesis in 1988. Together with James Cruickshank and Shin-ichi Tanigawa, we recently adapted Fogelsanger's proof technique to characterise when a triangulation of a surface is globally rigid in $\mathbb{R}^3$.

I will describe Fogelsanger's ingenious proof technique and then outline how it can be adapted to work for global rigidity.

Percolation through isoperimetry

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(This talk is based on joint work with Joshua Erde, Mihyun Kang, and Michael Krivelevich.)

Let G be a d-regular graph of growing degree on n vertices. Form a random subgraph G_p of G by retaining each edge independently with probability $p = p(d)$. Which conditions suffice to observe a phase transition at $p = 1/d$, similar to that in the binomial random graph $G(n, p)$?

We argue that in the supercritical regime $p = (1+\epsilon)/d, \epsilon > 0$ a small constant, it suffices that every subset S of G of at most $n/2$ vertices has edge-boundary of size at least $C|S|$, for some large enough constant $C = C(\epsilon) > 0$, to guarantee the likely appearance of the giant component in G_p. Moreover, its asymptotic order is equal to that in the random graph $G(n, (1 + \epsilon)/n)$, and all other components are typically much smaller.

We further give examples demonstrating the tightness of this result in several key senses.

Towards an edge-coloured version of the Corrádi–Hajnal theorem

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(This talk is based on joint work with Ella Williams.)

A classical result of Corrádi and Hajnal states that any graph G on n vertices with $n \in 3\mathbb{N}$ and $\delta(G) \geq 2n/3$ contains a triangle-factor. In this talk, we will explore a generalization of this result to edge-coloured graphs. Let G be an edge-coloured graph on $n \in 3\mathbb{N}$ vertices. The minimum colour degree $\delta^c(G)$ of G is the largest integer k such that, for every vertex $v \in V(G)$, there are at least k distinct colours on edges incident to v. We show that if $\delta^c(G) \geq 5n/6$, then G has a spanning set of vertex-disjoint rainbow triangles. On the other hand, we found an example showing the bound should be at least $5n/7$. We will also discuss a related tiling problems on digraphs, which may be of independent interest.

Erdős–Ko–Rado type problems in root systems

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(This talk is based on joint work with P. Ó Catháin,Q. R. Gashi.)

Given a Lie algebra, two roots are said to be strongly orthogonal if neither their sum nor difference is a root. In this talk, we investigate sets of mutually strongly orthogonal roots. In particular, those such that any two such sets have the property that the difference between their sums can itself be expressed as the sum of a strongly orthogonal set of roots. We discuss this property and its relationship to Erdős–Ko–Rado type problems and finally discuss applications in terms of the existence of finite projective planes of certain orders. This is joint work with Qëndrim R. Gashi, University of Prishtina and P. Ó Catháin at DCU.

Applications of hypergraph containers in finite geometry

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(This talk is based on joint work with Ishay Haviv, Aleksa Milojević, Yuval Wigderson, and Geertrui Van de Voorde.)

We will present some applications of the highly influential method of (hyper)graph containers, as developed by Balogh-Morris-Samotij [2] and Saxton-Thomason [5] independently. Together with Ishay Haviv, Aleksa Milojević and Yuval Wigderson [3], we were able to use this in conjunction with an argument due to Alon and Szegedy [1] to the problem of k-nearly orthogonal sets over fields of finite characteristic. These are sets of non-self-orthogonal vectors (with respect to the standard 'inner product') such that every subset of $k+1$ vectors contains an orthogonal pair. We prove that for every prime p there exists some $\delta = \delta(p) > 0$, such that for every field $\mathbb{F}$ of characteristic p and for all integers $d \geq k \geq 2$, there exists a k-nearly orthogonal set of at least $d^{\delta \cdot k/\log k}$ vectors of $\mathbb{F}^d$. The size of the set is optimal up to the $\log k$ term in the exponent. We further prove two extensions of this result.

Furthermore, in joint work with Geertrui Van de Voorde [4], we were able to use the method of hypergraph containers to prove asymptotically sharp upper bounds for a range of special substructures in projective and polar spaces including, but not limited to, spreads, ovoids and caps.

[1] N. Alon, M. Szegedy. Large sets of nearly orthogonal vectors. *Graphs Comb* 15 (1999), 1–4.

[2] J. Balogh, R. Morris, and W. Samotij. Independent sets in hypergraphs. *J. Amer. Math. Soc.* 28 (3) (2015), 669–709.

[3] I. Haviv, S. Mattheus, A. Milojević, Y. Wigderson. Larger nearly orthogonal sets over finite fields. Submitted, arXiv:2404.01057.

[4] S. Mattheus, G. Van de Voorde. Upper bounds for the number of substructures in finite geometries from the container method. Submitted, arXiv:2404.05305.

[5] D. Saxton and A. Thomason. Hypergraph containers. *Invent. Math.* 201 (3) (2015), 925–992.

Motions of hypergraphs realised in the projective plane

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(This talk is based on joint work with L.Berman, B.Schulze, B.Servatius, H.Servatius, K.Stokes and W.Whiteley.)

In this talk, we will discuss projective motions of realisations of incidence geometries as points and lines in the projective plane. By a projective motion, we mean a motion that preserves only the incidences between points and lines. Theorems in projective geometry, like the classical theorems by Pappus and Desargues, appear as dependencies in the projective rigidity matroid. Such dependencies make characterising rigidity with respect to projective motions a difficult, but interesting, problem.

Glauber dynamics for the hard-core model on bounded-degree H-free graphs

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The hard-core model has as its configurations the independent sets of some graph instance G. The probability distribution on independent sets is controlled by a 'fugacity' $\lambda > 0$, with higher λ leading to denser configurations. We investigate the mixing time of Glauber (single-site) dynamics for the hard-core model on restricted classes of bounded-degree graphs in which a particular graph H is excluded as an induced subgraph. If H is a subdivided claw then, for all λ, the mixing time is $O(n \log n)$, where n is the order of G. This extends a result of Chen and Gu for claw-free graphs. When H is a path, the set of possible instances is finite. For all other H, the mixing time is exponential in n for sufficiently large λ, depending on H and the maximum degree of G.

Perfect Conflict-Free Hypergraph Matchings and Applications

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(This talk is based on joint work with Felix Joos and Dhruv Mubayi.)

A classic problem in combinatorics asks when we can find a perfect, or even almost-perfect, matching $\mathcal{M}$ in a hypergraph $\mathcal{H}$; the problem can be extended by further requiring that the matching avoids certain subsets of the edges (*conflicts*). We show that, assuming certain simple degree and codegree conditions on the hypergraph $\mathcal{H}$ and the conflicts to be avoided, we can extend a conflict-free almost-perfect matching to one covering all of the vertices in a particular subset of $V(\mathcal{H})$, by using an additional set of edges; in particular we ensure that our matching avoids all of a further set of conflicts, which may consist of both old and new edges. This setup is useful for various applications, such as finding upper bounds for generalised Ramsey numbers, and our main theorem provides a black box which encapsulates many long and tedious calculations and simplifies the proof of existing results. We apply it to prove that K_n can be coloured with $\frac{n}{\ell-2} + o(n)$ colours in such a way that every copy of C_ℓ receives at least three distinct colours, extending the known result for 4-cycles.

New method for old results of Rédei, Lovász and Schrijver

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Eötvös Loránd University and Alfréd Rényi Institute of Mathematics

(This talk is based on joint work with sfs Gergely Kiss, Ádám Markó and Zoltán Nagy.)

Rédei proved that a set S of cardinality p in $\mathbb{F}_p^2$ determines at least $\frac{p+3}{2}$ directions or S is a line.

We managed find a short proof for Rédei's result avoiding the theory of lacunary polynomials by proving the following statement. Let f be a polynomial over the finite field $\mathbb{F}_p$. Consider the elements of the range as integers in $\{0, 1, \ldots, p-1\}$. Assume that $\sum_{x \in \mathbb{F}_p} f(x) = p$. Then either $f = 1$ or $deg(f) \geq \frac{p-1}{2}$.

The uniqueness (up to affine transformations) of the sets of size p in $\mathbb{F}_p^2$ determining exactly $\frac{p+3}{2}$ directions was proved by Lovász and Schrijver. The same result follows from the following theorem, from the almost uniqueness of the polynomials of degree $\frac{p-1}{2}$ of range sum p.

Theorem 1. *Let f be a polynomial over $\mathbb{F}_p$, where p is a prime, which is large enough. Consider the corresponding function from $\mathbb{F}_p$ to $\{0, 1, \ldots, p-1\} \subset \mathbb{Z}$. Assume that the degree of f is $\frac{p-1}{2}$, then f can be obtained from $\pm x^{\frac{p-1}{2}} + 1$ using suitable affine transformations.*

The result of Lovász and Schrijver follows using Fourier techniques.

Point-hyperplane incidences via extremal graph theory

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(This talk is based on joint work with Benny Sudakov and István Tomon.)

In this talk, I will discuss a new combinatorial approach to the problem of estimating the maximum number of incidences between m points and n hyperplanes over arbitrary fields, under the standard non-degeneracy assumption that no s hyperplane contain s points. This approach, based on the framework of induced Turán problems, matches the best known bounds in $\mathbb{R}^d$ for many interesting values of m, n, d. Moreover, in finite fields our bounds are sharp as a function of m and n in every dimension.

This talk is based on joint work with Benny Sudakov and István Tomon.

Contributed Talk Mon 1 Jul 16:05 Bancroft 2.40

Minimum degree conditions for graph rigidity

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(This talk is based on joint work with Michael Krivelevich and Alan Lew.)

A graph is called d-**rigid** if there exists a "generic" embedding of its vertices into $\mathbb{R}^d$ such that every continuous motion of the vertices that preserves the lengths of all edges also preserves the distances between all pairs of vertices. In this talk, we will discuss new minimum degree conditions for d-rigidity. We will see that for $d = O(\sqrt{n})$, if an n-vertex graph has a minimum degree of $\delta(G) \geq (n+d)/2-1$, then it is d-rigid, and this is optimal. Additionally, for $d = O(n/\operatorname{polylog} n)$, meeting this degree condition ensures $\lfloor d/2 \rfloor$-rigidity, which is optimal up to a constant factor of 2.

The proofs utilise, in particular, methods from a recent paper by Soma Villányi [Vil23], which established an optimal connectivity condition for rigidity, as well as from our prior work on "rigid partitions" [KLM23], which we have shown to be sufficient structural conditions for rigidity. As a byproduct of our arguments, we also obtain, in the corresponding minimum degree regime, a sharp lower bound on the **pseudo-achromatic number** of a graph — the maximum size of a vertex partition such that there is an edge between any two distinct parts — in terms of its minimum degree.

[KLM23] Krivelevich, M., Lew, A., and Michaeli, P., "Rigid partitions: from high connectivity to random graphs", *arXiv e-prints*, 2023. doi:10.48550/arXiv.2311.14451.

[Vil23] Villányi, S., "Every $d(d+1)$-connected graph is globally rigid in $\mathbb{R}^d$", *arXiv e-prints*, 2023. doi:10.48550/arXiv.2312.02028.

Graphon Branching Processes and Fractional Isomorphism

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(This talk is based on joint work with Jan Hladký and Eng Keat Hng.)

In their study of the giant component in inhomogeneous random graphs, Bollobás, Janson, and Riordan introduced a class of branching processes parametrized by an L^1-graphon. We prove that two such branching processes have the same distribution if and only if the corresponding graphons are fractionally isomorphic, a notion introduced by Grebík and Rocha.

A different class of branching processes was introduced Hladký, Nachmias, and Tran in relation to uniform spanning trees in finite graphs approximating a given graphon. We characterize which graphons yield the same distribution.

Counting perfect matchings in Dirac k-uniform hypergraphs

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(This talk is based on joint work with Matthew Kwan and Roodabeh Safavi.)

A celebrated theorem of Cuckler and Kahn [1, 2] states that the number of perfect matchings in a Dirac graph ($\delta(G) \geq n/2$) is tightly controlled up to $\exp(o(n))$ by a quantity named graph entropy. We extend this result to k-uniform Dirac hypergraph.

A k-uniform hypergraph is called d-Dirac if its minimum d-degree exceeds the d-Dirac threshold by a constant factor. For all $1 \leq d \leq k-1$, we prove the number of perfect matchings in a d-Dirac k-uniform hypergraph is tightly controlled up to $\exp(o(n))$ by (hyper)graph entropy. This result answers a question raised by Glock, Gould, Joos, Kühn and Osthus [3] on whether the entropy approach can be extended to hypergraphs.

For $d \geq k/2$, combined with a lower bound on graph entropy, we prove the number of perfect matchings in a d-Dirac k-uniform hypergraph matches the expected number of perfect matchings in a random hypergraph with corresponding density up to $\exp(o(n))$. Previously, Ferber, Hardiman and Mond [4] obtained the same result for $d = k-1$ and asked if the same is true for all d. This result gives a positive answer in the range of $d \geq k/2$.

[1] Bill Cuckler and Jeff Kahn. Entropy bounds for perfect matchings and Hamiltonian cycles. *Combinatorica*, 29(3):327–335, 2009.

[2] Bill Cuckler and Jeff Kahn. Hamiltonian cycles in Dirac graphs. *Combinatorica*, 29(3):299–326, 2009.

[3] Stefan Glock, Stephen Gould, Felix Joos, Daniela Kühn, and Deryk Osthus. Counting Hamilton cycles in Dirac hypergraphs. *Combin. Probab. Comput.*, 30(4):631–653, 2021.

[4] Asaf Ferber, Liam Hardiman, and Adva Mond. Counting Hamilton cycles in Dirac hypergraphs. *Combinatorica*, 43(4):665–680, 2023.

Weighted Intersecting Families

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(This talk is based on joint work with Dr Richard Mycroft.)

For natural numbers k and n, a k-uniform intersecting family is a subfamily of $[n]^{(k)}$ such that any two elements have non-empty intersection. The much-celebrated Erdős–Ko–Rado Theorem [1] states that when $n \geq 2k > 0$, any intersecting family $\mathcal{F} \subset [n]^{(k)}$ satisfies $|\mathcal{F}| \leq \binom{n-1}{k-1}$. The k-uniform intersecting families that attain this bound are the stars centred at one element (for example, the star centred at 1 is $S_1 := \{x \in [n]^{(k)} : 1 \in x\}$).

Given an increasing weight function on the ground set, and some collection of maximal left-compressed intersecting families (MLCIFs) that we call 'canonical', we show the following extension of the Erdős–Ko–Rado Theorem:

Theorem 1. *The canonical families are the only MLCIFs that can be made optimal under an increasing weight function, with n sufficiently large. Furthermore, each canonical family can be made uniquely optimal.*

[1] P. Erdős, Chao Ko, and R. Rado. Intersection theorems for systems of finite sets. *Quart. J. Math. Oxford Ser. (2)*, 12:313–320, 1961. doi: 10.1093/qmath/12.1.313.

Intersection patterns in point sets of a projective plane

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(This talk is based on joint work with Tamás Héger.)

Let $\Pi = \Pi_q$ denote a projective plane of order q with point set $\mathcal{P}$ and line set $\mathcal{L}$. A planar set S of points in Π is said to be of *type* $(a_1, \ldots, a_r)$ if the set $\{a_i \mid i = 1 \ldots r\}$ collects the line intersection numbers of the set S, i.e., $\{a_i : i = 1 \ldots r\} = \{|S \cap \ell| : \ell \in \mathcal{L}\}$.

The most studied planar sets are those with only two intersection numbers. In desarguesian planes $\mathrm{PG}(2, q)$ over the q-element field $\mathbb{F}_q$, these sets often have some algebraic structure if such sets exist, and their size $|S|$ is often well defined by the properties. Notable examples are the so called maximal $(k; n)$-arcs and the Baer subplanes.

Our aim is to study sets S, for which some fixed value $k \in \{0, 1, \ldots, q + 1\}$ is *missing from the type* of S:

Problem 1. *Given an integer number $m \in [1, q^2 + q + 1]$ and an integer k. Is it true that there is an m-set H of the desarguesian plane $\mathrm{PG}(2, q)$ which does not have a k-secant, i.e., a line ℓ with $|\ell \cap H| = k$?*

For specific values of k, this problem has been studied before. The case $k = 1$ requires sets avoiding tangent lines. Such sets were called *sets without a tangent or untouchable sets.*When q is odd, Blokhuis, Seress, Wilbrink showed via polynomial techniques that 1-secants always exist for sets S of the size m, when $0 < m \leq q + 0.25\sqrt{2q} + 1$ holds [1]. The higher dimensional generalisation of Problem 1., which include the famous cap-set problem, is discussed in [2].

In the talk, we show an affirmative answer for Problem 1. for the majority of the cases using algebraic contructions. Then we present an extension where avoidable intersection sizes are prescribed independently for each line. Let $f : \mathcal{L} \to \mathbb{N}$ be a function which assigns a non-negative integer to every line $\ell \in \mathcal{L}$ of a projective plane Π_q of order q. An *f-avoiding set S in the projective plane* is such that for each line $\ell \in \mathcal{L}$, $|S \cap \ell| \neq f(\ell)$. Using polynomial methods we show

Theorem 2. *There exists an f-avoiding set in every projective plane Π_q for every function f.*

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Higher circuits in rigidity matroids

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(This talk is based on joint work with John Hewetson, Bill Jackson and Ben Smith.)

Lovász 1980, and then Dress and Lovász 1987, introduced the concept of a double circuit in a matroid in order to analyse the matroid matching problem. We generalise this notion to higher circuits to derive some new results on the generic d-dimensional rigidity matroid $\mathcal{R}_d$. This matroid encodes whether a framework (G, p), consisting of a graph $G = (V, E)$ and a map $p : V \to \mathbb{R}^d$, is rigid or flexible when considered as a structure comprising of rigid bars (the edges) joined at universal joints (the vertices).

When $d = 1$, $\mathcal{R}_1$ is nothing but the cycle matroid of a graph. In simpler terms, a framework is rigid if and only if the underlying graph is connected. When $d = 2$, an elegant result of Pollaczek-Geiringer 1927 characterises combinatorially when a generic framework is rigid. Extending these results to $d \geq 3$ is a longstanding open problem. The main result I will describe is an extension of a coning lemma of Whiteley 1983, and of Garamvölgyi, Gortler and Jordán 2022, which allows us to characterise rigidity in $\mathbb{R}^d$ for graphs with a sufficiently high degree vertex.

Majority Dynamics On Finite Trees

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Majority dynamics is a process on graphs and is described as follows. Initially each individual $i \in V$ considered as a vertex of a graph G on a set of vertices V has an *initial opinion* $\xi_0(i) \in \{-1, 1\}$. Then, at every time step $t \in \mathbb{N}$, they adopt the majority opinion of their neighbours, i.e. $\xi_t(i) = \operatorname{sign} \sum_{j \in N_G(i)} \xi_{t-1}(j)$, where $N_G(i)$ denotes the set of neighbours of i in G. For convenience, we only consider graphs with odd degrees, so $\sum_{j \in N_G(i)} \xi_t(j)$ is never equal to 0. We study the time until stabilisation for majority dynamics on *finite* trees:

$$\tau(T; \xi_0) = \min\{t : \xi_{t+2}(i) = \xi_t(i) \text{ for all } i \in V(T)\}.$$

First of all, for any finite tree T, we found the exact value of the worst-case stabilisation time

$$\tau(T) := \max_{\xi_0 \in \{-1,1\}^{V(T)}} \tau(T; \xi_0).$$

We call a rooted tree T *perfect k-ary*, if all vertices other than leaves have $k+1$ neighbours, and all leaves are at the same distance from the root. We showed that the stabilisation time on a perfect 2-ary tree with a uniformly random vector of initial opinions ξ_0 on its vertices is linear with high probability with respect to the diameter of the tree and differs by a non-trivial constant factor from the worst-case time. In addition, we proved that, for a fixed even $k > 2$ and a uniformly random vector of initial opinions ξ_0 on vertices of a perfect k-ary tree T with diameter D, $\tau(T; \xi_0) = \Omega(\sqrt{D})$ with high probability.

This is joint work with Itai Benjamini and Maksim Zhukovskii.

The asymptotic behaviour of $sat(n, \mathcal{F})$

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(This talk is based on joint work with Andrea Freschi.)

Given a family $\mathcal{F}$ of graphs, we say that a graph G is $\mathcal{F}$-*saturated* if it is maximally $\mathcal{F}$-free, meaning G does not contain a graph in $\mathcal{F}$ but adding any new edge to G creates a graph in $\mathcal{F}$. We then define $sat(n, \mathcal{F})$ to be the minimum number of edges in an $\mathcal{F}$-saturated graph on n vertices. In 1986, Kászonyi and Tuza showed that $sat(n, \mathcal{F}) = O(n)$ for all families $\mathcal{F}$ and Tuza conjectured that for singleton families $sat(n, \mathcal{F})/n$ converges. Tuza's Conjecture remains wide open. In this talk, I will discuss recent results about the asymptotic behaviour of $sat(n, \mathcal{F})$, mostly in the sparse regime $sat(n, \mathcal{F}) \leq n + o(n)$, in each of the cases when $\mathcal{F}$ is a singleton, when $\mathcal{F}$ is finite and when $\mathcal{F}$ is possibly infinite.

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Discrepancy of spanning trees in dense edge-coloured graphs

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(This talk is based on joint work with Lyuben Lichev, Adva Mond, and Julien Portier.)

The goal of discrepancy theory is to study how unbalanced the induced colouring of a substructure of a larger coloured structure can be, subject to some restrictions. Such problems date back to Erdős, Füredi, Loebl, and Sós [1], who proved in 1995 that for a tree T on n vertices with maximum degree Δ, any red-blue colouring of K_n contains a copy of T where one colour is given to $c(n-1-\Delta)$ more edges than the other.

While much work has since been done on discrepancy in numerous settings, in this talk we return to this original framework, and answer a question of Erdős, Füredi, Loebl, and Sós [1], generalising their work to allow many colours. In particular, we can prove that for a fixed tree T and r colouring of K_n where each colour is given to a linear proportion of the edges, there is an embedding of T where colour i is over-represented, for any i.

We can then generalise our results further, insisting that not only are edges of colour i over-represented, but they are more over-represented than any other colour, provided that the r-colouring of K_n is balanced, we are "not too close" to a single example for which the result fails, and that $\Delta \leq \varepsilon n$ for some small $\varepsilon > 0$. Moreover, in the case where Δ is constant we can generalise from the setting where our host graphs are complete to where they are merely sufficiently dense.

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Building graphs with high minimum degree on a budget

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(This talk is based on joint work with Shoham Letzter.)

We consider the problem of constructing a graph of minimum degree $k \geq 1$ in the following controlled random graph process, introduced recently by Frieze, Krivelevich and Michaeli. Suppose the edges of the complete graph on n vertices are permuted uniformly at random. A player, Builder, sees the edges one by one, and must decide irrevocably upon seeing each edge whether to purchase it or not.

Suppose Builder purchases an edge if and only if at least one endpoint has degree less than k in her graph. Frieze, Krivelevich and Michaeli observed that this strategy succeeds in building a graph of minimum degree at least k by τ_k, the hitting time for having minimum degree k. They conjectured that any strategy using εn fewer edges, where $\varepsilon > 0$ is any constant, fails with high probability.

In this work we disprove their conjecture. We show that for $k \geq 2$ Builder has a strategy which purchases $n/9$ fewer edges and succeeds with high probability in building a graph of minimum degree at least k by τ_k. For $k = 1$ we show that any strategy using εn fewer edges fails with probability bounded away from 0, and exhibit such a strategy that succeeds with probability bounded away from 0.

Bounds on the Higher Degree Erdős-Ginzburg-Ziv Constants over $\mathbb{F}_q^n$

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(This talk is based on joint work with Stefano Della Fiore.)

The classical Erdős-Ginzburg-Ziv constant of a group G denotes the smallest positive integer ℓ such that any sequence S of length at least ℓ contains a zero-sum subsequence of length $\exp(G)$.

In a recent paper Caro and Schmitt generalized this concept, using the m-th degree symmetric polynomial $e_m(S)$ instead of the sum of the elements of S and considering subsequences of a given length t. In particular, they defined the higher degree Erdős-Ginzburg-Ziv constants $EGZ(t, R, m)$ of a finite commutative ring R and presented several lower and upper bounds to these constants.

In this talk we will provide lower and upper bounds for $EGZ(t, R, m)$ in case $R = \mathbb{F}_q^n$. The lower bounds here presented have been obtained, respectively, using Lovász Local Lemma and the Expurgation method and, for sufficiently large n, they beat the lower bound provided by Caro and Schmitt for the same kind of rings. Finally, we prove closed form upper bounds derived from the Ellenberg–Gijswijt and Sauermann results for the cap-set problem assuming that $q = p^k$, $t = p$, and $m = p - 1$. Moreover, using the Slice Rank method we derive a convex optimization problem that provides the best bounds for $q = 3^k$, $t = 3$, $m = 2$ and $k = 2, 3, 4, 5$.

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Weighted domination models and randomized heuristics

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(This talk is based on joint work with Lukas Dijkstra and Vadim Zverovich.)

Given a simple vertex-weighted graph $G = (V, E, w \to \mathbb{R})$, a set X of vertices in G is a *dominating set* of G if every vertex in $V \setminus X$ is adjacent to at least one vertex in X. The minimum cardinality of a dominating set of G is called the *domination number* $\gamma(G)$. The minimum weight of a dominating set of G is denoted by $\gamma_w(G)$. The problems of finding the values of $\gamma(G)$, $\gamma_w(G)$, and the corresponding dominating sets in G are well-known to be computationally intractable. Denote by $\gamma_w^*(G)$ the smallest weight of a minimum cardinality dominating set of G. The problem of finding $\gamma_w^*(G)$ is a particular case of the two-objective optimization problem, where the objective of minimizing the size of the dominating set can be considered as a constraint, i.e., a particular case of finding Pareto-optimal solutions. Clearly, $\gamma_w(G) \leq \gamma_w^*(G)$, and finding a dominating set of size $\gamma(G)$ and weight $\gamma_w^*(G)$ is at least as difficult as finding a dominating set of size $\gamma(G)$. A similar kind of classic two-objective optimization problem could be, e.g., finding a maximum cardinality matching of the maximum weight in an edge-weighted graph, where the first optimization criterion is by the matching's cardinality (efficiently solvable).

We focus on the problems of finding dominating sets of weight equal or close to $\gamma_w(G)$ and $\gamma_w^*(G)$ in G. Finding a dominating set corresponding to $\gamma_w^*(G)$ in G is different from the classic minimum weight dominating set problem which does not require optimizing the set's cardinality. First, we show how to reduce the problem of finding $\gamma_w^*(G)$ to the minimum weight dominating set problem by using Integer Linear Programming formulations, and how to find Pareto-optimal solutions to the two-objective optimization problem in general. Then, under different assumptions, the probabilistic method [1] is applied to obtain three new upper bounds on the minimum weight dominating sets in graphs, generalizing the classic results for $\gamma(G)$. The corresponding randomized heuristics for finding small weight dominating sets in graphs are described and studied with respect to finding solutions reasonably close to the parameters $\gamma_w(G)$ and $\gamma_w^*(G)$ in G. Computational experiments are used to illustrate the results for two different types of random graphs. The experiments show that the simple randomized heuristics are very efficient, and, by taking vertex weights into consideration, the heuristics usually become more effective.

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Weak saturation rank

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(This talk is based on joint work with Nikolai Terekhov.)

Given graphs F and G, the *weak F-saturation number* $\mathrm{wsat}(G,F)$ *of the graph G* is the minimum number of edges in $H \subset G$ such that, for a certain sequence of graphs $H_0 \subset H_1 \subset \cdots \subset H_m$, every H_i is obtained from H_{i-1} by adding an edge that belongs to a copy of F in H_i. It took years to prove [1, 3, 4] the conjecture of Bollobás [2] that $\mathrm{wsat}(K_n, K_s) = \binom{n}{2} - \binom{n-s+2}{2}$. All known proofs of this fact are linear algebraic. Since then, linear algebra was applied for other pairs (G,F), and it is certainly one of the most efficient tools. In [4], Kalai suggested a general method of finding $\mathrm{wsat}(G,F)$ using matroids. For a matroid $M = (E,\mathcal{I})$ and a set $A \subseteq E$, let $\mathrm{rk}_M(A)$ be the rank of the submatroid of M induced on A. Let us call a matroid $M = (E(G),\mathcal{I})$ on the edges of G *weakly F-saturated*, if every copy $\tilde{F}$ of F in G is a *cycle* in M, i.e. for every edge $e \in E(\tilde{F})$, $\mathrm{rk}_M(E(\tilde{F}) \setminus e) = \mathrm{rk}_M(E(\tilde{F}))$. The *weak F-saturation rank* $\text{rk-sat}(G,F)$ of G is the maximum rank of a weakly F-saturated matroid M on $E(G)$. Kalai proved that for any G, F, $\mathrm{wsat}(G,F) \geq \text{rk-sat}(G,F)$. In most known applications, linear matroids are sufficient to get the exact value of $\mathrm{wsat}(G,F)$: the lower bound on $\mathrm{wsat}(G,F)$ is then represented in terms of rank of a vector space spanned by the edges of G.

We prove that for every graph F, there exists an *integer* $a_F \geq 0$ such that $\text{rk-sat}(K_n,F) = a_F n + O(1)$. Thus, linear algebraic method cannot be used to get even the asympotics (in n) of $\mathrm{wsat}(K_n,F)$ since there are graphs F such that $\lim_{n\to\infty} \mathrm{wsat}(K_n,F)/n$ is not an integer. Further, we introduce a new method that allows to apply the approach of Kalai even when c_F is not an integer by considering matroids on edges of multigraphs. We also generalise our results to a random host graph $G = G(n,1/2)$ and to hypergraphs.

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Colour-bias perfect matchings in hypergraphs

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(This talk is based on joint work with József Balogh and Andrew Treglown.)

Given a k-uniform hypergraph H and an r-colouring of its edges, we look for a minimum ℓ-degree condition that guarantees the existence of a perfect matching in H that has significantly more than n/rk edges in one colour. We refer to it as a *colour-bias* perfect matching.

For 2-coloured graphs, a result of Balogh, Csaba, Jing and Pluhár yields the minimum degree threshold that ensures a perfect matching of significant colour-bias. In this talk, I will present an analogous of this result for k-uniform hypergraphs. More precisely, for each $2 \leq \ell < k$ and $r \geq 2$ we determined the minimum ℓ-degree threshold for forcing a perfect matching of significant colour-bias in an r-coloured k-uniform hypergraph. With our result, the presented problem is solved almost in its totality, leaving only the minimum vertex degree version open.

The burning number conjecture holds for large p-caterpillars

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(This talk is based on joint work with Danielle Cox and Kerry Ojakian.)

Graph burning is a deterministic, discrete-time process that models the spread of information or contagion in a graph. At time-step $t = 0$, a single vertex is chosen to be burned. Once a vertex is burned, it remains in that state at future time-steps. At time-step $t > 0$, one new vertex (called a source vertex) is chosen to be burned; and any unburned vertex that has a burned neighbour at step $t - 1$, becomes burned at step t. The *burning number* of a graph is the minimum number of time-steps for all vertices to be burned. The Burning Number Conjecture states that for any graph G on n vertices, the burning number is at most $\lceil \sqrt{n} \rceil$. It is well-known that to settle the conjecture, it suffices to settle the conjecture on trees. However, this has been done for only a few classes of trees. We prove the Burning Number Conjecture for sufficiently large p-caterpillars.

Turán Densities for Daisies and Hypercubes

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(This talk is based on joint work with David Ellis and Imre Leader.)

The Turán density of an r-uniform hypergraph H, denoted by $\pi(H)$, is the limit of the maximum density of an n-vertex r-uniform hypergraph not containing a copy of H, as n tends to infinity. An r-daisy is an r-uniform hypergraph consisting of the six r-sets formed by taking the union of an $(r-2)$-set with each of the 2-sets of a disjoint 4-set. Bollobás, Leader and Malvenuto, and also Bukh, conjectured that the Turán density of the r-daisy is zero. A folklore Turán-type conjecture for hypercubes states that for fixed d the smallest set of vertices of the n-dimensional hypercube Q_n that meets every copy of Q_d has density $1/(d+1)$ as n goes to infinity. In this talk, we show that the Turán density for daisies is positive, and, by adapting our construction, we also disprove the hypercube conjecture mentioned above.

Online Capacitated Team Orienteering Problem with Potentially Unreliable Predictions

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(This talk is based on joint work with Vahid Akbari and Ali Hassanzadeh.)

The Capacitated Team Orienteering Problem (CTOP) is a formidable challenge within the domain of combinatorial optimization, necessitating the orchestration of a fleet of vehicles through multiple sites, each characterized by diverse location-specific parameters. The primary aim is to devise optimal routes for these vehicles, maximizing the total prize collection while respecting stringent capacity and time constraints. The CTOP boasts applications across various domains, including disaster response, maintenance operations, marketing strategies, tourism initiatives, and surveillance endeavors, where coordinated team efforts are instrumental for efficient exploration and prize attainment.

However, in many instances, the CTOP landscape is further complicated by inherent uncertainties in predicting location-specific attributes, thereby posing significant challenges to route planning. This is particularly pronounced when employing subjective predictions, as precise attribute values often remain elusive until a site is physically accessed. Consequently, in such scenarios, there arises an urgent need for innovative online optimization strategies capable of dynamically adapting to evolving information while adhering to predefined constraints.

In response to this imperative, this paper conducts a comprehensive analysis leveraging both theoretical frameworks and empirical assessments of competitive ratios. We establish an exact tight upper bound on the competitive ratio of online algorithms and introduce three pioneering online algorithms. Notably, two of these algorithms achieve optimal competitive ratios. The third algorithm, underpinned by polynomial time approximation techniques, achieves a competitive ratio of $\frac{1}{3.53}$ relative to the tight upper bound.

Empirical evaluations conducted across a spectrum of randomly generated instances and literature-based cases underscore the efficacy of our proposed methodologies across diverse simulation scenarios, thereby heralding promising advancements in addressing the complexities inherent in the CTOP landscape.

Covering edge-coloured $K_n^{(3)}$ with vertex-disjoint monochromatic tight cycles

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(This talk is based on joint work with Allan Lo.)

Let $K_n^{(3)}$ be the complete 3-uniform hypergraph on n vertices. A tight cycle is a 3-uniform graph with its vertices cyclically ordered so that every 3 consecutive vertices form an edge, and any two consecutive edges share exactly 2 vertices. A result by Bustamante, Corsten, Frankl, Pokrovskiy and Skokan shows that all r-edge coloured $K_n^{(k)}$ can be partitioned into $c_{r,k}$ vertex disjoint monochromatic tight cycles. However, the constant $c_{r,k}$ is of tower-type. In this work, we show that $c_{r,3}$ is a polynomial in r.

Oriented Diameter and Domination

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(This talk is based on joint work with Jane Morgan and Emily Rivett-Carnac.)

Let G be a connected graph. An orientation of G is a digraph obtained from G by assigning to each edge of G a direction. The oriented diameter of G is the minimum diameter of an orientation of G. It is known that only bridgeless graphs have a strong orientation, i.e., an orientation of finite diameter.

Relationships between the oriented diameter and other graph parameters such as diameter, minimum degree, maximum degree and domination number have been well studied in the literature. There are also several results on the oriented diameter of graphs from special graph classes or of product graphs.

In this talk we present upper bounds on the oriented diameter in terms of domination-type parameters. A set S of vertices of G is a dominating set if every vertex of G is either in S, or adjacent to some vertex in S, and the minimum cardinality of such a set is the domination number $\gamma(G)$ of G. The connected domination number $\gamma_c(G)$ is the minimum cardinality of a dominating set of G which induces a connected graph, and for $d \in \mathbb{N}$, the d-distance domination number $\gamma_d(G)$ is the minimum cardinality of a set S such that every vertex of G is within distance at most d from some vertex in S.

The currently best upper bound on the oriented diameter of a bridgeless graph G in terms of its domination number is $4\gamma(G)$, but no sharp bound is known. In our talk we present a sharp upper bound on the oriented diameter in terms of connected domination number. We also show that the oriented diameter of a bridgeless graph G is at most $(2d + 1)(d + 1)\gamma_d(G) + O(d)$, and this bound is best possible apart from a factor of at most about 2.

New constructions of generalized Heffter arrays

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Relative Heffter arrays were introduced in [3] as a generalization of the original concept of Heffter arrays, defined by Archdeacon in [1]. Although many constructions of these arrays were given throughout the years (including complete solutions for some families of Heffter arrays, see for instance [5] and references therein), there are still many open cases for the existence of relative Heffter arrays. In this talk, we present a construction of a family of relative Heffter arrays that has been recently obtained in a work in progress with L. Johnson and A. Pasotti [4], and some existence results, proved in a joint work with S. Costa and A. Pasotti [2], regarding a variant of Heffter arrays introduced in [1], called weak Heffter arrays.

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Permutations that separate close elements

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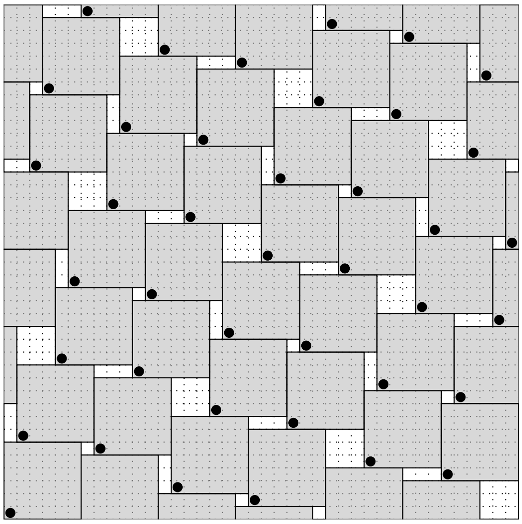
(This talk is based on joint work with Tuvi Etzion.)

Look at the diagram below. It is drawn on an $n \times n$ torus (here with $n = 40$). There are n non-overlapping 6×6 squares; more generally, we will consider rectangles that are s cells wide and k cells high. The dots in the lower left-hand corners form a permutation: there is one dot in each row and each column. For fixed n and k, what is the largest value $\sigma(k, n)$ of s where such a construction is possible?



I proved (*J. Combin. Theory Ser. A*, 2023) that $\sigma(n, k)$ can only take one of two values: $\sigma(n, k) \in \{\lfloor (n-1)/k \rfloor - 1, \lfloor (n-1)/k \rfloor\}$. This establishes a conjecture of Mammoliti and Simpson from 2020. Tuvi Etzion and I (*arXiv*:2306.03685) have recently shown which of these two values $\sigma(n, k)$ takes, determining the value of $\sigma(n, k)$ for all values of n and k. In this talk, I will discuss these results and some of the techniques we use.

Continued fractions using a Laguerre digraph interpretation of the Foata–Zeilberger bijection and its variants

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In the combinatorial theory of continued fractions, the Foata–Zeilberger bijection [3] and its variants have been extensively used to derive various continued fractions enumerating several (sometimes infinitely many) simultaneous statistics on permutations (combinatorial model for factorials) and D-permutations (combinatorial model for Genocchi and median Genocchi numbers). We will begin this talk by taking a look at some of these multivariate continued fractions. We will then sketch out the Foata–Zeilberger bijection. We will then introduce a Laguerre digraph which is a digraph in which each vertex has in- and out-degrees 0 or 1. Using Laguerre digraphs, we provide a new interpretation of the Foata–Zeilberger bijection, which enables us to count cycles in permutations. This interpretation enables us to prove some conjectured continued fractions due to Sokal and Zeng [5] in the case of permutations, and Randrianarivony and Zeng [4] (from 1996) and Deb–Sokal [2] in the case of D-permutations.

This talk will be based on work done in [1].

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Dirac's theorem for linear hypergraphs

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(This talk is based on joint work with Hyunwoo Lee.)

Dirac's theorem states that any n-vertex graph G with even integer n satisfying $\delta(G) \geq n/2$ contains a perfect matching. In this talk, we present a generalized result of Dirac's theorem to k-uniform linear hypergraphs as the following.

Any n-vertex k-uniform linear hypergraph H with minimum degree at least $\frac{n}{k} + \Omega(1)$ contains a matching that covers at least $(1 - o(1))n$ vertices.

This minimum degree condition is asymptotically tight and obtaining perfect matching is impossible with any degree condition. Furthermore, we show that if $\delta(H) \geq (\frac{1}{k} + o(1))n$, then H contains almost spanning linear cycles, almost spanning hypertrees with $o(n)$ leaves, and "long subdivisions" of any $o(\sqrt{n})$-vertex graphs.

On the Wiener index of graphs with given maximum degree

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(This talk is based on joint work with Alex Alochukwu, Paul Horn, Simon Mukwembi, Janet Osaye, Katherine Perry.)

In a connected, finite graph G of order n, the distance $d_G(u, v)$ between two vertices u and v is the length of a shortest $u - v$ path in G. The *Wiener index* $W(G)$ of G is the sum of the distances between all unordered pairs of vertices of G, that is, $W(G) = \sum_{\{u,v\}\subseteq V(G)} d_G(u, v)$. For a graph, the *maximum degree*, $\Delta(G)$ is the greatest of the degrees of the vertices of G.
Bounds on the Wiener Index for 2-connected and for 2-edge-connected graphs in terms of order were given by Plesník in 1984. In 2008, Stevanović proved upper bounds on Wiener index of a graph in terms maximum degree by showing that a bound on trees is sufficient.

In this talk, we settle a question posed by Alochukwu and Dankelmann in 2021 on whether a strengthening of the bounds by Plesník and Stevanović is possible. We show that these bounds can be improved significantly for 2-connected graphs and for 2-edge-connected graphs containing a vertex of large degree.

A connection between Johnson graphs and Clebsch–Gordan coefficients of $U(\mathfrak{sl}_2)$

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The Johnson graph $J(D, k)$ is a finite simple connected graph whose vertices are all k-element subsets of a D-element set and two distinct vertices are adjacent when their intersection contains $k-1$ elements. The universal enveloping algebra $U(\mathfrak{sl}_2)$ of the Lie algebra $\mathfrak{sl}_2$ is a unital associative algebra over $\mathbb{C}$ generated by E, F, H subject to the relations $[H, E] = 2E$, $[H, F] = -2F$ and $[E, F] = H$. The Clebsch–Gordan coefficients of $U(\mathfrak{sl}_2)$ are used to describe the transition from the uncoupled basis to the coupled basis for a finite-dimensional irreducible $U(\mathfrak{sl}_2) \otimes U(\mathfrak{sl}_2)$-module. In this talk, I will link the two seemingly irrelevant subjects: Johnson graphs and Clebsch–Gordan coefficients of $U(\mathfrak{sl}_2)$. The q-analog version is available at arXiv:2308.07851.

Well quasi-order for combinatorial structures under consecutive orders

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(This talk is based on joint work with Nik Ruškuc.)

A poset is well quasi-ordered (wqo) if it contains no infinite antichains or infinite descending chains; this property provides a way of distinguishing between those posets which are 'tame' and 'wild'. We consider posets where combinatorial structures are related when one can be embedded in the other while respecting an underlying linear order. Given such a poset, avoidance sets are subsets which are defined by their forbidden substructures. The wqo problem asks if it is decidable, given a finite set, whether its avoidance set is wqo. We will discuss this problem for a range of structures under consecutive orders, including equivalence relations and graphs. This involves adapting methods used by McDevitt and Ruškuc for permutations and words, which associate structures with paths in certain digraphs. We will highlight interesting differences between the approaches required for different structures and indicate how these affect the results.

The determinant of the distance matrix of a tree

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(This talk is based on joint work with E. Briand, L. Esquivias, A. Lillo, and M. Rosas.)

Let $T = ([n], E)$ be a tree, let $M(T)$ be its distance matrix. A remarkable formula due to Graham and Pollak (1971) is

$$\det M(T) = (-1)^{n-1}(n-1)2^{n-2}.$$

We provide the first fully combinatorial proof of this formula (using Lindström–Gessel–Viennot involutions).

Moreover, our methods provide a combinatorial proof of *every* generalisation of this formula found in the literature.

Almost partitioning every 2-edge-coloured complete k-graph into k monochromatic tight cycles

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(This talk is based on joint work with Allan Lo.)

A *k-uniform tight cycle* is a k-graph (k-uniform hypergraph) with a cyclic order of its vertices such that every k consecutive vertices from an edge. We show that for $k \geq 3$, every red-blue edge-coloured complete k-graph on n vertices contains k vertex-disjoint monochromatic tight cycles that together cover $n - o(n)$ vertices.

Remoteness of graphs and digraphs with given size and connectivity constraints

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(This talk is based on joint work with Peter Dankelmann and Sonwabile Mafunda.)

In a connected, finite graph G of order n and size m, the distance $d(u,v)$ between two vertices u and v is the length of a shortest $u-v$ path in G. The average distance $\bar{\sigma}(v)$ of a vertex v is the average of the distances from v to all other vertices in G, that is, $\bar{\sigma}(v) = \frac{1}{n-1} \sum\limits_{x \in V(G)} d(v,x)$. The remoteness $\rho(G)$ of a graph G is defined as $max\{\bar{\sigma}(v) | v \in V(G)\}$.

Bounds on proximity and, or, remoteness in terms of order were given by Aouchiche and Hansen in [1]. Stronger bounds, that consider also the size were given by Entringer et al. [2]. and the same bound in [1] was extended to digraphs in [3]. In this talk, we give sharp upper bounds on the remoteness of a κ-connected graph of given order and size. We also show that our proof strategy can be used to determine corresponding bounds when vertex connectivity is replaced by edge connectivity for $\lambda \in \{2,3\}$. We extend our results to bipartite graphs. We further extend our results to a large class of digraphs containing all graph, the Eulerian digraphs. In each case we show that our results are best possible.

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New non-extremal triple arrays

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(This talk is based on joint work with Lars-Daniel Öhman.)

A $(v, e, \lambda_{rr}, \lambda_{cc}, \lambda_{rc} : r \times c)$-*triple array* is a special type of a *row-column design*: it is an $r \times c$ array on v symbols such that each symbol occurs exactly e times, no symbol appears more than once in any row or column, and any two rows, two columns, a row and a column have exactly λ_{rr}, λ_{cc}, λ_{rc} common symbols, respectively.

Triple arrays were introduced by Agrawal [1] in 1966 while studying *designs for two-way elimination of heterogeneity*. McSorley et al. [2] showed that every triple array satisfies $v \geq r + c - 1$. Almost all known triple arrays are *extremal*, i.e. have $v = r + c - 1$. McSorley et al. [2] found the only non-extremal ($v > r + c - 1$) triple array known so far using computer search. Yucas [3] described the structure of this 7×15 triple array on 35 symbols, and showed how it can be constructed using the Fano plane (i.e. the unique symmetric $(7, 3, 1)$-BIBD) and a certain resolution of $PG(3, 2)$.

We generalize this observation of Yucas and propose a new method of constructing triple arrays by using a symmetric BIBD and a resolution of another BIBD with compatible parameters. This method can potentially be used to construct both extremal and non-extremal triple arrays. In particular, we obtain 85 pairwise non-isotopic 7×15 triple arrays on 35 symbols, including the previously known example. We also construct non-extremal triple arrays on a new set of parameters: 15×21 triple arrays on 63 symbols.

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Partial Shuffles by Lazy Swaps

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(This talk is based on joint work with Barnabás Janzer and Imre Leader.)

Suppose we generate a random permutation using a sequence of random swaps – that is, we perform a sequence of moves each of which involves swapping a pair of elements in given positions with given probability.

How many such moves are needed to make sure that at the end we have a uniformly random permutation? What if we just require that every element is equally likely to be in any position? And what if we insist that every pair, or just a single fixed pair, of elements is uniformly distributed?

I will discuss some problems and results on these questions and related ones.

Combinatorial enumeration of lattice paths by flaws with respect to a linear boundary of rational slope

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(This talk is based on joint work with Federico Firoozi and Amarpreet Rattan.)

Let a, b be fixed positive coprime integers. For a positive integer g, write $N_k(g)$ for the set of lattice paths from the startpoint $(0, 0)$ to the endpoint (ga, gb) with steps restricted to $\{(1, 0), (0, 1)\}$, having exactly k flaws (lattice points lying above the linear boundary joining the startpoint and endpoint). We wish to determine $|N_k(g)|$.

The enumeration of lattice paths with respect to a linear boundary while accounting for flaws has a long and rich history, dating back to the 1949 results of Chung and Feller. The only previously known values of $|N_k(g)|$ are the extremal cases $k = 0$ and $k = g(a+b)-1$, determined by Bizley in 1954. We derive a recursion for $|N_k(g)|$ whose base case is given by Bizley's result for $k = 0$. We solve this recursion to obtain a closed form expression for $|N_k(g)|$ for all k and g. Our methods are purely combinatorial.

Tight Hamiltonicity from dense links of triples

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(This talk is based on joint work with Mathias Schacht and Jan Volec.)

An old theorem of Dirac states that every graph on $n \geq 3$ vertices with minimum degree at least $n/2$ contains a Hamilton cycle. Our main result extends this to hypergraphs as follows. We show that for all $k \geq 4$, $\epsilon > 0$ and n sufficiently large, every k-uniform hypergraph on n vertices in which each set of $k-3$ vertices is contained in at least $(5/8+\epsilon)\binom{n}{3}$ edges contains a tight Hamilton cycle. This is asymptotically best possible.

Spectral theory of excess one digraphs

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A d-regular digraph G has excess one if for each vertex u there is a unique vertex $o(u)$, called the *outlier* of u, such that $d(u, o(u)) > k$ (and it turns out that o will be an automorphism of G). Such digraphs are analogous to undirected cages with excess one, and we conjecture that they do not exist, except in trivial cases. It was shown in [2] that there are no 2-regular digraphs with excess one, whilst [3] showed that there are no such digraphs for $k \in \{3, 4\}$ and for $k = 2$ there are no such digraphs for $d \geq 8$.

In this talk we present an inductive approach to ruling out the existence of digraphs with excess one that comes from a classification of the subdigraphs fixed by the outlier automorphism, and combine this with spectral theory. We describe three main results that appeared in [3]:

- There are no *involutory* digraphs with excess one, i.e. for which the outlier function is a transposition.
- We fill in the gaps for $k = 2$ in [1] to show that there are no digraphs with excess one for $k = 2$ and $d \geq 2$.
- We place strong divisibility and structural conditions on a 3-regular digraph with excess one, which imply that the set of such digraphs is asymptotically sparse.

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Cycle switching in Steiner triple systems of order 19

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(This talk is based on joint work with Terry Griggs.)

Cycle switching is a particular form of transformation applied to isomorphism classes of a Steiner triple system of a given order v (an STS(v)), yielding another STS(v). This relationship may be represented by an undirected graph. An STS(v) admits cycles of lengths $4, 6, \ldots, v-7$ and $v-3$. In the particular case of $v = 19$, it is known that the full switching graph, allowing switching of cycles of any length, is connected. We show that if we restrict switching to only one of the possible cycle lengths, in all cases the switching graph is disconnected (even if we ignore those STS(19)s which have no cycle of the given length). Moreover, in a number of cases we find intriguing connected components in the switching graphs which exhibit unexpected symmetries. Our method utilises an algorithm for determining connected components in a very large implicitly defined graph which is more efficient than previous approaches, avoiding the necessity of computing canonical labellings for a large proportion of the systems.

Centrality of monotone and star factorisations

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(This talk is based on joint work with Jesse Campion Loth.)

Let $\mathfrak{S}_n$ be the symmetric group on the set $\{1, 2, \ldots, n\}$. For a permutation ω, we let $c(\omega)$ be the number of cycles of ω. We consider two different types of factorisation problems in $\mathfrak{S}_n$. We resolve a problem of Goulden and Jackson from 2009, which we describe below.

A *monotone double Hurwitz factorisation* of length k of a permutation $\omega \in \mathfrak{S}_n$ is a sequence $\sigma, \tau_1, \tau_2, \ldots \tau_k$ of permutations whose product is ω that satisfies the following properties: (1) σ is an n-cycle; (2) $\tau_i = (a_i\, b_i)$, where $a_i < b_i$, for $1 \le i \le k$ (that is, each τ_i is a transposition); and (3) $b_1 \le b_2 \le \cdots \le b_k$ (the monotone condition). These factorisations were considered in depth in by Goulden, Guay-Paquet and Novak [2].

A *star factorisation* of length m of ω is a sequence of transpositions $(a_1\, n), \ldots, (a_m\, n)$ such that their product is ω and they generate a transitive subgroup of $\mathfrak{S}_n$. Goulden and Jackson [3] (generalizing earlier results of Irving and the speaker) found that conjugate elements have the same number of star factorisations, a remarkable fact given the asymmetric role the symbol n plays in these factorisations. We call a factorisation problem that displays this property — that conjugate elements have the same number of factorisations — *central*. The previous results did not expose why star factorisations are central *combinatorially* – centrality was obtained as a corollary of their enumerative formulae. This led Goulden and Jackson to ask for a combinatorial proof of centrality.

It transpires that the monotone double Hurwitz factorisations are also central, a fact that essentially follows from a classic result on symmetric functions and Jucys-Murphy elements. In this talk I present the results of [1], where Campion Loth and I show for suitable factorisation lengths that the set of monotone double Hurwitz and star factorisations of ω are equinumerous. We give a combinatorial proof of this. From this, we also obtain a combinatorial proof of the centrality of star factorisations, thus answering the question of Goulden and Jackson.

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Congruences involving the binomial coefficient and some applications

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(This talk is based on joint work with Laid Elkhiri and Abdallah Derbal.)

Abstract

In this presentation, we give some new identities and super-congruences involving binomial coefficient, and some interesting congruences related to Fibonacci and Lucas sequence.

Theorem 1. *Let $p > 3$ be a prime and $x \in \mathbb{Z}_p$. Then*

$$\sum_{m=0}^{\frac{p-1}{2}} x^{-m}\binom{p-m-1}{\frac{p-1}{2}} \equiv \frac{1+x^p}{x^{\frac{p-1}{2}}(1-x)^{\frac{p+1}{2}}} - \frac{x^{\frac{p+1}{2}}}{(1-x)^{\frac{p+1}{2}}}\left(\left(\sqrt{\frac{x-1}{x}}+1\right)^p - \left(\sqrt{\frac{x-1}{x}}-1\right)^p\right) \pmod{p^2}. \tag{1}$$

Now we present two consequences of this Theorem .

Corollary 2. *Let $p > 3$ be a prime and $x \in \mathbb{Z}_p$. Then*

$$\sum_{k=0}^{\frac{p-1}{2}} x^k\binom{2k}{k} \equiv \frac{1-4^p x^p}{(1-4x)^{\frac{p+1}{2}}} \pmod{p} \tag{2}$$

Corollary 3. *Let $p > 3$ be a prime and $m \in \{0, 1, \ldots, p-1\}$. Then*

$$\sum_{k=m}^{p-1} L_k\binom{k}{m} \equiv (-1)^{m+1}\left(L_{2m+1} - L_{2m+p+1}\right) \pmod{p} \tag{3}$$

$$\sum_{k=m}^{p-1} F_k\binom{k}{m} \equiv (-1)^{m+1}\left(F_{2m+1} - F_{2m+p+1}\right) \pmod{p} \tag{4}$$

Keywords. Super-congruences, inverse binomial coefficients, binomial coefficients, harmonic numbers.

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Resolution of the Kohayakawa-Kreuter Conjecture

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(This talk is based on joint work with A. Martinsson, R. Steiner and Y. Wigderson.)

A graph G is said to be *Ramsey* for a tuple of graphs $(H_1, \ldots, H_r)$ if every r-coloring of the edges of G contains a monochromatic copy of H_i in color i, for some i. A fundamental question at the intersection of Ramsey theory and the theory of random graphs is to determine the threshold at which the binomial random graph $G_{n,p}$ becomes a.a.s. Ramsey for a fixed tuple $(H_1, \ldots, H_r)$, and a famous conjecture of Kohayakawa and Kreuter predicts this threshold. Earlier work of Mousset-Nenadov-Samotij, Bowtell-Hancock-Hyde, and Kuperwasser-Samotij-Wigderson has reduced this probabilistic problem to a *deterministic* graph decomposition conjecture. We show that a similar decomposition statement is true, resolving the conjecture.

Chords in Longest Cycles

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(This talk is based on joint work with Alexey Pokrovskiy.)

Let G be a graph on n vertices, and let C be a cycle in G. A *chord* in C is an edge in G between two vertices of C which does not already form part of the cycle. The following 1976 conjecture of Thomassen has garnered a lot of attention over the years:

Conjecture 1. *Let G be 3-connected. Every cycle of maximum order in G has a chord.*

Thomassen himself proved that the conjecture holds in the case that G is cubic (see [1]), and several other authors have looked at variations of the problem in various other classes of graphs; and in general these graphs are usually quite sparse. Indeed, for very dense graphs—even without any connectivity assumptions—the result is obvious, e.g., if $\delta(G) \geqslant \frac{n}{2}$ then by Dirac's theorem G is Hamiltonian and thus (provided $n \geqslant 5$), any cycle of maximum order has a chord. On the other hand, the following construction of

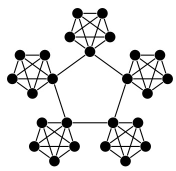

Figure 1: Illustration of Harvey's construction with $t = 5$.

Harvey shows that graphs having $\delta(G) < \sqrt{n}$ can avoid chords: take t copies of K_t, and join them around an t-cycle, as illustrated in figure 1. This graph has $\delta(G) = \sqrt{n} - 1$, yet the central cycle is both of maximum length and chordless. In [2], he conjectures that this bound is tight:

Conjecture 2. *Let G be a graph on n vertices with $\delta(G) > \sqrt{n} - 1$. Every cycle of maximum order in G has a chord.*

In the same paper, he shows that this is true with the slightly stronger assumption that $\delta(G) \geqslant \frac{3+\sqrt{17}}{2\sqrt{2}}\sqrt{n} \approx 2.52\sqrt{n}$. We prove the following asymptotic form of the conjecture:

Theorem 3. *Fix $\epsilon > 0$, and let G be an graph on n vertices satisfying $\delta(G) \geqslant (1+\epsilon)\sqrt{n}$. Then if n is large enough, every cycle of maximum order in G has a chord.*

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Interpretations of some transforms on binary functions

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A *binary function* is a function $f : 2^E \to \mathbb{C}$ for which $f(\emptyset) = 1$, where E is a finite ground set. Binary functions generalise binary matroids in the sense that any indicator function of a linear space over GF(2) is a $\{0,1\}$-valued binary function (using the natural correspondence between subsets of E and their characteristic vectors in $\mathrm{GF}(2)^E$). The author showed in 1993 that binary functions have deletion and contraction operations and extend arbitrary matroids, with duality corresponding to the Hadamard transform, and admit a generalisation of the Whitney rank generating function (a close relative of the Tutte polynomial). Subsequent papers (2004–2019) provided a family of transforms $L^{[\mu]}$ and associated minor operations, indexed by complex numbers μ, and developed their theory, with the identity transform and Hadamard transform corresponding to $\mu = 1$ and $\mu = -1$ respectively.

In this talk, we look at properties of transforms $L^{[\mu]}$ when $|\mu| = 1$. We use these transforms to characterise those binary functions for which the Hadamard transform is just the elementwise complex conjugate. We then give an interpretation of $L^{[\mu]}f$, for $|\mu| = 1$: it yields an appropriate quantum superposition of all the partial Hadamard transforms of f. We discuss the interpretation for the special case of plane graphs.

Balanced colourings and equitable partitions of triangular association schemes

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(This talk is based on joint work with P. J. Cameron, D. Ferreira, S. S. Ferreira and C. Nunes.)

In some experiments, such as those in Human-Computer Interaction, the experimental units are all pairs from a set of n individuals and each "treatment" is an activity undertaken by such a pair. The set of such pairs forms a triangular association scheme, which can also be regarded as a strongly regular graph, with an edge between two pairs if they have an individual in common.

If we regard each "treatment" as a colour, then the experimental design is a colouring of the vertex-set of the graph. There are two, unrelated, desirable statistical conditions that can be expressed as combinatorial conditions of the colouring. The first is called a balanced colouring of the graph. The second is called an equitable partition of the graph.

This talk will describe both types of colouring for the strongly regular graph defined by a triangular association scheme.

On orientations of planar mixed graphs

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(This talk is based on joint work with Ahmad Abdi.)

Let $D = (V, A, E)$ be a mixed graph with directed arcs A and undirected arcs E. A *pseudo-dicut* is a cut $\delta(U) \subseteq A \cup E$ for some nonempty proper vertex subset U such that $\delta_A^-(U) = \emptyset$, i.e, U does not have any incoming arc from A. Suppose every pseudo-dicut has at least two edges from E and suppose the undirected arcs E form one connected component, not necessarily spanning. Chudnovsky, et al. [1] conjectured that there exists an orientation $\overrightarrow{E}$ for the undirected arcs such that both digraphs with arcs $A \cup \overrightarrow{E}$ and $A \cup \overleftarrow{E}$ are strongly connected. We prove this conjecture for planar mixed graphs. The crux of the proof is to eliminate vertices incident only to directed arcs whilst preserving the planarity of the mixed graph.

Our result proves another conjecture of Chudnovsky et al. [1]. Consider a planar mixed graph $D = (V, A, E)$ where the directed arcs A form a Hamilton cycle that does not separate E, and no undirected edge has a directed path between its ends. Then there is an orientation $\overrightarrow{E}$ of the undirected edges such that both digraphs with arcs $A \cup \overrightarrow{E}$ and $A \cup \overleftarrow{E}$ are acyclic.

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Typical Ramsey properties of abelian groups

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(This talk is based on joint work with Robert Hancock and Andrew Treglown.)

A classical result of Rado characterises all those integer matrices A for which any finite colouring of $\mathbb{N}$ yields a monochromatic solution to the system of equations $Ax = 0$. Rödl and Ruciński [3] and Friedgut, Rödl and Schacht [1] proved a random version of Rado's theorem where one considers a random subset of $\{1, 2, \ldots, n\}$ instead of $\mathbb{N}$. In this talk, we consider the analogous random Ramsey problem in the more general setting of abelian groups.

Given a matrix A with integer entries, a subset S of an abelian group and $r \in \mathbb{N}$, we say that S is (A, r)-Rado if any r-colouring of S yields a monochromatic solution to the system of equations $Ax = 0$. Given a sequence $(S_n)_{n\in\mathbb{N}}$ of finite subsets of abelian groups, let $S_{n,p}$ be a random subset of S_n obtained by including each element of S_n independently with probability p. We are interested in determining the probability threshold $\hat{p} := \hat{p}(n)$ such that

$$\lim_{n\to\infty} \mathbb{P}[S_{n,p} \text{ is } (A, r)\text{-Rado}] = \begin{cases} 0 & \text{if } p = o(\hat{p}); \\ 1 & \text{if } p = \omega(\hat{p}). \end{cases}$$

Our main result is a general black box to tackle problems of this type. Using this tool in conjunction with a series of supersaturation results, we determine the probability threshold for a number of different cases. For example, a consequence of the Green–Tao theorem [2] is that every finite colouring of the primes contains arbitrarily long monochromatic arithmetic progressions. Using our machinery, we obtain a random version of this result. We also prove a novel supersaturation result for $S_n := [n]^d$ and use it to prove an integer lattice generalisation of the random version of Rado's theorem.

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[2] B. Green and T. Tao, The primes contain arbitrarily long arithmetic progressions, *Ann. Math.* **167(2)** (2008), 481–547.

[3] V. Rödl and A. Ruciński, Rado partition theorem for random subsets of integers, *Proc. London Math. Soc.* **74(3)** (1997), 481–502.

Asymmetric graph removal lemmas

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(This talk is based on joint work with António Girão, Eoin Hurley, and Lukas Michel.)

Asymmetric removal lemmas were introduced by Csaba and Gishboliner, Shapira, Wigderson as a natural extension of the classical graph removal lemma. This says that if an n-vertex graph cannot be made F-free by deleting εn^2 edges, then it contains at least $\delta n^{|F|}$ copies of F where $\delta > 0$ depends only on ε and F. The dependence of δ on ε is poorly understood in general but using Behrend's construction of large subsets of $\{1, \ldots, N\}$ with no 3-term arithmetic progression one can show that if F is not bipartite, then δ must be sub-polynomial in ε.

In an asymmetric removal lemma different graphs appear in the assumption and conclusion: the statement is of the form 'if an n-vertex graph cannot be made F-free by deleting εn^2 edges, then it contains at least $\delta n^{|H|}$ copies of H'. Gishboliner, Shapira, and Wigderson showed that surprisingly, for some pairs (H, F), δ can be a polynomial in ε; if this is the case, one says that H is *F-abundant*. They showed that $C_{2\ell+1}$ is C_{2k+1}-abundant for all $k < \ell$, and asked whether there are many other examples of pairs (H, F) such that H is F-abundant, tentatively conjecturing that the answer is no.

We prove that in fact there are many such pairs, showing for example that for every graph F there are F-abundant graphs with the same chromatic number as F.

In this talk I will discuss the proof and the relationship between asymmetric graph removal lemmas and integer solutions to linear equations.

The weird and wonderful world of near-vector spaces

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Near-vector spaces differ from traditional near-vector spaces in that they possess less linearity as a result of one of the distributive laws not holding in general. This talk will focus on the near-vector spaces first defined by Johannes André. I will give a brief introduction to the theory and discuss some recent results.

Alternating Sign Matrices, Latin-like Squares, and the ASM Polytope

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An *alternating sign matrix (ASM)* is a square matrix with entries from $\{0, 1, -1\}$ for which the non-zero entries in each row/column alternate in sign, beginning and ending with 1. ASMs arise in many different contexts as a natural generalisation of permutation matrices. A Latin square is isomorphic to a 3-dimensional analogue of a permutation matrix.

$$\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 2 & 3 & 1 \\ \hline 3 & 1 & 2 \\ \hline \end{array} \iff \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \nearrow \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \nearrow \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Brualdi and Dahl [1] introduced a 3-dimensional analogue of an ASM called an *alternating sign hypermatrix (ASHM)*, and used this to define a generalisation of a Latin square. Given an ASHM A, its corresponding Latin-like square L is given by $L(A)_{ij} = \sum_k kA_{ijk}$.

$$\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 2 & 2 & 2 \\ \hline 3 & 2 & 1 \\ \hline \end{array} \iff \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \nearrow \begin{pmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix} \nearrow \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

There is currently no set of necessary and sufficient conditions to determine if a given $n \times n$ array of integers from $\{1, \ldots, n\}$ is a Latin-like square without finding a corresponding ASHM. As a step towards this, Brualdi and Dahl introduced the *weighted projection* of an ASM. Each plane of an ASHM A is an ASM, and the weighted projection of that ASM is its corresponding row or column in $L(A)$.

Brualdi and Dahl conjectured that a vector v of length n with entries from $\{1, ..., n\}$ is the weighted projection of some ASM A if and only if v is *majorised* by the vector $(1, 2, \ldots, n)$. This talk presents a proof of this conjecture [2], and outlines links between this topic and the *ASM-polytope*.

[1] R. Brualdi, G. Dahl. *Alternating Sign Matrices and Hypermatrices, and a Generalization of Latin Squares.* Advances in Applied Mathematics, **95**(10): 1016, 2018.

[2] C. O'Brien. *Weighted projections of alternating sign matrices: Latin-like squares and the ASM polytope.* Electronic Journal of Combinatorics, **31**: 1.10, 2024.

Orientable embeddings of dense graphs and digraphs with eulerian faces

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(This talk is based on joint work with Joanna Ellis-Monaghan.)

Surface embeddings of graphs with faces bounded by euler circuits arise in several situations, such as building DNA models of graphs that can be scanned easily, and finding maximum genus orientable directed embeddings of digraphs. We show that given a circuit (closed trail) decomposition of the edges of an n-vertex eulerian graph (or digraph) where all vertices have at least $(4n+2)/5$ distinct neighbours, there is an orientable embedding where the faces are bounded by the circuits of the given decomposition and just one or two additional circuits. When there is only one additional circuit it is necessarily an euler circuit. Thus, if the number of vertices and the number of edges have the same parity, such a graph has an orientable embedding with two faces bounded by euler circuits, where one of the euler circuits can be specified in advance. Our main theorem generalizes previous results on embeddings of tournaments and of Steiner triple systems.

Contributed Talk Thu 4 Jul 11:20 Arts Two LT

Thresholds for constrained Ramsey and anti-Ramsey problems

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(This talk is based on joint work with Natalie Behague, Joseph Hyde, Shoham Letzter and Natasha Morrison.)

Let H_1 and H_2 be graphs. A graph G has the constrained Ramsey property for (H_1, H_2) if every edge-colouring of G contains either a monochromatic copy of H_1 or a rainbow copy of H_2. We give a 0-statement for the threshold for the constrained Ramsey property in $G(n, p)$, whenever $H_1 = K_{1,k}$ for some $k \geq 3$ and H_2 is not a forest. Along with previous work of Kohayakawa, Konstadinidis and Mota, this resolves the constrained Ramsey property for all non-trivial cases with the exception of $H_1 = K_{1,2}$, which is equivalent to the anti-Ramsey property for H_2.

For a fixed graph H, we say that G has the anti-Ramsey property for H if any proper edge-colouring of G contains a rainbow copy of H. We show that the 0-statement for the anti-Ramsey problem in $G(n, p)$ can be reduced to a (necessary) colouring statement, and use this to find the threshold for the anti-Ramsey property for some particular families of graphs.

Subgraphs with a positive minimum semidegree in digraphs with large outdegree

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(This talk is based on joint work with Andrzej Grzesik and Vojta Rödl.)

Given a directed graph G, we define *the minimum semidegree of* G to be the smallest integer s such that every vertex of G has both the indegree and the outdegree at least s. We prove that every d-out-regular directed graph G has a subgraph that has the minimum semidegree at least $\frac{d(d+1)}{2|V(G)|}$.

On the other hand, for every $\varepsilon > 0$ we construct infinitely many tournaments T with the minimum outdegree d satisfying that every subgraph of T has the minimum semidegree at most $(1+\varepsilon) \cdot \frac{d(d+1)}{2|V(T)|}$. In particular, there are n-vertex tournaments with the minimum outdegree $d = \Theta\left(\sqrt{n}\right)$ such that all their subgraphs have the minimum semidegree at most 1, and there are n-vertex tournaments with the minimum outdegree $d = \gamma n$ such that all their subgraphs have the minimum semidegree at most $(\frac{1}{2} + \varepsilon) \cdot \gamma^2 n$.

Recurrence properties of polynomial sumsets

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A set of integers S is called *chromatically intersective* if, whenever the integers are coloured with finitely many colours, there exist two distinct integers x_1 and x_2 with the same colour such that $(x_1 - x_2) \in S$. There are many equivalent definitions of chromatic intersectivity; in dynamics, chromatically intersective sets are called *sets of (topological) recurrence* and have been widely-studied in the context of dynamical recurrence.

Partitioning the integers into residue classes reveals that chromatically intersective sets must contain a multiple of any positive integer. Using a similar argument, for any finite collection of real numbers $\alpha_1, \ldots, \alpha_m$ and any $\rho > 0$, one can show that chromatically intersective sets contain a non-zero integer x such that every $x\alpha_i$ is within a distance ρ from an integer. A famous open problem popularised by Katznelson and Ruzsa asks whether these simple necessary conditions are also sufficient. In the language of dynamics, a positive answer would imply that S is a set of recurrence for all topological dynamical systems if and only if S is a set of recurrence for all finite-dimensional toral rotations.

In this talk, I will discuss ongoing research into variations of chromatic intersectivity concerning iterated polynomial sumsets. More precisely, we investigate sets S such that, for any finite colouring of the positive integers, S must contain an integer of the form $(a_1h(x_1) + \cdots + a_sh(x_s))$, where the a_i are fixed non-zero integers, h is a fixed integer polynomial of degree $d \geqslant 1$, and the variables x_i are all distinct and share the same colour. Provided there are at least $(1 + o(1))d^2$ variables, we provide a positive answer to the analogue of the Katznelson-Ruzsa problem. Additionally, when $a_1 + \cdots + a_s = 0$, we can show that S contains such integers where the x_i all lie in a given arbitrary set of integers with positive upper density. By incorporating recent transference techniques developed by myself and Sam Chow (University of Warwick), we also establish analogous results for monochromatic or dense subsets of the prime numbers.

Disjoint and external partial difference families

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(This talk is based on joint work with Laura Johnson.)

Disjoint and external partial difference families are recently-introduced combinatorial structures which simultaneously generalize partial difference sets, disjoint difference families and external difference families, and have applications to communications, information security and experiment design. In this talk I will introduce and motivate these objects, and present various construction methods including a cyclotomic framework in finite fields.

Saturated Partial Embeddings of Planar Graphs

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(This talk is based on joint work with Alexander Clifton.)

In this talk, we study how far one can deviate from optimal behavior when embedding a planar graph. For a planar graph G, we say that a plane subgraph $H \subseteq G$ is a *plane-saturated subgraph* if adding any edge (possibly with new vertices) to H would either violate planarity or make the resulting graph no longer a subgraph of G. For a planar graph G, we define the *plane-saturation ratio*, $\mathrm{psr}(G)$, as the minimum value of $\frac{e(H)}{e(G)}$ for a plane-saturated subgraph H of G and investigate how small $\mathrm{psr}(G)$ can be. While there exist planar graphs where $\mathrm{psr}(G)$ is arbitrarily close to 0, we show that for all twin-free planar graphs, $\mathrm{psr}(G) > 1/16$, and that there exist twin-free planar graphs where $\mathrm{psr}(G)$ is arbitrarily close to $1/16$. In fact, we study a broader category of planar graphs, focusing on classes characterized by a bounded number of degree 1 and degree 2 twin vertices. We offer solutions for some instances of bounds while positing conjectures for the remaining ones.

The k-Ramsey Number of Two Five Cycles

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(This talk is based on joint work with Elizabeth Jonck and Ronald J. Maartens, University of the Witwatersrand, Johannesburg, South Africa.)

Given any two graphs F and H, the Ramsey number $R(F, H)$ is defined as the smallest positive integer n such that every red-blue coloring of the edges of the complete graph K_n of order n, there will be a subgraph of K_n isomorphic to F whose edges are all colored red (a red F) or a subgraph of K_n isomorphic to H whose edges are all colored blue (a blue H). If F and H are bipartite graphs, then the k-Ramsey number $R_k(F, H)$ is defined as the smallest positive integer n such that for any red-blue coloring of the edges of the complete k-partite graph of order n in which each partite set is of order $\lfloor \frac{n}{k} \rfloor$ or $\lceil \frac{n}{k} \rceil$ there will be a subgraph isomorphic to F whose edges are all colored red (a red F) or a subgraph isomorphic to H whose edges are all colored blue (a blue H). Andrews, Chartrand, Lumduanhom and Zhang found the k-Ramsey number $R_k(F, H)$ for $F = H = C_4$, and for $F = K_{1,t}$ and $H = K_{1,s}$ where $s, t \geq 2$. We continue their work by investigating the case where the graphs F and H are both C_5.

Lower Bounds for Maximum Weight Bisections of Graphs with Bounded Degrees

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(This talk is based on joint work with Stefanie Gerke, Gregory Gutin and Anders Yeo.)

A *cut* is a partition of the vertices of a graph into two disjoint subsets. A *bisection* is a cut in which the number of vertices in the two parts differs by at most 1. It is well known that if $\Delta(G) \leq k$, then G has a cut with at least $\frac{k+1}{2k}|E(G)|$ edges when k is odd, and $\frac{k+2}{2(k+1)}|E(G)|$ edges when k is even. We conjecture that the same bound holds not only for cut but also for weighted bisections:

Conjecture 1. *Let $G = (V(G), E(G), w)$ be a weighted graph. If $\Delta(G) \leq k$, then there exists a bisection of weight at least $\frac{k+1}{2k}w(G)$ if k is odd and $\frac{k+2}{2(k+1)}w(G)$ if k is even.*

Bollobás and Scott [1] showed that Conjecture 1 holds for unweighted regular graphs. Lee, Loh and Sudakov proved that if $\Delta(G) \leq k$, then there exists a bisection of size at least $\frac{k+1}{2k}|E(G)| - \frac{k(k+1)}{4}$ if k is odd and $\frac{k+2}{2(k+1)}|E(G)| - \frac{k(k+2)}{4}$ if k is even. In this talk, I will provide proofs for Conjecture 1 when k is even or equal to 3. I will also talk about edge-weighted triangle-free subcubic graphs and show that they admit a bisection with weight at least $\theta \cdot w(G)$ (where $\theta \approx 0.716959$) with only one exception graph ($K_{1,3}$).

[1] B. Bollobás and A. Scott, Judicious Partitions of Bounded-Degree Graphs, Journal of Graph Theory, 46(2004), 131–143.

[2] C. Lee, P. Loh and B. Sudakov, Bisections of graphs, Journal of Combinatorial Theory, Series B, 103(2013), 599–629.

Hidden structures in (higher) Euler characteristic invariants

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We will discuss the gamma vector, which was originally considered in the context of the combinatorics of Eulerian polynomials and later resurfaced in a special case of the Hopf conjecture on Euler characteristics of (piecewise Euclidean) nonpositively curved manifolds in work of Gal and properties of poset Eulerian polynomials in work of Brändén. Since then, it has appeared in many different combinatorial applications. We find explicit formulas which give a local-global interpretation and complement/contrast lower bound properties stated earlier by Gal. In addition, a formula involving Catalan numbers and binomial coefficients hints at connections to noncrossing partitions and Coxeter groups in existing positivity examples. Finally, we note that considering characteristic classes directly lead to log concavity and Schur positivity properties.

Latin hypercubes realizing integer partitions

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(This talk is based on joint work with Diane Donovan and James Lefevre.)

L. Fuchs asked the following question: If n is any positive integer and $n = n_1+n_2+\cdots+n_k$ any fixed partition of n, is it possible to find a quasigroup Q of order n which contains subquasigroups $Q_1, Q_2, \ldots, Q_k$ of orders $n_1, n_2, \ldots, n_k$ respectively, whose set theoretical union is Q? Such a quasigroup is equivalent to a latin square of order n with disjoint subsquares of orders $n_1, n_2, \ldots, n_k$, and the existence of these latin squares is a partially solved problem. This problem of realizing a partition in a latin square can be extended to latin cubes with disjoint subcubes of the same orders. In this talk, we will discuss the existence problem for latin cubes and how the problem changes as the number of dimensions increases.

A Deterministic Rotation Method for Coloring Plane Graphs

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(This talk is based on joint work with Andrew Bowling, abowling@umn.edu.)

The Four Color Theorem famously states that the regions of any plane graph can be colored with four or fewer such that no two adjacent regions have the same color. Both proving this claim and obtaining a four coloring for an arbitrary graph are nontrivial tasks. Here we describe several algorithms based on a systematic rotation method that are entirely deterministic which we have demonstrated to work on a very large number of diverse graphs (over 175 million), including at least one algorithm which has not failed to color every region of the graphs.

Partitioning Problems via Random Processes

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(This talk is based on joint work with Oliver Cooley, Mihyun Kang, and Matthew Kwan.)

There are a number of well-known problems and conjectures about partitioning graphs to satisfy local constraints. For example, the majority colouring conjecture of Kreutzer, Oum, Seymour, van der Zypen and Wood states that every directed graph has a majority 3-colouring i.e. a colouring of its vertices with 3 colours such that for every vertex v, at most half of the out-neighbours of v have the same colour as v. We prove that this conjecture holds for almost all digraphs of any given density: For every $p = p(n) \in [0, 1]$, the binomial random digraph $D(n, p)$ has a majority 3-colouring with high probability. Our proof relies on a carefully designed randomised colouring procedure that iteratively converges to a majority 3-colouring.

Ramsey goodness of loose paths

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(This talk is based on joint work with Allan Lo.)

The *Ramsey number* of a pair of graphs (G, H), denoted by $R(G, H)$, is the smallest integer n such that, for every red/blue-colouring of the edges of the complete graph K_n, there exists a red copy of G or a blue copy of H. In the 1980s, Burr showed that, if G is large and connected, then $R(G, H)$ is bounded below by $(v(G) - 1)(\chi(H) - 1) + \sigma(H)$, where $\chi(H)$ is the chromatic number of H and $\sigma(H)$ stands for the minimum size of a colour class over all proper $\chi(H)$-colourings of H. We say that G is H-*good* if $R(G, H)$ is equal to this general lower bound. This notion was first studied systematically by Burr and Erdős and has received considerable attention from researchers since its introduction. Among other results, it was shown by Burr that, for any graph H, every sufficiently long path is H-good.

These concepts generalise in the natural way to k-graphs, and in this talk we will explore the notion of Ramsey goodness when G is an ℓ-path for some $1 \leq \ell \leq k-1$. We will show that, while long loose paths are not always H-good, they are very close to being H-good for every k-graph H. As we will see, this is in stark contrast to the behaviour of ℓ-paths for larger ℓ.

A new construction of the Biggs–Smith distance-regular graph

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Around 1970, Biggs and Smith described a "remarkable" cubic distance-transitive graph on 102 vertices arising from the group $\mathrm{PSL}(2,17)$, which was later shown by Brouwer to be the unique distance-regular graph with its parameters. Various constructions of the graph are known. From the graph, one may construct an incidence structure whose points and lines are indexed by the vertices of the graph, and where each point is incident with the three lines labelled by its neighbours. Using the GAP package FinInG, I recently stumbled across this geometry inside the symplectic polar space $W(7,2)$. In this talk, we will try to make some sense of this observation.

Layered subgraphs of the hypercube

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(This talk is based on joint work with Imre Leader, Natasha Morrison, Kada Williams.)

A subgraph of the n-dimensional hypercube is called 'layered' if it is a subgraph of a layer of some hypercube. There are many graphs that are subgraphs of some hypercube, but not layered – a simple example is the 4-cycle. However, the known examples all tend to fail to be layered for 'local' reasons: they contain 4-cycles, or else some other dense cube-like local structure. In this talk, I will present a short and sweet constructive proof that there exist subgraphs of the cube of arbitrarily large girth that are not layered. This answers a question of Axenovich, Martin and Winter. Perhaps surprisingly, these subgraphs may even be taken to be induced.

Local Central Limit Theorem for triangle counts in sparse random graphs

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(This talk is based on joint work with Letícia Mattos.)

Let X_H be the number of copies of a fixed graph H in $G(n,p)$. In 2016, Gilmer and Kopparty conjectured that a local central limit theorem should hold for X_H as long as H is connected, $p \gg n^{-1/m(H)}$ and $n^2(1-p) \gg 1$, while proving that it holds for the case $H = K_3$ and p is constant. We extend their theorem to the sparse setting. More precisely, we show that there exists $C > 0$ such that if $p \in (Cn^{-1/2}, 1/2)$, then

$$\sup_{x \in \mathcal{L}} \left| \frac{1}{\sqrt{2\pi}} e^{-x^2/2} - \sigma \cdot \mathbb{P}(X^* = x) \right| \to 0,$$

where $\sigma^2 = \mathbb{V}\mathrm{ar}(X_{K_3})$, $X^* = (X_{K_3} - \mathbb{E}(X_{K_3}))/\sigma$ and $\mathcal{L}$ is the support of X^*. By combining our result with the results of Röllin–Ross and Gilmer–Kopparty, we confirm the aforementioned conjecture for triangle counts. As a corollary, we obtain an optimal anti-concentration result for triangle counts.

Orientations of graphs with at most one directed path between every pair of vertices

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(This talk is based on joint work with Barbora Dohnalová, Jiří Kalvoda, Sophie Spirkl.)

Given a graph G, we say that an orientation D of G is a *KT orientation* if, for all $u, v \in V(D)$, there is at most one directed path (in any direction) between u and v. Graphs that admit such orientations have been used by Kierstead and Trotter (1992), Carbonero, Hompe, Moore, and Spirkl (2023), Briański, Davies, and Walczak (2023), and Girão, Illingworth, Powierski, Savery, Scott, Tamitegami, and Tan (2024) to construct graphs with large chromatic number and small clique number that served as counterexamples to various conjectures.

Motivated by this, we consider which graphs admit KT orientations (named after Kierstead and Trotter). In particular, we construct a graph family with small independence number (sub-linear in the number of vertices) that admits a KT orientation. We show that the problem of determining whether a given graph admits a KT orientation is NP-complete, even if we restrict ourselves to planar graphs. Finally, we provide an algorithm to decide if a graph with maximum degree at most 3 admits a KT orientation. Whereas, for graphs with maximum degree 4, the problem remains NP-complete.

Ramsey numbers of hypergraphs of a given size

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(This talk is based on joint work with Jacob Fox and Benny Sudakov.)

The q-color Ramsey number of a k-uniform hypergraph H is the minimum integer N such that any q-coloring of the complete k-uniform hypergraph on N vertices contains a monochromatic copy of H. The study of these numbers is one of the central topics in Combinatorics. In 1973, Erdős and Graham asked to maximize the Ramsey number of a graph as a function of the number of its edges. Motivated by this problem, we study the analogous question for hypergaphs. For fixed $k \geq 3$ and $q \geq 2$ we prove that the largest possible q-color Ramsey number of a k-uniform hypergraph with m edges is at most $\mathrm{tw}_k(O(\sqrt{m}))$, where tw denotes the tower function. We also present a construction showing that this bound is tight for $q \geq 4$. This resolves a problem by Conlon, Fox and Sudakov. They previously proved the upper bound for $k \geq 4$ and the lower bound for $k = 3$. Although in the graph case the tightness follows simply by considering a clique of appropriate size, for higher uniformities the construction is rather involved and is obtained by using paths in expander graphs.

[1] P. Erdős and R. L. Graham. On partition theorems for finite graphs. In *Infinite and finite sets (Colloq., Keszthely, 1973; dedicated to P. Erdős on his 60th birthday), Vols. I, II, III*, Colloq. Math. Soc. János Bolyai, Vol. 10, pages 515–527. North-Holland, Amsterdam, 1975.

[2] D. Conlon, J. Fox, and B. Sudakov. Ramsey numbers of sparse hypergraphs. *Random Structures & Algorithms*, 35(1):1–14, 2009.

A NEW LOOK AT THEOREMS OF CAUCHY AND CAYLEY

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(This talk is based on joint work with David Craven, Hamid Reza Dorbidi, Scott Harper and Benjamin Sambale.)

Theorems on group theory by Cauchy and Cayley date from the early days of the subject and are in every undergraduate course on groups. But they give rise to questions which appear not to have been looked at before.

Cauchy proved that, if a prime p divides the order of a group G, then G embeds the cyclic group of order p. (The converse is also true, by Lagrange's theorem.) We say that the positive integer n is a *Cauchy number* if there is a finite set $\mathcal{F}$ of finite groups such that the order of a group G is divisible by n if and only if G embeds some member of $\mathcal{F}$. We have determined all the Cauchy numbers: they are the prime powers, 6, and numbers of the form $2p^a$ where p is a Fermat prime greater than 3 and $a \geq 1$.

Cayley proved that every group of order n can be embedded in the symmetric group S_n. This suggests the obvious question: what is the smallest order of a group which embeds every group of order n? We cannot answer this – our bounds are quite far apart – but we have found a formula for the order of the smallest abelian group which embeds every abelian group of order n. It is defined in terms of the function f given by

$$f(n) = \sum_{k=1}^{n} \lfloor n/k \rfloor,$$

which was investigated by Dirichlet.

Peter J. Cameron, David Craven, Hamid Reza Dorbidi, Scott Harper, Benjamin Sambale, Minimal cover groups, arXiv 2311.15652

New lower bounds for cap sets

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A cap set is a subset of $\mathbb{F}_3^n$ with no solutions to $x+y+z=0$ other than when $x=y=z$. The cap set problem asks how large a cap set can be, and is an important problem in additive combinatorics and combinatorial number theory. In this talk, I will introduce the problem, give some background and motivation, and describe how I was able to provide the first progress in 20 years on the lower bound for the size of a maximal cap set. Building on a construction of Edel, we use improved computational methods and new theoretical ideas to show that, for large enough n, there is always a cap set in $\mathbb{F}_3^n$ of size at least 2.218^n. I will then also discuss recent developments, including an extension of this result by Google DeepMind.

Optimal Hamilton covers and linear arboricity for random graphs

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(This talk is based on joint work with Stefan Glock, David Munhá Correia, and Benny Sudakov.)

In his seminal 1976 paper, Pósa showed that for all $p \geq C \log n/n$, the binomial random graph $G(n,p)$ is with high probability Hamiltonian. This leads to the following natural questions, which have been extensively studied: How well is it typically possible to cover all edges of $G(n,p)$ with Hamilton cycles? How many cycles are necessary? We show that for $p \geq C \log n/n$, we can cover $G \sim G(n,p)$ with precisely $\lceil \Delta(G)/2 \rceil$ Hamilton cycles. Our result is clearly best possible both in terms of the number of required cycles, and the asymptotics of the edge probability p, since it starts working at the weak threshold needed for Hamiltonicity. This resolves a problem of Glebov, Krivelevich and Szabó, and improves upon previous work of Hefetz, Kühn, Lapinskas and Osthus, and of Ferber, Kronenberg and Long, essentially closing a long line of research on Hamiltonian packing and covering problems in random graphs.

Contributed Talk Fri 5 Jul 11:20 Arts Two LT

Powers of Hamilton cycles in oriented and directed graphs

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(This talk is based on joint work with Louis DeBiasio, Jie Han, Allan Lo, Theodore Molla and Simón Piga.)

The Pósa–Seymour conjecture determines the minimum degree threshold for forcing the kth power of a Hamilton cycle in a graph. After numerous partial results, in 1998 Komlós, Sárközy and Szemerédi proved the conjecture for sufficiently large graphs. In this talk we focus on the analogous problem for digraphs and for oriented graphs. We asymptotically determine the minimum degree threshold for forcing the square of a Hamilton cycle in a digraph. We also give a conjecture on the corresponding threshold for kth powers of a Hamilton cycle more generally. For oriented graphs, we provide a minimum semi-degree condition that forces the kth power of a Hamilton cycle; although this minimum semi-degree condition is not tight, it does provide the correct order of magnitude of the threshold.

Spanning loose trees in k-uniform hypergraphs

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(This talk is based on joint work with Allan Lo.)

A classic result of Komlós, Sárközy and Szemerédi states that every large n-vertex graph with minimum degree at least $(\frac{1}{2}+\gamma)n$ contains all spanning trees of bounded degree. We consider a generalization of it to loose spanning hypertrees in k-uniform hypergraphs, that is, linear hypergraphs obtained by subsequently adding edges sharing a single vertex with a previous edge.

We show that for all $k \geq 4, \gamma$, and Δ, every large n-vertex k-uniform hypergraph with minimum $(k-2)$-degree at least $(\frac{1}{2}+\gamma)\binom{n}{2}$ contains every spanning loose tree with maximum vertex degree Δ. This bound is asymptotically tight, since some loose trees with bounded vertex degree contain a perfect matching.

This generalises a result of Pehova and Petrova [1], who proved the corresponding result when $k=3$.

[1] Y.Pehova and K.Petrova. Minimum vertex degree conditions for loose spanning trees in 3-graphs. *arXiv preprint arXiv:2301.09630*, 2023.

Monomial Representations and Combinatorics

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(This talk is based on joint work with Santiago Barrera Acevedo, Heiko Dietrich and Ronan Egan.)

D. G. Higman observed that the centraliser algebra of a rank 3 permutation group is spanned by the incidence matrix of a strongly regular graph on which the permutation group acts by automorphisms. Quite generally, incidence matrices of combinatorial structures invariant under a permutation group live in the centraliser algebra of the corresponding permutation representation.

In this talk, I will explain how monomial representations act on objects such as Complex Hadamard matrices and sets of equiangular lines. Central extensions of groups and cocycles arise naturally: this interpretation ties together the theory of cocyclic Hadamard matrices with classical techniques from algebraic combinatorics. To illustrate the theory, I will show how Grobner basis techniques can be used to construct complex Hadamard matrices in the centraliser algebra of a monomial representation.

The number of sum-free subsets of the square

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The problem of finding the largest sum-free subset of the square $[n]^2 = \{1, 2, \ldots, n\}^2$ was communicated by Harout Aydinian to Oriol Serra and posed as an open problem at the 19th British Combinatorial Conference in 2001 [Cam05]. Cameron [Cam05; Cam02] conjectured that the maximal size of a sum-free subset of $[n]^2$ is $0.6n^2 + O(n)$, which was proved in 2017 by Elsholtz and Rackham [ER17]. Generalising the Cameron-Erdős conjecture [CE90] to 2 dimensions, Elsholtz and Rackham conjectured that the number of sum-free subsets of $[n]^2$ is $2^{0.6n^2+O(n)}$. We present a proof of the slightly weaker bound $2^{0.6n^2+O(n\log(n))}$.

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[ER17] Christian Elsholtz and Laurence Rackham. "Maximal sumfree sets of integer lattice grids". In: *Journal of the London Mathematical Society* 95.2 (2017), pp. 353–372.

Characterising flip process rules with the same trajectories

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Garbe, Hladký, Šileikis and Skerman [2] recently introduced a broad class of discrete-time random graph processes called flip processes. A flip process starts with a given n-vertex graph, and each step of a flip process involves randomly selecting a k-tuple of distinct vertices and modifying the induced subgraph according to a predetermined rule. Garbe, Hladký, Šileikis and Skerman showed that the typical behaviour of flip processes is neatly captured by certain continuous-time deterministic graphon trajectories. It was observed [1] that seemingly different flip process rules may sometimes have the exact same collection of trajectories.

We [3] obtain a complete characterisation of equivalence classes of flip process rules with the same trajectories. As an application, we show that the symmetric deterministic rules and the rules of order 2 are precisely the rules which are unique in their equivalence classes. These include complementing rules, component completion rules, clique removal rules, and extremist rules.

[1] P. Araújo, J. Hladký, E. K. Hng, and M. Šileikis. Prominent examples of flip processes. *Random Structures Algorithms*, 64(3):692–740, 2024.

[2] F. Garbe, J. Hladký, M. Šileikis, and F. Skerman. From flip processes to dynamical systems on graphons. *Ann. Inst. Henri Poincaré Probab. Stat.*, 2023. Accepted, available on journal website.

[3] E. K. Hng. Characterization of flip process rules with the same trajectories, 2023. arXiv.2305.19925.

Alternating paths in oriented graphs with large semidegree

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(This talk is based on joint work with Jozef Skokan.)

In new progress on conjectures of Stein, and Addario-Berry, Havet, Linhares Sales, Reed and Thomassé, we prove that every oriented graph with all in- and out-degrees greater than $5k/8$ contains an alternating path of length k. This improves on previous results of Klimošová and Stein, and Chen, Hou and Zhou.

Large Matchings in Multipartite Hypergraphs

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(This talk is based on joint work with Candida Bowtell.)

A common introductory exercise for students studying matchings in graphs is to prove the following statement: if G is a bipartite graph whose vertex classes A and B each have size n, with $\deg(x) \geq a$ for every $x \in A$ and $\deg(y) \geq b$ for every $y \in B$, then G admits a matching of size at least $\min(n, a+b)$.

We prove the natural counterpart of this result for k-partite k-uniform hypergraphs. Specifically, if H is a k-partite k-graph whose vertex classes $V_1, \ldots, V_k$ each have size n, and for each $i \in [k]$ we have $\deg(S) \geq a_i$ for each crossing $(k-1)$-tuple S of vertices of H which avoids V_i, then H admits a matching of size at least $\min(n-1, \sum_{i\in[k]} a_i)$. This answers a question asked recently by Han, Zang and Zhao, who proved the result in the case where at least two of the a_i are large, and who gave examples to show that for $k \geq 3$ the bound we prove is best possible in both terms (i.e. neither $n-1$ nor $\sum_{i\in[k]} a_i$ can be reduced).

As is typical of hypergraph matching problems, much more sophisticated methods are required than for the corresponding problem in graphs. Specifically, our proof is achieved by providing lower bounds on the size of largest rainbow matchings in families of k-partite k-graphs which satisfy a combination of minimum degree and multiplicity conditions.

Rank distributions of matrix representations of graphs over $\mathbb{F}_2$

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(This talk is based on joint work with Rachel Quinlan and Cian O'Brien.)

Over a finite field $\mathbb{F}$, the number of $n \times n$ matrices of rank r typically increases as r increases, $0 \leq r \leq n$. However, over the field of two elements $\mathbb{F}_2$, the most frequently occurring rank in $M_n(\mathbb{F}_2)$ is not n but $n-1$. The numbers of symmetric $\mathbb{F}_2$-matrices of rank n and $n-1$ coincide if n is odd and differ marginally if n is even. This opens the door to a more thorough investigation of the distribution of the matrix ranks over the field of two elements.

Let Γ be a simple undirected graph. A symmetric matrix $M(\Gamma)$ with entries in a field $\mathbb{F}$ represents Γ if the off-diagonal entries of $M(\Gamma)$ correspond to edges of Γ in the sense that $M_{ij}(\Gamma) \neq 0_{\mathbb{F}}$ if and only if x_i and x_j are adjacent in Γ. The diagonal entries of $M(\Gamma)$ are not subject to any conditions, and therefore there are many matrices representing Γ over $\mathbb{F}$. This project aims to identify and characterize simple graphs of order n with more $\mathbb{F}_2$-matrix representations of rank $n-1$ than rank n, a property rare over other finite fields.

We restrict our attention to graphs of order $n \geq 3$ with an induced sub-graph isomorphic to P_{n-1} or C_{n-1}. This poster presents results on the rank distributions of matrix representations of such graphs over $\mathbb{F}_2$.

Counting antichains in the Boolean lattice

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(This talk is based on joint work with Matthew Jenssen and Jinyoung Park.)

An old question of Dedekind asks for the number of antichains (monotone Boolean functions) in the Boolean lattice on n elements. After a long series of increasingly precise results, Korshunov determined this number up to a multiplicative factor of $(1 + o(1))$. We revisit Dedekind's problem and study the typical structure of antichains using tools from probability and statistical physics. This yields a number of results which include refinements of Korshunov's asymptotics, asymptotics for the number of antichains of a fixed size, and a 'sparse' version of Sperner's Theorem.

For example, we prove the following.

Theorem 1. *Let $\beta \in (0, 1)$ be a constant. The following is true as $n \to \infty$, n even.*

If $\beta > 3/4$, then almost all antichains of size $\beta\binom{n}{n/2}$ are entirely contained in $\binom{[n]}{n/2}$.

If $\beta < 3/4$, then almost no antichains of size $\beta\binom{n}{n/2}$ are entirely contained in $\binom{[n]}{n/2}$.

Along the way we also discuss some new variations and improvements on Sapozhenko's classical graph container method.

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Reconstructing point sets in $\mathbb{R}$ and $\mathbb{R}^d$ from randomly revealed pairwise distances

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(This talk is based on joint work with Douglas Barnes, Richard Montgomery, Rajko Nenadov, Jan Petr, Benedict Randall Shaw, Alan Sergeev, Tibor Szabó.)

We first consider the Erdős-Rényi random graph process $\{G_m\}_{m\geq 0}$ in which we start with an empty graph G_0 on the vertex set $[n]$, and in each step form G_i from G_{i-1} by adding one new edge chosen uniformly at random. Resolving a conjecture by Benjamini and Tzalik, we give a simple proof that w.h.p. as soon as G_m has minimum degree 2 it is globally rigid in the following sense: For any function $d\colon E(G_m) \to \mathbb{R}$, there exists at most one injective function $f\colon [n] \to \mathbb{R}$ (up to isometry) such that $d(ij) = |f(i) - f(j)|$ for every $ij \in E(G_m)$. We also resolve a related question of Girão, Illingworth, Michel, Powierski, and Scott in the sparser regime for the random graph.
In the second part of the talk, we let V be a set of n points in $\mathbb{R}^d$, and show that if the distance between each pair of points is revealed independently with probability $p > n^{-2/(d+4)}$, then we can reconstruct almost all of V up to isometry, with high probability. We do this by relating it to a polluted graph bootstrap percolation result, for which we adapt the methods of Balogh, Bollobás, and Morris.

Maximum degree of induced subgraphs of the Kneser graph

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(This talk is based on joint work with David Ellis, Ehud Friedgut, and Noam Lifshitz.)

For a graph G with independence number $\alpha(G)$, García-Marco and Knauer [2] defined its *sensitivity* $\sigma(G)$ to be the minimal maximum degree of any subgraph induced by $\alpha(G)+1$ vertices. The name comes from the Sensitivity Conjecture, whose graph-theoretic formulation proven by Huang [4] is $\sigma(Q_n) \geq \sqrt{n}$ for the n-dimensional hypercube graph Q_n. We consider the sensitivity of Kneser graphs.

For integers $n \geq k \geq 1$, the *Kneser graph* $K(n,k)$ is the graph with vertex-set consisting of all the k-element subsets of $[n]$, where two vertices are adjacent in $K(n,k)$ if the corresponding k-element subsets are disjoint. Using spectral methods, we show that if $n > 10000k$, then $K(n,k)$ has sensitivity at least

$$\left(1/2 - O\left(\sqrt{k/n}\right)\right)\binom{n-k-1}{k-1}.$$

This solves (up to a multiplicative constant) a problem of Gerbner, Lemons, Palmer, Patkós and Szécsi [3].

We also show that, if $n > 10000s^5k$, then the minimal maximum degree of an m-vertex induced subgraph of $K(n,k)$ exhibits s 'jumps' as m increases.

Previously, Frankl and Kupavskii [1] have shown similar results when n is at least a quadratic in k.

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A characterization of simultaneous optimization, majorization, and (bi-)submodular polyhedra

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Motivated by resource allocation problems (RAPs) in power management applications, we investigate the existence of solutions to optimization problems that simultaneously minimize the class of Schur-convex functions, also called least majorized elements. For this, we introduce a generalization of majorization and least majorized elements, called (a, b)-majorization and least (a, b)-majorized elements, and characterize the feasible sets of problems that have such elements in terms of base and (bi)submodular polyhedra. Hereby, we also obtain new characterizations of these polyhedra that extend classical characterizations in terms of optimal greedy algorithms from the 1970s. We discuss the implications of our results for RAPs in power management applications and derive a new characterization of convex cooperative games and new properties of optimal estimators of specific regularized regression problems. In general, our results highlight the combinatorial nature of simultaneously optimizing solutions and provide a theoretical explanation for why such solutions generally do not exist.

Sharp stability for the Brunn-Minkowski inequality for arbitrary sets

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(This talk is based on joint work with Alessio Figalli and Peter van Hintum.)

The Brunn-Minkowski inequality is a fundamental geometric inequality, closely related to the isoperimetric inequality. It states that for (open) sets A and B in $\mathbb{R}^d$, we have $|A+B|^{1/d} \geq |A|^{1/d} + |B|^{1/d}$. Here $A+B = \{x+y : x \in A, y \in B\}$. Equality holds if and only if A and B are convex and homothetic sets (one is a dilation of the other) in $\mathbb{R}^d$. The stability of the Brunn-Minkowski inequality is the principle that if we are *close* to equality, then A and B must be *close* to being convex and homothetic. Using a combinatorial approach, we prove a sharp stability result for the Brunn-Minkowski inequality, establishing the exact dependency between the two notions of closeness, thus concluding a long line of research on this problem.

The Critical Group of an Embedded Graph

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(This talk is based on joint work with Criel Merino, Iain Moffatt.)

Critical groups are finite Abelian groups associated with graphs. The order of the critical group of a connected graph is equal to its number of spanning trees, a fact equivalent to Kirchhoff's Matrix–Tree Theorem. They are well-established in combinatorics and arise in several ways such as chip firing, through the Laplacian and via fundamental cycles and cocycles.

We show that each of these three ways of defining the critical group of a graph may be lifted to embedded graphs and crucially all three yield isomorphic groups.

A characterization of g_2-minimal normal 3-pseudomanifolds with at most four singularities

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(This talk is based on joint work with Raju Kumar Gupta and Sourav Sarkar.)

The Lower Bound Theorem (LBT), established by Walkup, Kalai, and others in various specialized contexts, provides fundamental insight into the structure of d-dimensional normal pseudomanifolds. It asserts that for such pseudomanifolds Δ, the inequality $g_2(\Delta) := f_1(\Delta) - (d+1)f_0(\Delta) + \binom{d+2}{2} \geq 0$ holds, with equality only when Δ resembles a stacked sphere, particularly for dimensions $d \geq 3$.

Building upon the previous works, it is deduced that $g_2(\Delta) \geq g_2(\mathrm{lk}(v, \Delta))$ for any vertex v of a 3-dimensional normal pseudomanifold Δ. This insight motivates the exploration of pseudomanifolds with relatively minimal g_2, where $g_2(\Delta)$ equals $g_2(\mathrm{lk}(v, \Delta))$ for some vertex v of Δ.

In this talk, we explore the combinatorial properties of g_2-minimal 3-dimensional normal pseudomanifolds Δ with three or four singular vertices, including specific $\mathbb{RP}^2$-singularities. Singular vertices are those whose link is not a sphere. We extend previous results that characterized Δ with at most two singular vertices to include these cases. Specifically, we show that when Δ has three singular vertices, one of which is an $\mathbb{RP}^2$-singularity, or four singular vertices, with two being $\mathbb{RP}^2$-singularities, Δ can be constructed from a one-vertex suspension of a surface and certain boundary complexes of 4-simplices. Our proofs rely on combinatorial operations such as connected sums, vertex foldings, and edge foldings.

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Queens on the Torus

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Define a queen on $\mathbb{Z}_n^d$ with admissible moves parallel to $\mathbf{x} \in \{-1, 0, 1\}^d$ at arbitrary length. How many queens can be placed on $\mathbb{Z}_n^d$ without any two in conflict? In 1918, Pólya proved that n queens cannot be placed on $\mathbf{Z}_n^2$ if n is a multiple of 2 or 3. We prove that $n-1$ queens are impossible if n is a multiple of 3 or 4. While no more than n^{d-1} queens can be placed in d dimensions, we shall discuss how to find constructions of $(n - \mathcal{O}_d(1))n^{d-2}$ independent queens on $\mathbf{Z}_n^d$.

Hamilton decompositions of regular tripartite tournaments

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(This talk is based on joint work with J. Lada, V. Patel, Y. Pehova, J. Skokan.)

A regular k-partite tournament is a regular orientation of the balanced complete k-partite graph. Kühn and Osthus proved that, for all $k \geq 4$, any large enough regular k-partite tournament can be decomposed into edge-disjoint Hamilton cycles, and conjectured the same to be true for all $k \geq 2$. Granet proved the conjecture for the case of $k = 2$ (also known as Jackson's conjecture) and, somewhat surprisingly, provided a counterexample for the case of $k = 3$. Our main result is a proof that an approximate version of the conjecture still holds for $k = 3$, i.e. that for all $\varepsilon > 0$, any large enough regular tripartite tournament on n vertices contains edge-disjoint Hamilton cycles covering all but at most εn^2 edges. We also prove that if $\alpha > 1/3$, any large enough balanced regular tripartite digraph with minimum semidegree at least αn is Hamilton decomposable.

Ruzsa's discrete Brunn-Minkowski inequality and locality in sumsets

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(This talk is based on joint work with Peter Keevash and Marius Tiba.)

We explore higher dimensional additive phenomena in the integers proving optimal bounds in Freiman type results. As an application of the technical framework we prove a Brunn-Minkowski type inequality conjectured by Ruzsa asserting the following. For all $d, t, \varepsilon > 0$, there exists an $n = O_{d,t}(\varepsilon^{-1})$ so that for $A, B \subset \mathbb{Z}$ if B is not contained efficiently in n generalised arithmetic progressions of dimension $d-1$ and $t|A| \leq |B| \leq t^{-1}|A|$, then $|A+B|^{1/d} \geq |A|^{1/d} + (1-\varepsilon)|B|^{1/d}$.

On the Brouwer's conjecture for the sum of the Laplacian eigenvalues of a graph

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Let $G(V,E)$ be a simple graph of order n, size m and having the vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(G) = \{e_1, e_2, \ldots, e_m\}$. The adjacency matrix $A = (a_{ij})$ of G is a $(0,1)$-square matrix of order n whose (i,j)-entry is equal to 1 if v_i is adjacent to v_j and equal to 0, otherwise. Let $D(G) = diag(d_1, d_2, \ldots, d_n)$ be the diagonal matrix associated to G, where $d_i = \deg(v_i)$, for all $i = 1, 2, \ldots, n$. The matrix $L(G) = D(G) - A(G)$ is called the Laplacian matrix and its eigenvalues are called the Laplacian eigenvalues of the graph G. Let $0 = \mu_n \leq \mu_{n-1} \leq \cdots \leq \mu_1$ be the Laplacian eigenvalues of G and $S_k(G) = \sum\limits_{i=1}^{k} \mu_i$, $k = 1, 2, \ldots, n$ be the sum of k largest Laplacian eigenvalues of G. For any k, $k = 1, 2, \ldots, n$, A. Brouwer conjectured that $S_k(G) = \sum\limits_{i=1}^{k} \mu_i \leq m + \binom{k+1}{2}$. We discuss the bounds for $S_k(G)$ and the recent developments of the Brouwer's conjecture.

Recent developments on energy of graphs

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Let G be a simple graph of order n and with vertex set $V(G) = \{v_1, v_2, \dots, v_n\}$. The adjacency matrix $A = (a_{ij})$ of G is a $(0,1)$-square matrix of order n whose (i,j)-entry is equal to 1 if v_i is adjacent to v_j and equal to 0, otherwise. If $\lambda_1, \lambda_2, \dots, \lambda_n$ are the eigenvalues of A, the energy of G is defined as $E(G) = \sum_{i=1}^{n} |\lambda_i|$. As the energy of a given graph is used to approximate the π-electron energy of a molecule, the graph energy is closely related to the concepts of theoretical chemistry. We present recent developments on graph energy.