MTH5117 Mathematical writing: Coursework 1

Franco Vivaldi

DEADLINE: Sunday of week 3, at 23.55.

ASSESSED PROBLEMS [with allocated marks].
Problem 1: 5 [10]. Problem 2: 4 [10].
Problem 3: 2, 4, 5, 7 [40]. Problem 4: 1, 2 [40]

— — —

In assignments, you will often see the symbol [$\not$]. It indicates that the written material should contain *no mathematical symbols whatsoever,* apart from numerals. For instance, in the expression 'The function f is continuous', the function's name f counts as a mathematical symbol.

The variant [$\not$, n] indicates that, in addition, the assignment should consist of approximately n words (thus [$\not$, 100] means 80–120 words). If [$\not$] does not appear, mathematical symbols may be used freely.

Problem 1. The following expressions define sets. Turn words into symbols, using standard or Zermelo definitions (or any other symbolic representation).
[*Represent geometrical objects, e.g., planar curves, by their cartesian equations. A set of curves then becomes a set of equations.*]

1. The set of natural numbers with three decimal digits.
2. The set of points in the closed unit disc that are not in the centre.
3. The set of complex numbers with integer imaginary part.
4. The set of parabolas in the plane, symmetrical with respect to the ordinate axis.
5. The set of rational points in the open unit cube.
6. The set of circles in the plane, going through the origin.

Problem 2. The following expressions define sets. Turn symbols into words. Be concise. [$\not\in$]

1. $\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots\}$
2. $2\mathbb{Z} \setminus 4\mathbb{Z}$
3. $\{\sin(x) : x \in \mathbb{Z}\}$
4. $\{x \in \mathbb{R} \setminus \mathbb{Q} : x^2 \in \mathbb{N}\}$
   [*Think about it.*]
5. $\{(x, y) \in \mathbb{Z}^2 : \gcd(x, y) > 1\}$

Problem 3. For each item, provide two levels of description. [$\not\epsilon$]

> (i) a coarse description, which only identifies the class to which the item belongs (set, function, polynomial, etc.);
> (ii) a finer description, which defines the object in question, or characterises its structure.

[*Familiarity with a relevant part of the web-book is essential.*]

1. $y - x(x+1) = 0$
2. $(1, 1+x, 1+x+x^2, 1+x+x^2+x^3, \ldots)$
3. $\ddot{x} - 3\,\dot{x} - 2 = 0$
4. $13 = (3 + 2\sqrt{-1})(3 - 2\sqrt{-1})$
5. $f(A \cap B)$
6. $A \cup B = B \cup A$
7. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2 + x^k}$
8. $X + Y \stackrel{\nabla}{=} \{z : z = x + y,\, x \in X,\, y \in Y\}$

Problem 4. Explain, clearly and plainly.

1. I have a positive integer, and I must decide if it's prime. What shall I do? [$\not\epsilon$, 50]
2. I have a function given by a quadratic polynomial, and I must verify that this function assumes only positive values. What shall I do? [$\not\epsilon$, 50]