School of Mathematical Sciences



ESSENTIAL MATHEMATICS EXAMINATION

Duration: 2 hours. Calculators are NOT permitted.

To pass the exam, you need 12 correct answers.

Record each answer by ticking the corresponding box in the answer form.

- 1. Determine the number of primes lying between 110 and 130, end-points included
 - $[\mathbf{a}]$ 2
- [b]
- [e]not in the list

not in the list

- [c]4
- $[\mathbf{d}]$ 5

3

- 2. Compute the greatest common divisor of 30^2 and 168
 - [a]
- [b]
- 6
- [c]12
- $[\mathbf{d}]$ 36
- Determine the fractional part of $\frac{1848}{39}$ 3.
 - [a]
- [b] 39
- $[\mathbf{c}]$
- [d]13
- not in the list [e]

4. Evaluate

$$-\frac{7}{3} + \left\lceil \left(\frac{17}{34} + \frac{1}{3}\right) \left(\frac{6}{5}\right)^2 - \left(\frac{6}{5} - \frac{9}{12}\right) \left(\frac{4}{3}\right) \right\rceil \div \frac{9}{5}$$

[a] $-\frac{28}{9}$

[b] $-\frac{94}{75}$

[e] not in the list

 $[\mathbf{c}]$ -2

[d] $-\frac{4}{3}$

5. Estimate

$$x = \frac{40001}{20} \times \frac{1500}{2999}$$

[a] $10^2 < x < 10^3$

[b] $10^3 < x < 10^4$

[c] $10^4 < x < 10^5$

[d] $10^5 < x < 10^6$

[e] not in the list

6. Simplify

$$\left(\frac{x^2 z^{-2}}{(1/y)^3}\right) \left(-\frac{y^2 z}{x y^{-1}}\right)^{-3}$$

 $[\mathbf{a}] \qquad \frac{x^5}{z^5 y^6}$

[b] $-\frac{x^5}{z^5y^{12}}$

[e] not in the list

 $[\mathbf{c}] \qquad -\frac{x^5}{z^5 y^3}$

[d] $\frac{zy^1}{x}$

7. Compute the quotient of the following division

$$(x^4 - 4x + 1) \div (x - 2)$$

[a] $x^3 - 4x + 12$

[b] $x^3 + 2x^2$

[e] not in the list

[c] $x^3 - 2x^2$

[d] $x^3 - 2x^2 + 4x - 4$

8. When $x^2y + 2x^2 - 9y - 18$ is factored completely, one of the factors is

[a]
$$x+3$$

$$[\mathbf{b}] \qquad xy - 2x - 3y + 6$$

[c]
$$xy - 2x + 3y - 6$$

[d]
$$x^2 - 9$$

9. Add and simplify

$$\frac{1}{y-3} - \frac{y+4}{2y^2 - 5y - 3}$$

[a]
$$\frac{3y+5}{2y^2-5y-3}$$

[b]
$$\frac{3y^2 - 4y - 9}{(y - 3)(2y^2 - 5y - 3)}$$

e not in the list

- $[\mathbf{c}] \qquad \frac{-y-3}{-2y^2+6y}$
- $[\mathbf{d}] \qquad \frac{1}{1+2y}$

10. Simplify

$$\left(\frac{x}{a} - \frac{1}{ax}\right)^2 \left(-x - \frac{1}{x}\right)^2 - \left(\frac{1}{x^4} - 2\right) \frac{1}{a^2}.$$

[a]
$$\frac{x^6 - 4x^4 + 8x^2 - 4}{a^2x^2}$$

$$[\mathbf{b}] \qquad \frac{-4+x^4}{a^2}$$

[c]
$$\frac{x^4}{a^2}$$

$$[\mathbf{d}] \qquad \frac{x^4 + 4}{a^2}$$

[e] not in the list

11. Compute $f(-2z^3)$, where

$$f(x) = -x + \frac{1}{2}x^2 - \frac{1}{4}x^3$$

[a]
$$2z^6 + 2z^3 + 2z^5$$

[b]
$$2z^6 - 2z^9 - 2z^3$$

[c]
$$-2z^3 + z^6 - \frac{1}{2}z^9$$

[d]
$$z^9 + 2z^3 + 2z^6$$

[e] not in the list

12. Simplify, eliminating radicals at denominators

$$\frac{30}{\sqrt{75}} - \frac{1}{(\sqrt{3} - 2)^2}$$

[a]
$$-7 - 2\sqrt{3}$$

[a]
$$-7 - 2\sqrt{3}$$
 [b] $\frac{-1 + 14\sqrt{3}}{7}$

[c]
$$-\frac{1}{7} - 2\sqrt{3}$$

[d]
$$2\sqrt{3} - \frac{7 + 2\sqrt{3}}{37}$$

not in the list

13. Simplify

$$\frac{z^2}{\sqrt{z^2+z^4}}\sqrt{1-\frac{1}{z^4}}$$

[a]
$$\frac{1}{|z|} \frac{\sqrt{z^4 - 1}}{z^2 + 1}$$
[c]
$$\frac{1}{z} \frac{\sqrt{z^4 - 1}}{\sqrt{z^2 + 1}}$$

[b]
$$\sqrt{\frac{z^6 - z^4}{(z^2 + z^4)z^4}}$$
[d]
$$\frac{\sqrt{z^2 - 1}}{|z|}$$

[c]
$$\frac{1}{z} \frac{\sqrt{z^4 - 1}}{\sqrt{z^2 + 1}}$$

[d]
$$\frac{\sqrt{(z^2 + z^4)}}{\sqrt{z^2 - 1}}$$

not in the list

14. The equation

$$\frac{8}{3x} + \frac{20}{2x+8} = \frac{6x}{x^2}$$

has, precisely

- $[\mathbf{a}]$ three solutions
- [b] two irrational solutions
 - $[\mathbf{e}]$ not in the list

- [c] one integer solution
- $[\mathbf{d}]$ one non-integer solution

15. The equation $2\sqrt{y+4} = y+1$ has, precisely

- [a]no real solution
- [b] a single solution
- no solution [c] two integer solutions $[\mathbf{d}]$
- not in the list

End of examination paper