School of Mathematical Sciences



ESSENTIAL MATHEMATICS EXAMINATION

Duration: 2 hours. Calculators are NOT permitted.

To pass the exam, you need 12 correct answers.

Record each answer by ticking the corresponding box in the answer form.

- 1. Factor 378 into primes
 - [a] $2 \cdot 189$
- [b] $2 \cdot 3 \cdot 63$
- [e] not in the list

- [c] $2^3 \cdot 47$
- [d] $2^3 \cdot 3^2 \cdot 7$
- **2.** Determine the least common multiple x of 24 and 126
 - [a] $100 \le x < 250$
- [b] $250 \le x < 500$
- [e] not in the list

- [c] $500 \le x < 1000$
- [d] $1000 \le x < 3000$
- 3. Determine the integer nearest to $\frac{1723}{13}$
 - [a] 121
- [b] 122
- [e] not in the list

- [c] 131
- [d] 132

4. Evaluate

$$\frac{5}{30} - \frac{1}{7} \times \left[\left(4 - \frac{18}{27} \right) \div \frac{8}{3} + \left(\frac{3}{8} \right)^2 \times \left(\frac{7}{4} \times \frac{7}{9} + \frac{5}{12} \right) \right]$$

[a] $\frac{19}{27}$

[b] $-\frac{67}{72}$

[e] not in the list

[c] $-\frac{1}{21}$

[d] $\frac{5}{42}$

 $5. Estimate x = 7\sqrt{3} + 1$

[a] 11 < x < 12

[b] 12 < x < 13

[e] not in the list

[c] 13 < x < 14

[d] 14 < x < 15

6. Simplify

$$\left(\frac{-a\,b^2c^3}{c\,b^3}\right)^3\left(\frac{a^5}{-b^7c}\right)^{-2}$$

[a] $-\frac{a^{13}c^4}{b^{17}}$

[b] $\frac{a^{13}c^4}{b^{17}}$

[e] not in the list

[c] $-\frac{b^{11}c^8}{a^7}$

[d] $-\frac{b^{12}c^7}{a^4}$

7. Compute the remainder of the following division

$$(-x^4 + 3x^2 + 2x - 1) \div (x^2 + 2)$$

[a] 2x - 3

[b] 2x + 11

[e] not in the list

 $[\mathbf{c}] \qquad 3x + 2$

[d] 2x - 11

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When $9y^2 - x^2y^2 - 9z^2 + (xz)^2$ is factored completely, one of the factors is 8.

not in the list

 $[\mathbf{c}]$ 3-y [d]3+y

9. Add and simplify

$$\frac{1}{y^2 - 2y - 15} + \frac{3}{y^2 - 10y + 25}$$

[a] $\frac{4y+4}{(y+3)(y-5)^2}$ [b] $\frac{3y+10}{(y+3)(y-5)}$

not in the list [e]

[c] $\frac{4y-2}{(y+3)(y-5)^2}$ [d] $\frac{4y-14}{(y+3)(y-5)^2}$

10. Simplify

$$-\frac{2}{3 x^2} \left[\left(x^2 - \frac{1}{3} y \right)^2 - \frac{1}{9} y^2 \right]^2 - \frac{1}{9} x^2 y \left(8x^2 - \frac{8}{3} y \right).$$

[a] $\frac{2}{3}x^6$

[b] $-\frac{2}{3}x^6$

[c] $-\frac{2}{9}x^4(3x^2+8y)$

[d] $\frac{2}{9}x^4(3x^2 - 8y)$

not in the list [e]

Compute $f\left(-\frac{1}{a^3}\right)$, where 11.

$$f(x) = \frac{x^2 - x - 1}{x - 1}$$

[a] $\frac{1 - a^3 - a^6}{a^6 - a^3}$

[b] $\frac{1+a^3-a^6}{a^3-1}$

[c] $\frac{1-a^3-a^6}{1+a^3}$

[d] $\frac{-1-a^3+a^6}{a^3(a^3+1)}$

not in the list

$$\frac{1}{\sqrt{5}} \, \frac{\sqrt{30} - \sqrt{12} \, \sqrt{15}}{(\sqrt{2} - \sqrt{3})^2}$$

to the form $m + n\sqrt{d}$, where m, n and d are integers.

[a]
$$-42 - 7\sqrt{6}$$

[b]
$$-18 - 7\sqrt{6}$$

[c]
$$6 + \sqrt{30}$$

[d]
$$18 - \sqrt{12}$$

13. Simplify
$$\sqrt{x^2 - x^3} - \sqrt{4 - 4x}$$

[a]
$$(x-2)\sqrt{1-x}$$

[b]
$$\sqrt{x^2(1-x)-4(1-x)}$$

$$[\mathbf{c}] \qquad (|x|-2)\sqrt{1-x}$$

$$[\mathbf{d}] \qquad |x+2|\sqrt{1-x}$$

$$\frac{2(3-x)}{3} - \frac{5(1-2x)}{6} < \frac{4-2x}{2} + 1.$$

$$[\mathbf{a}] \qquad \frac{11}{12} > z$$

[a]
$$\frac{11}{12} > x$$
 [b] $x > -\frac{11}{6}$

$$[\mathbf{c}] \qquad \frac{1}{2} < x$$

[c]
$$\frac{1}{2} < x$$
 [d] $-x < \frac{33}{24}$

$$4x - \frac{x^2 - 9}{x + 3} = \frac{6}{3x - 1}.$$

[a]
$$\frac{2 \pm \sqrt{7}}{3}$$

[b]
$$\frac{-1 \pm \sqrt{10}}{3}$$

[c]
$$\frac{7 \pm \sqrt{9}}{15}$$

[d]
$$\frac{2}{3}$$
 and $-\frac{4}{3}$

$$[e]$$
 not in the list

End of examination paper