School of Mathematical Sciences



ESSENTIAL MATHEMATICS EXAMINATION

Duration: 2 hours. To pass the exam, you need 12 correct answers.

Record each answer by ticking the corresponding box in the answer form.

Calculators are NOT permitted in this examination.

- 1. Determine the number of primes lying between 89 and 109, end-points included
 - $[\mathbf{a}]$
- [b]
- $[\mathbf{e}]$ not in the list

- [c]
- $[\mathbf{d}]$

5

- 2. Determine the greatest common divisor x of 100^3 and 880
 - $[\mathbf{a}]$ $10 \le x < 25$
- [b] $25 \le x < 50$
- [e] not in the list

- $50 \le x < 75$ [c]
- [d] $75 \le x < 150$
- Determine the fractional part of $\frac{2019}{11}$ 3.
 - $\frac{9}{11}$ [a]
- $\frac{6}{11}$
- not in the list

- [c]
- [d]

4. Evaluate

$$\left[-\frac{16}{3} \times \left(2 + \frac{3}{5} \right) \times \left(-\frac{1}{2} \right)^3 - \left(\frac{1}{2} - \frac{1}{3} \right) \right] \times \frac{21}{3} \times \left(-\frac{2}{47} \right) \div \left(\frac{2}{3} - 2 \right)$$

- [a]
- [b] $\frac{4}{47}$
- [e] not in the list

[c] $\frac{11}{47}$

[d] $\frac{7}{20}$

5. Estimate

$$x = \frac{10^{-3} \times 999}{8001^2} \times 2^6$$

- [a] $10^{-3} < x < 10^{-2}$
- $[\mathbf{b}] \qquad 10^{-4} < x < 10^{-3}$
- [c] $10^{-5} < x < 10^{-4}$
- [d] $10^{-6} < x < 10^{-5}$
- [e] not in the list

6. Simplify

$$\frac{(-1)^2}{(-x)^{10}} \left(\frac{(-1/y)^3}{-x^{-3}}\right)^3$$

- [a] $\frac{1}{xy^9}$
- $[\mathbf{b}] \qquad -\frac{1}{xy^9}$
- [e] not in the list

- [c] $\frac{1}{x^{19}y^9}$
- [d] $-\frac{1}{x^4y^6}$

7. Compute the quotient of the following division

$$(4y^5 - y^4 + y^2 + 1) \div (y^3 + y^2 - 3)$$

- [a] $4y^2 5y + 5$
- [b] $4y^2 3y + 3$
- [e] not in the list

- [c] $4y^2 3y + 5$
- [d] $4y^2 5y 3$

2

8. When $35a^5b + 21a^3b^2 - 10a^6 - 6a^4b$ is factored completely, one of these factors is

[a]
$$7b + 2a$$

[b]
$$5a^2 + 3b$$

[e] not in the list

[c]
$$-2a^2 + 7b$$

[d]
$$7a^2 + 2b$$

9. Simplify

$$\frac{2 - 14x}{1 - 14x + 49x^2} - \frac{4 - x^2}{7x - 1}$$

$$[\mathbf{a}] \qquad -\frac{x^2+6}{7x-1}$$

$$[\mathbf{b}] \qquad \frac{x^2 - 2}{1 - 7x}$$

[c]
$$\frac{x^2-6}{7x-1}$$

[d]
$$-\frac{x^2+2}{1-7x}$$

not in the list

10. Simplify

$$\left(\frac{a}{3} - \frac{b}{9}\right) \left[\frac{1}{4} \left(3a - \frac{2}{3}b \right)^2 + \left(\frac{3}{2}a + \frac{1}{3}b \right)^2 + \left(3a - \frac{2}{3}b \right) \left(\frac{3}{2}a + \frac{1}{3}b \right) \right]$$

$$[\mathbf{a}] \qquad (3a+b)a^2$$

[b]
$$3a^3 - \frac{1}{3}a^2b - \frac{2}{9}ab^2$$

[c]
$$3a^3 - a^2b$$

[d]
$$(3a-b)\left(a^2+\frac{4}{9}b^2\right)$$

11. Compute $f(-x/y^2)$, where

$$f(\beta) = \frac{1}{-\beta + \frac{1}{\beta^3 - 3}}$$

[a]
$$-\frac{y^2(x^3+3y^6)}{x^4+3xy^6+y^8}$$

[b]
$$\frac{y^2(x^3 - 3y^6)}{x^4 - 3xy^6 - y^8}$$

[c]
$$\frac{y^2(x^3+3y^6)}{-x^4+3xy^6-y^8}$$

[b]
$$\frac{y^2(x^3 - 3y^6)}{x^4 - 3xy^6 - y^8}$$
[d]
$$\frac{y^2(x^3 + 3y^6)}{x^4 + 3xy^6 - y^8}$$

not in the list [e]

12. Simplify, eliminating radicals at denominator

$$\left(\sqrt{20} - \sqrt{\frac{49}{5}}\right) \frac{4}{3 + \sqrt{5}}$$

[a]
$$-3 + \frac{9}{5}\sqrt{5}$$

[a]
$$-3 + \frac{9}{5}\sqrt{5}$$
 [b] $\frac{15 + 9\sqrt{5}}{5}$

[c]
$$17 + \frac{51}{5}\sqrt{5}$$

[c]
$$17 + \frac{51}{5}\sqrt{5}$$
 [d] $\frac{-75 + 9\sqrt{5}}{5}$

not in the list

13. Simplify, eliminating radicals at denominator (assuming all radicands positive)

$$\sqrt{a^{-1} - 4a^3} \sqrt{(1 + 2a^2)^{-1}}$$

[a]
$$\frac{\sqrt{a}\sqrt{1+2a^2}}{a}$$
 [b] $\frac{\sqrt{a(1-2a^2)}}{a}$ [c] $\frac{\sqrt{a(1-4a^4)}}{1+2a^2}$ [d] $\frac{\sqrt{a(1-4a^2)}}{a}$

$$[\mathbf{b}] \qquad \frac{\sqrt{a(1-2a^2)}}{a}$$

[c]
$$\frac{\sqrt{a(1-4a^4)}}{1+2a^2}$$

$$[\mathbf{d}] \qquad \frac{\sqrt{a(1-4a^2)}}{a}$$

[e] not in the list

14. Solve

$$\left(k + \frac{1}{2}\right)^2 - (k - 1)^2 - 6k < 5k - 3$$

[a]
$$k > \frac{9}{32}$$

$$\mathbf{p}] \qquad k > \frac{3}{16}$$

not in the list

[a]
$$k > \frac{9}{32}$$
 [b] $k > \frac{3}{16}$ [c] $k < -\frac{15}{32}$ [d] $k > \frac{17}{40}$

[d]
$$k > \frac{17}{40}$$

15. Find all solutions of

$$\frac{1-x}{3\sqrt{x-2}} = 1$$

$$[\mathbf{a}] \qquad \frac{11 + 3\sqrt{5}}{2}$$

$$[\mathbf{b}] \qquad \frac{11 - 3\sqrt{5}}{2}$$

$$[\mathbf{d}] \qquad \frac{7 \pm 3\sqrt{3}}{2}$$

not in the list

End of examination paper