MAS111 Convergence and Continuity

Sample of in-term test questions. In the test there are 10 questions. For further info, see the course's web page.

Some questions consist of three propositions, labeled 1), 2), 3), which are either *True* or *False.* The notation T,F,F in an answer means:

1) is *True,* 2) is *False,* 3) is *False*

etc.

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1. True or false?

1) $\forall x \in \mathbf{R} \quad x^4 \geq 0$
2) $\exists r \in \mathbf{Q}$ s.t. $r = r^2 + 1$
3) $\forall y \in \mathbf{R} \quad y^2 \neq y - 1$

[a] T, T, F **[b]** T, F, F **[c]** T, F, F **[d]** F, T, T **[e]** not in the list

2. True or false?

1) $\forall y \in \mathbf{R} \quad y^4 \geq y^2$
2) $\exists s \in \mathbf{Q}$ s.t. $6s^2 = s + 1$
3) $\forall y \in \mathbf{R} \quad -y^2 \neq 2y + 5$

[a] F, F, T **[b]** T, T, F **[c]** T, F, F **[d]** F, T, T **[e]** not in the list

3. True or false?

1) $\forall y \in \mathbf{N} \quad y^2 = y \Longrightarrow y = 1$
2) $\exists y \in \mathbf{R}$ s.t. $y^3 < -1$
3) $\forall a \in \mathbf{Z} \quad a^2 < 4 \Longrightarrow a < 2$

[a] T, T, F **[b]** T, F, T **[c]** T, F, F **[d]** F, T, T **[e]** not in the list

4. True or false?

1) $\forall a, b, c \in \mathbf{R} \quad |a+b+c| \leq |a|+|b|+|c|$
2) $\exists x, y \in \mathbf{R}$ s.t. $||x|-|y|| > |x-y|$
3) $\forall x, y \in \mathbf{R} \quad |x-y| < 3 \iff y-3 < x < y+3$

[a] T, T, F **[b]** T, F, T **[c]** T, F, F **[d]** F, T, T **[e]** not in the list

5. True or false?

1) $\forall a \in \mathbf{N} \quad a^2 < 4 \iff a < 2$
2) $\forall n \in \mathbf{Z} \quad n = n+1 \Longrightarrow n = 0$
3) $\forall r \in \mathbf{R} \quad r^2 \leq 2r^2 \iff r = 0$

[a] T, F, F **[b]** T, T, T **[c]** F, T, T **[d]** T, T, F **[e]** not in the list

6. True or false?

1) $\forall r \in \mathbf{Q}\, \exists b \in \mathbf{N}$ s.t. $\forall m \in \mathbf{Z} \quad mrb \in \mathbf{Z}$
2) $\forall r \in \mathbf{R}\, \exists b \in \mathbf{Q}$ s.t. $rb \in \mathbf{Q}$
3) $\forall x, y \in \mathbf{Q}\, \exists z \in \mathbf{Q}$ s.t. $x = yz$

[a] F, F, T **[b]** F, T, T **[c]** T, T, F **[d]** T, F, F **[e]** not in the list

7. Let $n \in \mathbf{N}$. True or false?

1) $\displaystyle\prod_{k=1}^{n}(1+k^2) \geq 2^n$
2) $\displaystyle\sum_{k=0}^{n} q^k = \frac{q^{n+1}-1}{1-q}$
3) $\displaystyle\sum_{i=0}^{n} \frac{1}{3^i} = \frac{3-3^{n+1}}{2 \cdot 3^{n+1}}$

[a] T, T, F **[b]** F, F, T **[c]** F, F, F **[d]** F, T, T **[e]** not in the list

8. Let $n \in \mathbf{N}$. True or false?

1) $\binom{100}{40} = \binom{100}{60}$

2) $\binom{n}{n+1} = \binom{n}{-1}$

3) $\binom{2n}{n} > 1$

[**a**] F, F, F [**b**] T, T, F [**c**] F, T, F [**d**] T, T, T [**e**] not in the list

9. Let $n \in \mathbf{N}$. True or false?

1) $\prod_{k=1}^{n} k(n-k+1) = (n!)^2$

2) $\prod_{k=1}^{n} k \leq \left(\frac{1}{n}\sum_{k=1}^{n} k\right)^n$

3) $\prod_{k=1}^{n} 2 \leq 2^n$

[**a**] T, T, F [**b**] T, F, F [**c**] T, F, T [**d**] F, F, T [**e**] not in the list

10. True or false?

1) $\lim_{n\to\infty} \frac{a+n}{b+n} = \frac{a}{b} \quad a, b, \in \mathbf{N}$

2) $\lim_{n\to\infty} \frac{n^2}{1+n^3} = 0$

3) $\lim_{n\to\infty} \left(\frac{n}{2} + \frac{1-n^2}{2n}\right) = 1$

[**a**] T, T, F [**b**] T, F, F [**c**] F, F, T [**d**] F, T, T [**e**] not in the list

11. True or false?

1) $\lim_{n\to\infty} \frac{n^2}{1+n^n} = 0$

2) $\lim_{n\to\infty} \frac{1}{1+\sqrt{n}} = 0$

3) $\lim_{n\to\infty} \frac{\sqrt{n}}{n+2} = 0$

[a] T, T, F [b] T, F, F [e] not in the list

[c] F, F, T [d] F, T, T

12. Compute

$$\lim_{n\to\infty}\left(\frac{2-3n^3}{4+6n^2}+\frac{1}{2}n\right)$$

[a] 0 [b] 1 [e] not in the list

[c] $\frac{1}{2}$ [d] 3

13. Compute

$$\lim_{n\to\infty}\frac{1}{(2n)!}\binom{2n}{n}$$

[a] ∞ [b] 0 [e] not in the list

[c] $\frac{1}{2}$ [d] 1

14. Compute

$$\lim_{n\to\infty}\frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n+2}+\sqrt{n}}$$

[a] 2 [b] 0 [e] not in the list

[c] $\frac{1}{2}$ [d] 1

15. Compute

$$\lim_{n\to\infty}\frac{3-\sqrt{n}}{\sqrt{n}+2}$$

[a] -1 [b] 0 [e] not in the list

[c] $\frac{3}{2}$ [d] 2

16. Compute

$$\lim_{n\to\infty} \frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n}}$$

[**a**] -2 [**b**] 0 [**e**] not in the list

[**c**] $\frac{1}{2}$ [**d**] -1

17. Compute

$$\lim_{n\to\infty} \left(\frac{n^2}{5-3n} + \frac{n}{3} \right)$$

[**a**] 2 [**b**] 0 [**e**] not in the list

[**c**] $\frac{1}{3}$ [**d**] 1

18. Compute

$$\lim_{n\to\infty} \frac{n}{n+\cfrac{1}{n+\cfrac{1}{n}}}$$

[**a**] 2 [**b**] 0 [**e**] not in the list

[**c**] $\frac{1}{2}$ [**d**] 1

19. Compute

$$\lim_{n\to\infty} \sum_{k=0}^{5} \frac{(-1)^k}{n^k}$$

[**a**] -1 [**b**] 0 [**e**] not in the list

[**c**] $\frac{1}{5}$ [**d**] 1

20. Let (a_n) be a real sequence. True or false?

1) (a_n) does not converge $\Longrightarrow$ (a_n) is not bounded
2) (a_n) converges $\Longrightarrow$ $(1/a_n)$ converges
3) (a_n) converges $\Longrightarrow$ (a_n^2) converges

[a] T, F, F **[b]** F, F, T **[c]** T, T, F **[d]** F, T, T **[e]** not in the list

21. Let (a_n) be a real sequence. True or false?

1) (a_n^2) converges $\Longrightarrow$ (a_n) converges
2) (a_{2n}) converges $\Longrightarrow$ (a_n) converges
3) (a_n) is not bounded $\Longrightarrow$ $(1/a_n)$ converges

[a] T, T, F **[b]** T, F, F **[c]** F, T, T **[d]** F, F, T **[e]** not in the list

22. Let (a_n) be a real sequence. True or false?

1) (a_n) converges $\Longrightarrow$ (a_{2n}) converges
2) (a_n) converges $\Longrightarrow$ $(a_n - a_n^2)$ converges
3) (a_n) does not converge $\Longrightarrow$ $(a_n - a_n^2)$ does not converge

[a] T, T, F **[b]** T, F, F **[c]** F, F, T **[d]** F, T, T **[e]** not in the list

23. Let (a_n) and (b_n) be real sequences. True or false?

1) $(a_n + b_n)$ converges $\Longrightarrow$ (a_n) converges
2) (a_n) converges $\Longrightarrow$ $(a_n + b_n)$ converges
3) (a_n) and (b_n) converge $\Longrightarrow$ $(a_n b_n^2)$ converges

[a] T, T, F **[b]** F, F, T **[c]** T, F, F **[d]** F, T, T **[e]** not in the list