cwork8b.tex 27/11/2003

MAS/202 Algorithmic Mathematics: Coursework 8

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DEADLINE: Wednesday of week 10, at 12:00 pm.

CONTENT: vectors

Problem 1. Let $w_1 = (1, 1, 0)$, $w_2 = (2, 1, 2)$ be vectors in $(\mathbb{Z}/(3))^3$, and let $\mathcal{W} = \langle w_1, w_2 \rangle$.

Write down all the elements of $\mathcal{W}$; for each of them determine the leading index and —if appropriate— the leading term.

Problem 2. Write an algorithm to the following specifications

`Algorithm ldindx`
`INPUT`: $v = (v_1, \ldots, v_n)$, an n-dimensional vector.
`OUTPUT`: l, where l is the leading index of v.

Problem 3. Let $v, w_1, w_2, w_3 \in \mathbb{Q}^3$, with

$$v = (4/7, -3/5, 5/7); \quad w_1 = (1, 1/2, 1), \quad w_2 = (0, 1, 2), \quad w_3 = (0, 0, 1).$$

(*a*) Trace `Sift`$(v, (w_1, w_2, w_3))$.

(*b*) Write v as a linear combination of w_1, w_2, w_3, verifying your calculation explicitly.

Problem 4. Let $F = \mathbb{Z}/(11)$, and let

$$w_1 = (1, 0, 7, 3) \qquad w_2 = (0, 1, 3, 4) \qquad w_3 = (0, 0, 0, 1) \in F^4.$$

Use the algorithm `Sift` to prove that $(5, 2, 4, 3) \notin \langle w_1, w_2, w_3 \rangle$.

Problem 5. Consider the following algorithm

`Algorithm Triangle`

`INPUT`: $a, b, c \in \mathbb{Q}^2$.

`OUTPUT`: TRUE, if a, b, c are vertices of an equilateral triangle,
FALSE otherwise.

(a) Write the algorithm, assuming $a = (a_1, a_2)$, etc. The algorithm must perform only rational operations, i.e., no square roots.
[*Hint:* the vertices are not necessarily distinct.]

(b) Describe the algorithm. [$\not\epsilon$, 50]

(c) Will the algorithm return the value TRUE for some input?
[*Hint:* translate one vertex to the origin, then use trigonometry.]