MAS/202 Algorithmic Mathematics: Coursework 6

Franco Vivaldi

DEADLINE: Wednesday of week 8, at 12:00 pm.

CONTENT: Recursive algorithms

MicroESSAY: Write an essay on recursive algorithms. [$\not{e}$, 50]

Problem 1. Apply the algorithm `GCD`, to determine a greatest common divisor of 11147 and 3763.

Problem 2. Let $F = \mathbb{Z}/(5)$, and let $c = x^4 + 2x^3 + 3x^2 + 2x + 2$ and $d = 2x^2 + 3$ be polynomials in $F[x]$.

(a) Apply the algorithm `ExtendedGCD` to determine $g, s, t \in F[x]$, such that g is a gcd of c and d, and $g = sc + td$.

(b) Verify the validity of the equation $g = sc + td$ in this case.

Problem 3. Consider the following algorithm

```
Algorithm M
INPUT: x, y ∈ ℕ
OUTPUT: ??
if x = 0 then
   return 0;
else
  if x ≤ y then
     return y + M(x − 1, y);
  else
     return  M(y, x);
  fi;
fi;
end;
```

(a) Trace M$(7, 3)$. (There should be 5 calls to the algorithm.)

(b) Explain in one sentence what this algorithm does. [¢]

(c) Explain what happens if the boolean expression $x \leq y$ is replaced by $x < y$.

(d) Write a non-recursive version of the above algorithm.
(The algorithm should perform an analogous computation using a loop, and produce the same output. In particular, no multiplications are allowed.)

Problem 4*. Find two integers a, b, with $0 \leq b < a$ such that the recursive computation of GCD(a, b) involves 10 calls to the function GCD. Explain what you are doing. (The smaller the value of a, the higher the mark.)

[*Hint:* start from the last call to GCD, and work your way backward, minimizing the size of the first argument of the previous call.]