cwork1b.tex 7/10/2003

MAS/202 Algorithmic Mathematics: Coursework 1

Franco Vivaldi

DEADLINE: Wednesday of week 3, at 12:00 pm.

CONTENT: Basics. Divisibility.

When writing essays, check grammar and spelling. The symbol $[\not\in, n]$ indicates that the essay should contain no mathematical symbols whatsoever (apart from numerals), and should consist of approximately n words. The length of the essay is not always specified.

MicroESSAY: Write an essay on boolean calculus. $[\not\in, 100]$

Problem 1. Use a table to trace the values of the variables x, y, and the boolean expressions $|x| \geq |y|$, and 'x is even', as the following statement sequence is executed:

```
x := 3;
y := 0;
while |x| ≥ |y| do
    if x is even then
        x := x/2 − y;
    else
        x := (x + 1)/2 − y;
        y := y − x;
    fi;
od;
```

(This was a 2001 final examination question. The table should have 4 columns and 4 rows. The correct answers were 30%; most students did the first two rows correctly, half of them made a mistake in row 3.)

Problem 2. Consider the following algorithm

```
Algorithm A
INPUT: a, b ∈ ℤ.
OUTPUT: ??
if NOT a > b then
   c := a;
   a := b;
   b := c;
fi
x := 0;
while a > b then
   x := x + 1;
   a := a − 1;
od
return x;
end;
```

(a) Determine $\mathtt{A}(-2,3)$, by tracing the algorithm.

(b) Write the output specifications, using symbols only. [*Hint:* compute $\mathtt{A}(a,b)$ for various values of a and b.]

(c) Describe input and output specifications in one sentence [$\not\in$].

Problem 3.

(a) Write an algorithm to the following specifications:

```
Algorithm Min3Int
INPUT: a, b, c ∈ ℤ.
OUTPUT: m, where m is the minimum of a, b, c.
```

(b) Explain in one sentence [$\not\in$] what goes wrong in your algorithm if the input specification is changed to `INPUT:` $a,b,c \in \mathbb{C}$, while leaving everything else unchanged. [*Hint:* there is only one place to look.]

Problem 4. Consider the following theorem

Theorem. *Let A and B be two finite sets with the same cardinality, and let $f : A \to B$ be a function. Then if f is injective, it is invertible.*

(a) Several mathematical terms appear in the formulation of this theorem. Select the three terms that you judge to be the most important, and give of each the precise definition.

(b) By means of a counterexample, show that the above theorem does not hold if the sets in question have different cardinality.
[*Hint:* make A, B and f *very* simple.]

(c) Explain in one sentence the meaning of the word *counterexample*. [$\not\in$]

Problem 5.

(a) Using the operators DIV and MOD, write an algorithm to the following specifications:

`Algorithm Nmul`
`INPUT`: $a, b \in \mathbb{Z}$, $b \neq 0$.
`OUTPUT`: c, where c is a multiple of b which is closest to a.

(Start with the case $a, b > 0$; if you find the general case difficult, deal with this case only.)

(b) Characterize the set of values of a and b for which such multiple is not unique (i.e., for which the expression 'a multiple of b' cannot be replaced by '*the* multiple of b').