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Beyond templates 2: exploring students' approaches to learning on a mathematical writing course

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This is the second of two articles on Developing Students' Learning through Mathematical Writing (a Maths, Stats & OR Network (MSOR Network) funded mini-project) at Queen Mary, University of London (QMUL). These articles report on the Mathematical Writing (MW) course at QMUL, devised by Professor Franco Vivaldi. The investigation into student learning on the MW course was carried out by staff in the Thinking Writing (TW) initiative at QMUL which works to develop the teaching and assessment of writing (see http://www.thinkingwriting.qmul.ac.uk/).

In this article, we begin by describing the MW course (see MW course page - http://www.maths.qmul.ac.uk/~fv/teaching/mw/). We then discuss the aim of our study and research methodology. The rest of the article focuses on a case study of one student's learning on the course. We explore this case in detail in order to explicate our interpretation of the data.

Details of the course

The Mathematical Writing (MW) course at QMUL aims to move students away from a reliance on using templates to solve mathematical problems – a practice typical of A level and 1st year university studies - towards a deeper understanding of underlying mathematical concepts. It is also essential preparation for any student who wants to do a final year project. At the time of the study, the MW course was delivered to a cohort of 71 students.

The course consisted of:

- Three lectures a week – the third lecture (on Fridays) gave feedback on the weekly coursework;
- A web book – which covers the course syllabus in detail. (see http://www.maths.qmul.ac.uk/~fv/books/mw/mwbook.pdf);
- Weekly exercise classes – with help from the professor and post graduate teaching assistants (PGTAs)
- Weekly coursework – which can be viewed at http://www.maths.qmul.ac.uk/~fv/teaching/mw/ ; and,
- Professor's office hours – where students could come along with queries.

The coursework was handed in on Friday, and marked and returned to students the following Thursday. The next day, in the Friday lecture, the professor went over the coursework and explained possible solutions. The writing tasks engaged students in explaining mathematical concepts and underlying theory. Examples of writing tasks are:

- explaining a concept e.g. prime numbers;
- translating mathematical words into symbols, and vice-versa;
- improving an unclear or faulty proof; and,
- writing a summary of a mathematical text.

Aim of study

The aim of this study was to investigate student learning on the MW course. We reasoned that if students engaged with mathematical concepts and attempted to understand mathematical symbols and notation, they would be taking a deep approach to learning. Hence we drew on the literature on approaches to learning in designing the study [1, 2, 3].

Research methodology

We wanted to find out whether the MW course helped students to move from using templates to solve problems and whether it helped them engage with mathematical theory. In terms of the literature on approaches to learning this would mean moving from a surface approach, where students show a 'failure to distinguish principles from practice' [3] to a deep approach to learning, where students engage with and attempt to understand the underlying theory.

To do this we carried out a small scale study. We followed the principles of naturalistic inquiry [4,5,6] and collected data through ethnographic observations of lectures and exercise classes, informal discussion with students in the exercise class, a focus group, stimulated recall interviews with a small group of students (2 one hour interviews with each student), interviews with the PGTAs (who mark work and help with student queries in the exercise class), and an analysis of students' written work.

The case study

In this section we present a brief case study of one student's account of his learning. This case study presents:

- a brief biography of the student;
- the MW task and the student's response;
- an extract from the interview data where the student explains his thinking; and,
- the researchers' interpretation of the data.

The student's written task and his explanation of how he approached the task are given in detail so that the reader can develop their own interpretation of the data.

Brief biography

Saifullah is British born and educated and his first language is English. He attended a competitive entry grammar school where he took his GCSEs, and then he went on to an FE College to take his A levels. He enjoys studying languages; he studied French and German at school and he speaks Bengali. At school he was always good at maths and also German and his goal is to become a fluent German speaker; hence he chose to study Maths and German at university. He likes this combination and likes being able to switch from mathematical work to language work. His average mark on the MW course is a first.

Maths as a language

Saifullah's language background appears to influence his view of mathematics. Indeed he reported that as a result of the MW course he viewed maths as a language and realised that he needed to translate mathematical symbols and notation into English (in much the same way as he would translate from a foreign language).

"'I'm looking at maths like a language, that's what I've noticed, and I never saw that before [. . .] it makes a lot more sense to me now, because I'm learning the language of maths itself, I'm learning that, I'm becoming more fluent in the language of maths."

The MW task

This task was set in coursework 6 (see http://www.maths.qmul.ac.uk/~fv/teaching/mw/ for all of the MW coursework) so students already had some experience of coursework and feedback on their work.

The mathematical writing task

You are given four distinct complex numbers. How do you decide whether or not these numbers lie at the vertices of a square in the complex plane?

[Do not use symbols or mathematical notation in your answer.]

Most students on the course found writing a mathematical problem without using symbols or mathematical notation in their answers challenging. Saifullah's language background seemed to be an advantage. This was his response to the task:

'A square is a quadrilateral of four equal sides with a right-angle at every vertice [vertex]. This can be used to decipher whether or not four distinct complex numbers lie at the vertices of a square on the complex plane. One monotonous method of solving this would be to calculate the distance from each of the four points to the other three points. If the four points do represent the vertice [vertices] of a square, then for each point you should find two distances which are equal and one distance which is a multiple of the square root of two of the two equal distances. Also, these values would be the same for each of the four points. If not, they do not lie at the vertice [vertices] of a square.'

Saifullah's explanation (interview data)

Saifullah pointed out that his method of solving this problem was 'monotonous' and he was aware that there was a more elegant way of solving the problem. This is illuminated in his description:

Saifullah – [The professor's] way was really smart, how he did it, like he showed us in the class. It was really, really smart, the way he's done it.

[…?] but I found it too difficult to think about what I'd be doing. I couldn't piece all of this together. So, I scrapped that and I went to the most simple way possible to show that four points would be at the corners of the square. And I got full marks for it.

This is, like, the really basic way to look at it, if you're an A Level student, this is the way you'd look at it, but, I couldn't articulate the other way to do it, because I wasn't sure.

Like, a lot of people would write, "you do this matrix and then you do this translation of it, and this and that, and you do 90 degree rotation by timing i by i or something."

"But I couldn't remember all the points of a complex plane. So, the way I know – because it's correct, that's why I got full marks for it, this is a completely mathematically correct proof that I gave of where – I even mentioned, it is a monotonous method of solving, because I know it's a long-winded way to do it, but it works, so, that's why he gave me full marks for it. Although, when I saw his proof, in the Friday class, because I really wanted to know the other way to do this, because the way I wrote it, I didn't like it, because it's not so pretty. But his way, it was so much more – I couldn't have written it in that many lines, it was really smartly done. But that's why I like the Friday class, because you see the better ways to do things. And it also refreshed my memory about complex numbers and things like that."

Commentary

Saifullah described the difficulty of trying to solve this problem and his solution was to find, for him. 'the most simple way possible'. He also said he couldn't 'remember all of the points of a complex plane.' This is an interesting point. He seemed to be aware that he would need this information in order to produce a more 'elegant' answer to the problem. However, he didn't look up this information, perhaps because of time constraints. This student often mentioned time problems (e.g. doing homework during the lecture and tutoring in evenings). Instead of trying to work through the problem he "went to the most simple way possible." Saifullah may be choosing the way that takes less time and guarantees task completion, over the more difficult way which is time consuming and involves risk and a much deeper engagement with mathematical concepts. What seems important to him is that he should get a good mark e.g

"…and I got full marks for it…So the way I know – because it's correct, that's why I got full marks for it…because I know it's a long-winded way to do it, but it works, so, that's why he gave me full marks for it.

However, the simple way isn't his preferred way ("I didn't like it.") but provides the next best solution, because at least it is correct. He seems to understand that the game is to gain marks, but he is also motivated by something else: "I really wanted to know the other way to do this." He describes this other way, the professor's solution, as "really smart", "pretty", and shorter: "I couldn't have written it in that many lines."

There is a sense that he gets pleasure from seeing the professor's way, in contrast to his own monotonous approach. He describes an awareness that he had while doing the coursework that there was another solution but this was something he couldn't grasp – both conceptually, (*"I found it too difficult to think about what I'd be doing."*) and expressively (*"I couldn't articulate the other way."*) He seems to have a desire to fully understand the problem and to advance his understanding of mathematical problem solving and this is why he likes the Friday feedback lecture: *"…because you see the better way to do things."*

For comparison, the professor's solution to this task is given below:

"The stated property is not changed by translation, so we translate the points in such a way that their barycentre (arithmetical mean) is zero. Choose one of the new points and multiply it by increasing powers of the imaginary unit. If this process gives the remaining points, in some order, then our points lie at the vertices of a square."

To sum up, in this short extract from the interview Saifullah explains the way he tackled one task, reveals glimpses of his approach to learning: his aspiration to comprehend a more elegant solution to the problem. Moreover, we learn a little about his life world [7], the time pressure he is under and his desire for good marks.

Discussion

This was a small scale study involving around 15 students, 3 PGTAs and one professor. It is difficult to generalise results from any study and with a small scale study it is especially important to be cautious. This study took place in a very specific context, an innovative course in MW, very different from other maths courses that participants had experienced and this may have influenced their learning. Additionally, and this is relevant also to phenomenographical studies (the predominant research methodology in the field of student approaches to learning), it is important to note that the data in this study consists of students' *reports* of their learning. We don't know how close these reports are to what they actually did [8].

Having stated these caveats, we now want to discuss what we found in the light of the established 'approaches to the learning' framework.

Entwistle [3] defines three categories for analysing students' reports of their approaches to learning (see Table 1).

Applying these categories to our interpretation of Saifullah's report, we notice immediately that it seems to contain features of all three approaches to learning and that no one approach, or indeed all three, seem completely to characterise Saifullah as a learner. Overall the best fit may be with the strategic approach if this is understood as taking both a deep and surface approach at different points; so that Saifullah demonstrates an 'intention to understand' when he

Deep Approach
Intention to understand
Vigorous interaction with content
Relate new ideas to previous knowledge
Relate concepts to everyday experience
Relate evidence to conclusions
Examine the logic of the argument
Surface Approach
Intention to complete task requirements
Memorise information needed for assessments
Failure to distinguish principles from examples
Treat task as an external imposition
Focus on discrete elements without integration
Unreflectiveness about purpose or strategies
Strategic Approach
Intention to obtain highest possible grades
Organise time and distribute effort to greatest effect
Ensure conditions and materials for studying appropriate
Use previous exam papers to predict questions
Be alert to cues about marking schemes

Table 1 - Approaches to learning. (Source: Entwistle [3].)

talks about his reasons for attending the Friday lecture, but strategically meets the task requirements to gain marks. At the same time he is able to reflect on the monotonousness of his approach and show dissatisfaction with it.

Yet, applied in a superficial and mutually exclusive way, Entwistle's [3] categories appear over simplistic. In the everyday discourse of higher education teaching, deep equals 'good' and surface equals 'bad'. (Haggis [9] has pointed out that deep also equals 'like me'.) By focussing on the relatively small set of criteria which define deep/surface/strategic approaches to learning, researchers risk overlooking the complexity of student approaches to learning – the mixing of approaches and also behaviour that is not contained within the criteria. Entwistle himself, in responding to Webb's [10] critique of the literature on approaches to learning, agrees that the 'defining features of the deep approach oversimplify and, to an extent "simply lie"' [3].

Ironically, it is the simplicity of the deep/surface metaphor which has led to it being widely adopted, researched and cited in the literature on student learning.

Notes

1. For detailed reports on this study see: – http://www.mathstore.ac.uk/index.php?pid=180 [Accessed 19 April 2010].

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