experimental mathematics with MAPLE

Help with exercises for chapter 9

Exercise 9.1.

(b) Usual strategy: replace 10 with 3, and do it by hand. The sum of the first 3 prime numbers is $2+3+5=10$. Now do it with Maple, and check the answer.

(d) Is $p(x)$ reducible?

(e) The task is to compute x_{20} *without displaying it,* and then check that is is equal to $2^{21}+1$ using `evalb` (how many decimal digits has $2^{21}+1$?). Even though in this case we can verify the correctedness of our computation directly, it is worthwhile to pretend that we cannot, and test our code with the more manageable computation of x_3.

Ours is a *second-order recursive sequence.* First, you must review its definition (page 84), and then example 4.15 and exercise 4.18, to familiarize yourself with this construct. We compute the first few terms by hand:

$$x_2 = f(x_1, x_0) = f(5,3) = 3\cdot 5 - 2\cdot 3 = 15 - 6 = 9$$
$$x_3 = f(x_2, x_1) = f(9,5) = 3\cdot 9 - 2\cdot 5 = 27 - 10 = 17.$$

So the answer is $x_3 = 17$. We check it with Maple, *without* the `do`-loop, using the straightforward strategy illustrated at page 85, 5 lines from the end. Only then we shall design the `do`-loop, verify the answer again, and finally, when everything works, compute x_{20}.

Exercise 9.2. The various pieces of this problem have been illustrated in various examples.

Exercise 9.3.

(a) Let us compute by hand all the positive cubes less than 100:

$$1^3 = 1, \quad 2^3 = 8, \quad 3^3 = 27, \quad 4^3 = 64, \quad 5^3 = 125.$$

There are four of them. How have we arrived to this result? Starting from $n=1$, and increasing n by 1 at each step, we have computed the cube of n, `while` the result was less than 100. When the result became *equal to 100, or greater,* we have stopped.

(b) Compute, by hand, the smallest integer greater than 1 which is relatively prime to 30. Obviously, we must know the meaning of *relatively prime*, given at page 23, second paragraph. Once that is clear, it will be equally clear to us that the answer is 7. Next we check the above answer with Maple, first without `do`-loop, then with the `do`-loop. Finally, we replace 30 with 9699690. (What is special about 30 and 9699690?)

Exercise 9.4. The difficulty here is to understand the question. For this, it may be necessary to perform several computations by hand, with small samples.