experimental mathematics with MAPLE

Help with exercises for chapter 8

Exercise 8.1. The difficulty here is to gain reasonable confidence that our solution is correct, without external help. The best strategy is to reduce drastically the size of the parameter(s) of the problem (typically, the upper limit of a sum or a product), in such a way that the calculation by hand becomes possible. Once we know what the answer should be, we verify it with Maple. When everything works, we restore the original parameter(s), and compute the value of the actual expression.

(a) We compute instead

$$\sum_{k=0}^{2} k(k+1) = 0\,(0+1) + 1\,(1+1) + 2\,(2+1) = 0 \cdot 1 + 1 \cdot 2 + 2 \cdot 3 = 0 + 2 + 6 = 8.$$

The answer is 8. We verify it with Maple

```
> a:=k->k*(k+1):
> add(a(k),k=0..2);
```

8

Our code must be right. All is left to do, is replace the upper limit 2 with 100 in the last expressions.

To deal with the computations by hand of double sums and products, such as (h), (i), (j), you must first have section 8.4 of the book at your fingertips.

Exercise 8.2. The two expressions must have the same value. What data type do you expect this value to be? (Integer, rational, polynomial, etc.)

Exercise 8.3. This problem must be done in two stages. First, compute the summations (products). These are symbolic computations, and even if they are rather simple, they require familiarity with section 8.3. The result of a symbolic computation will be a Maple *expression.* For instance, in part (a), this expression is a *polynomial* of degree 3, with rational coefficients, in the indeterminate n (think about it). In part (b), the expression is a *rational function,* in part (c) is the product of a polynomial and an *exponential function,* etc.

A correct symbolic computation will produce the same expressions, but typically not in the desired form. For instance, part (a) could give something like this

$$\frac{1}{3}\,(n+1)^3 - \frac{1}{2}\,(n+1)^2 + \frac{1}{6}\,n + \frac{1}{6}.$$

The task is to show that the above is equal to the right hand side of equation (a). This is now a question of manipulating and simplifying algebraic expressions (section 7.4).

Exercise 8.6. Before getting started, you clearly must know what is a characteristic function (page 52–55).