experimental mathematics with MAPLE

Help with exercises for chapter 3

Exercise 3.3. Is the question clear? The primes $p = 2, 3$ do not belong to that set, because in both cases $p - 2$ is not prime. If $p = 5$, then $p - 2$ and $p + 2$ are prime, so the prime 5 belongs to the desired set. For $p > 5$, then , $p + 2$ is divisible by 3 if $p = 7,\ 13, 19, 31, \ldots$, and $p - 2$ is divisible by 3 for $p = 11, 17, 23, \ldots$. We must show that these two sequences exhaust all primes greater than 5.

What do $7, 13, 19, 31, \ldots$ have in common? Consider the remainder of division by 6. Then do the same for $p = 11, 17, 23, 29 \ldots$.

Exercise 3.5. The difficulty consists in translating an English expression into a boolean expression, the hard bit being 'but not by both'. We start with a straightforward approach. Let $p = 43$, $q = 47$, and $n = 11396333$. Because 'but' is not a boolean operator, we must replace it with 'and', and our sentence becomes 'and (not (divisible by both))'. Now, 'divisible by both' translates into '(p divides n) and (q divides n)', so the boolean equivalent of 'but not by both' is

`and (not ((`p` divides `n`) and (`q` divides `n`))`

For a more concise solution, note that p and q are *prime,* and an integer is divisible by two primes precisely when it is divisible by their product, from the fundamental theorem of arithmetic (page 32). (Primality is crucial here; 12 is divisible by 6 and by 4, but it is not divisible by their product 24.)

Exercise 3.6. In the last arithmetical expression, you must use the function h twice. Notice that such expression is the difference of two squares, which can be factored.

Exercise 3.7.

(b) Surjective means that image and co-domain are the same (where is this concept defined in the book?). The co-domain is given, and `map` is the Maple procedure required to construct the image. Make sure you verify that the two sets are the same without displaying them explicitly.

Exercise 3.8.

(c) First use it to decide whether or not $p_3 - p_2 = 2$, which is something you can test by hand.

Exercise 3.9. Similar difficulties as in exercise 3.5, above.

(a) The expression 'even non-negative', referred to a quantity x, means '(x gives remainder zero when divided by 2) and ($x \geq 0$)'.

(d) If p is twice a prime q plus one, how do you characterize q in terms of p?

(e) See exercise 3.5.

Exercise 3.10. Once you have such function, test it using `nextprime` and `prevprime`.

Exercise 3.11. If there is an integer between r and s, what can you say about the respective integer parts?