experimental mathematics with MAPLE

Help with exercises for chapter 2

Exercise 2.1. If in doubt, try various arrangements of parentheses, e.g.,

```
> a+b/c+d,(a+b)/c+d,a+b/(c+d),(a+b)/(c+d);
```

For more help, see the worksheet '`first steps`', available on-line.

Exercise 2.2. In both expressions, the answer is a small integer; this should surprise you, given the complexity of the expressions and the size of the integers involved.

If you suspect there is an error in your expression, first make sure you have not typed $\cdot$ instead of `*`, for multiplication. If that is not the problem, try replacing the integers with letters, i.e.,

$$a^b \cdot c - \frac{d}{e + \frac{f^{g^h - i} - j}{k \cdot l}} \cdot m.$$

The architecture of the expression will then become obvious, and with it any mistake.

Exercise 2.3. For inspiration, look at the discussion in the second paragraph of page 10.

Exercise 2.4.

(*a*) The answer is 61591. If you have problems with substitutions, do the following warm-up exercise. Let $h(z) = z + z^2$, and compute by hand

$$h(-z), \quad h(z^2), \quad h(-z^2), \quad -h(-z), \quad h(a), \quad h(1/a), \quad h(b/a^2).$$

Exercise 2.5.

(*a*) We begin with a simpler instance of the same problem, for which the answer can be computed by hand, so we can check.

For example, let us put in ascending order the integers $a = 3$, $b = -1$ and $c = 5$, using Maple, and without displaying any digit. This requires verifying the truth of two inequalities, that is, evaluating two boolean expressions. Now pretend you did not know the values of a, b and c, and attempt to put them in order, by just looking as the value of boolean expressions.

Once you have done it with three small integers, without displaying them, you can certainly do it with five large integers.

(*b*) Same strategy as above. First determine by hand the largest integer whose 3rd power is smaller than 10^2. Then verify it with Maple. When you are confident that your procedure works, apply it to the actual data.

Exercise 2.8. According to the procedure explained at page 24, p and q must be coprime and of opposite parity. So we test q in the range $970 \leq q < 987$, with q even, and coprime to p. This will give you 5 values of q. Then you just apply the formula.

Exercise 2.9. One strategy is to let p be even, and $q = 1$. Then $p > q$, p and q are relatively prime, and p and q have opposite parity. We have

$$x = 2p, \qquad y = p^2 - 1, \qquad z = p^2 + 1.$$

It is sufficient to check that the *smallest* among x, y and z has at least 20 digits. Persuade yourself that for large p the smallest of them is x. Then choose p such that $2p$ has at least 10 digits.

Alternatively, let $q = p - 1$. Then $p > q$, p and q are relatively prime, and p and q have opposite parity. We have

$$x = 2\,p\,(p-1), \qquad y = p^2 - (p-1)^2 = 2p - 1, \qquad z = p^2 + (p-1)^2 = 2p^2 - 2p + 1.$$

Which one is the smallest now?

Exercise 2.13. Remember the rule for constructing *nested expressions*: always start from the inside, which in this case is the end.

Exercise 2.14.

(b) The rightmost expression consists of *two* inequalities, which must be tested separately.

(c) For every divisor there is a twin divisor.

Exercise 2.15. Usual strategy: what integer is the closest to 18/5? What steps are involved in proving this with Maple *without displaying any digit?* Note that it would be incorrect to speak of *the* integer closest to 5/2? What happens in this case?

Exercise 2.16. The quantities x, y and z are *very* close to one another. This is what makes part (b) tricky. Once you understand the relative position of x, y and z, you must deal with the problem of finding rationals close to a/b, both to the right and to the left of it, while increasing b as little as possible.

Exercise 2.17. Everything can be established from prime factorization, so the only difficulty is in part (c), because we are not allowed to look at it. One possibility is to explore the Maple function `nops`.

Exercise 2.19.

(a) Writing p_n for the nth prime, we rewrite the above as

$$p_{25} \leq 100 \qquad \text{and} \qquad p_{26} > 100$$

Note the use of strict and non-strict inequalities.