Essential mathematics

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Abstract

The Essential Mathematics initiative has addressed successfully a difficult problem: an alarming number of mathematics students reach university with inadequate basic skills in elementary arithmetic and algebra. This programme has been running for nearly a decade at the School of Mathematical Sciences, Queen Mary, University of London. Essential Mathematics is based on the principle of *no compromise on minimal standards for basic skills.* It relies on a vigorous assessment method, transparent quality assurance, and web-based teaching material.

1 The problem

In continental Europe, a 12-year old student is normally expected to handle arithmetical expressions such as[1]

$$\frac{1}{6}+\left[\frac{5}{21}\div\left(1+\frac{7}{3}\right)+\left(-\frac{1}{2}-\frac{4}{7}\right)\times\left(\frac{2}{4}-\frac{4}{5}\right)\right]\div\left[-\frac{2}{5}\times\left(1-\frac{1}{4}\right)\right]$$

and at age 13 algebraic expressions such as

$$y\left[\frac{1}{2}x\left(2x-\frac{4}{3}y\right)-\left(x+\frac{1}{3}y\right)^2\right]\div\left[\left(x-\frac{1}{3}y\right)\left(x+\frac{1}{3}y\right)-x^2\right].$$

Similar expectations are found elsewhere, for instance in south-east Asia.

By contrast, only a minority of British students are exposed to such a level of computational complexity, even among those who specialise in mathematics. The roots of this problem are deep and complex; the result is a dramatic lack of fluency, stamina and confidence in basic manipulations, whose repercussions are felt across the entire curriculum. These deficiencies undermine the understanding advanced constructs, deprive the students

[1]From a textbook for the Italian national curriculum, 1996.

of a vital support for abstraction, limit their ability to use computers effectively. Above all, too many students are denied the *pleasure* of doing calculations.

Currently, a student can sail through mathematics A-levels without this problem being detected. Universities must therefore assume a new responsibility. Yet there is a straightforward solution: to have the students do *lots of exercises.* The difficulty lies in persuading a beginning university student to engage in an activity which seems unglamourous and unrelated to higher mathematics (it is neither), and where the time scales for reward are invariably long.

2 The initiative

The Essential Mathematics (EM) initiative —arguably, the most vigorous in the UK— was introduced in the academic year 2001/02, along with the supporting course MAS010[2]. Its success motivated the introduction, in 2004, of the twin course SEF026 for the Science and Engineering Foundation Programme.

The EM programme is designed for a large student population, currently over 250. Its main ingredients are:

- high expectations
- transparent procedures
- web-based learning material.

To be admitted to the second year, all Queen Mary students with Mathematics as home department must pass an examination on basic arithmetic and algebra (integers, fractions, square roots, polynomials, rational functions, linear and quadratic equations). This scheme was introduced following two failed attempts to embed remedial work within a first semester module. These attempts were either ineffective, or resulted in unacceptably high failure rates.

The EM exam has little in common with the other examinations. It is offered *seven times* during the first academic year, and it must be taken repeatedly until passed; students who do not pass do not progress to the second year, *irrespective of their performance in all other first year modules.* The EM exam does not count towards the final degree, but it appears in the students' transcripts.

The exam is substantial, with pass mark at 80%. The level of difficulty is determined by absolute criteria, not by considerations on progression. The exam adopts a pass/fail multiple-choice format, with 15 questions, and 12 correct answers to pass. Such a sharp pass/fail criterion is essential, given the uncompromising nature of the scheme. The exam

[2] now labelled MTH3100

is predictable, and questions never change in contents or style: only the numbers change. Two sample exam questions are displayed below.

1. Compute $f(-1/(3Y))$, where

$$f(b) = 6b - \frac{1}{3b^2} - \frac{1-b}{9b^3}$$

[a] $\dfrac{3Y^4 + 4Y^3 - 2}{Y}$ [b] $-\dfrac{3Y^4 + 4Y^3 + 2}{Y}$

[c] $3Y^3 - 4Y^2 - \dfrac{2}{Y}$ [d] $3Y^3 - 2Y^2 - \dfrac{2}{Y}$

[e] not in the list

2. Simplify, eliminating radicals at denominator

$$\frac{1}{7+3\sqrt{5}} - \frac{30}{\sqrt{20}}$$

[a] $\dfrac{7-15\sqrt{5}}{4}$ [b] $\dfrac{7-9\sqrt{5}}{4}$

[c] $\dfrac{7+9\sqrt{5}}{4}$ [d] $\dfrac{14-15\sqrt{5}}{8}$

[e] not in the list

The *quality assurance* of this examination is based on *total openness,* rather than on *certification*. As a result, the examination bureaucracy is virtually non-existent. The students get to know the exam results before they leave the examination room, and retain a copy of their submission for their own record. There is no stigma attached to failure. The examination process is open to inspection: past exam papers with answers are posted on the web, and so is the examination statistics. Abundance of examination opportunities and deterministic marking eliminate many burdens of examining, such as the need for blind marking, agonising on the pass/fail borderline, handling extenuating circumstances, appeals. Because any mistake in the exam would become public, we implement a rigorous procedure for checking the correctness of the examination paper. The examiners have no discretion at their disposal, so the external examiner's role becomes largely irrelevant.

The students attend a presentation of the programme during induction week. They are then given two weeks to get acquainted with the syllabus and the web-book (see section 4), to attempt mock exams, and to seek help from their advisers. Then they sit the first exam, which has no surprise element in it, and for which they are given plenty of time: two hours. Most students fail it (see below), and are hence enrolled in the supporting course, which runs alongside the other first semester courses. This is a complex operation, with several parallel tutorial sessions of approximately 20 students each, complemented by weekly tests on the material being covered. The course is partially supported by the Widening Participation programme at Queen Mary, which provide some of the teaching. All teaching material is available on the web (see below), and self-study is strongly encouraged. Staff is needed to identify the weakest students, to coordinate support activities (of which the peer support programme PASS is a recent addition), and to organise action against absenteeism from classes and tests, which are compulsory. The course is repeated in the second semester, for those who have not managed to pass by the beginning of January.

Essential Mathematics was praised as being ‘courageous’ in a past Teaching Quality Assessment exercise. It has been praised by the externals, and it was briefly reported by the THES on 28/06/02.

3 The results

The results speak for themselves. During the first few years of implementation, on average, well over 90% of the students failed the first exam, but eventually, over 90% of them passed (figure 1). These data refer to a student population with (nominally) a B in A-level mathematics, although many students were recruited during clearing.

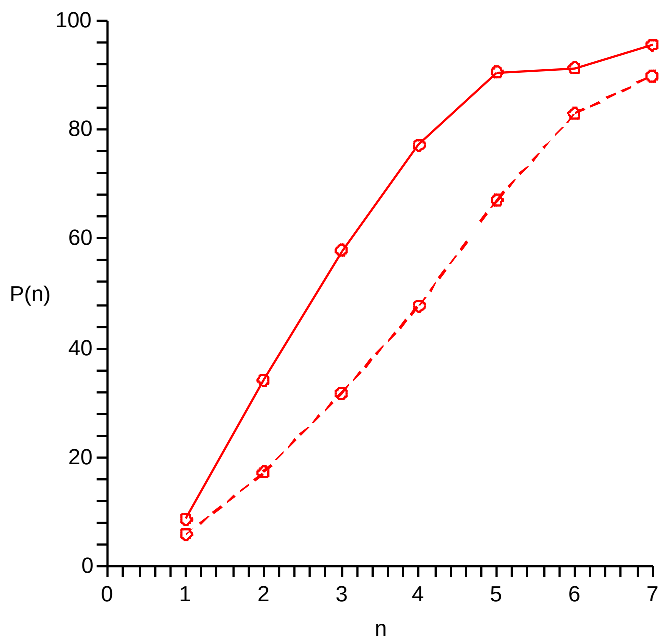

Figure 1: Cumulative percentage progression in EM exams. Exam 1 takes place in October of the first year, exam 7 the following August. The dotted curve represents an average over the three academic years 2001-2003, which include over 400 students. The solid curve refers to the academic year 2004/5. The markedly improved performance is linked to the introduction to compulsory attendance to lectures and tests. The current (2009) failure rate in Exam 1 is 85%, following an improvement in the students' entry qualifications.

After 2005, the entry qualifications of our students began to improve. Currently, roughly half of our intake have an A in A-level mathematics, but the failure rate in the first EM exam remains very high: 85%. It is clear that the underlying deficiencies are deeply rooted, and cannot be remedied by a burst of concentrated effort, even for competent students. The students who pass at the first attempt form a heterogeneous group; students from South-East Asia tend to do well.

Once a quorum of students have passed, progression accelerates. Over two-thirds of the students now pass by the end of the first semester (exam 4, held in early January). Students are encouraged to pass as quickly as possible. Until recently, those who did not pass by January were forced to drop one first year unit, and formally registered for EM as a level-0 unit in the second semester. To minimise the number of students losing a first year unit, in the academic year 2004/5 we introduced compulsory attendance to lectures and to weekly tests, which markedly improved the students' performance (figure 1). Over the past two years, we have opted for a softer deterrent to procrastination; whereas the students who pass by January get 100/100 on their course transcript, those who pass at later exams get a bare pass mark: 40/100. Almost invariably, the students who never make it have strong deficiencies elsewhere. However, we've had a handful of students who did not progress solely because of failure in this module; these students usually transfer to another institution. This phenomenon gives credibility to the scheme, and also raises important questions —see section 5.

4 The web-book

The centrepiece of the learning material is the course's web-book, freely available at

`http://www.maths.qmul.ac.uk/~fv/books/em/embook.pdf`

This book is designed for self-study. Besides developing the basic theory, the book contains *over one thousand exercises* of gradually increasing difficulty, each supplied with answer for immediate feedback. Some difficult exercises, which lie beyond the requirements of the course, are provided to challenge the best students. The book is also sprinkled with references to interesting arithmetical phenomena, to raise the students' interest and curiosity, and to give the teacher material for interaction with the more inquisitive students.

The web-book took several months to produce, and it was initially proofread by a team of postgraduate students. Over the following years, the book was improved and expanded.

5 Discussion

Let us consider the issue of *minimal standards* in university degrees, in connection with the EM programme. Given that our graduates will be the teachers of the future generation, what is the minimum a student needs to know in order to graduate?

When spelling out basic graduate attributes, an analogy with the driving test seems pertinent. If we were told that many people fail the driving test at their first attempt, we would be *reassured* rather than *concerned;* we wouldn't make the driving test easier just to improve 'progression'.

By contrast, the majority of UK university students now pass most exams at their *first attempt,* including key foundational courses. This is due to a combination of cultural, bureaucratic, and financial reasons. First, high failure rates are equated to poor teaching, not to high standards. Second, a heavy bureaucracy makes the examination process slow and inflexible; it is difficult to design assessment methods able to accommodate repeated failures. Finally, there are stark financial pressures: failing students means loss of income. The resulting conflict of interests is very obvious, yet seldom acknowledged, partly because some key exam procedures (e.g., scaling of marks) remain protected by confidentiality.

To raise minimal standards in a basic skill, we had to design an assessment system that as to ethos, practice, and quality assurance, clashed with the current examination culture. Getting it past university regulations was quite laborious. Yet the uncompromising nature of the assessment forced upon us a considerable level of rigour, not only procedural, but also educational. The urgency to improve learning made us place our teaching and supervision under close scrutiny.

Essential Mathematics also makes a clear statement about the value of *raising expectations.* If you demand more from your students, they will give you more. According to a study reported by the THES in 2007, the average working week of a UK university student is the shortest in Europe, so there is plenty of scope for raising standards by stretching our students more. At the Essential Mathematics examinations, I found students who had already passed, but asked to be allowed to sit again, just for the challenge of improving their score, or "to check if I'm still fit". (In one exam, these students accounted for 10% of the candidates!) This unexpected phenomenon gives us an opportunity for reflection.