Modelling of Data Networks using Chaotic Maps as Traffic Sources Supplementary Figures and Captions

M. Woolf,[†*] D.K. Arrowsmith, [†] R.J. Mondragon-C, [*] J.M. Pitts. [*]

Queen Mary, University of London,
LONDON E1 4NS,U.K.

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1 Figure Captions

Figure 1. The lattice configuration: squares represent hosts where packets are created, received, stored, and forwarded. Circles represent routers where packets are only stored and forwarded. The lattice has periodic boundary conditions.

Figure 2. Comparison of onset of congestion in two otherwise identical networks, one with Poisson-like traffic sources and the other with *LRD* sources. There is a phase transition at the onset of congestion with a critical load value of $\lambda_c \approx 0.39$. *(a)* Average packet lifetimes below the critical point are much greater than those for the *LRD* traffic sources. *(b)* The lower peak value in throughput for the *LRD* traffic sources reflects the longer average lifetimes below the critical point.

Figure 3. The effect of changing the degree of intermittency of the traffic sources. Plots similar to fig. 2 are shown for different values of the parameters $m = m_1 = m_2$ keeping their values equal in each case. For the highest degree of intermittency considered, $m_1 = m_2 = 1.95$, the differences seen previously between *LRD* and Poisson-like traffic sources are most pronounced. When $m_1 = m_2 = 1.5$, equivalent to there being no intermittency, the two traffic sources produce almost identical sources (cf. Schuster, Deterministic Chaos).

Figure 4. Comparison between *LRD* and Poisson-like traffic sources for a range of host densities, ρ. Differences in average lifetime and throughput similar to those shown in fig. 2 are seen for all values of ρ. The value of the load at the critical point, λ_c, is inversely proportional to ρ.

Figure 5. Comparison of the theoretical expression for the critical load, λ_c, with data from the simulation. There is good agreement between the two. The line $\lambda_c = 2/\rho L$ also represents a divide between the free and congested phases.

Figure 6.(a) Time series of average host and router queue lengths for a network with Poisson-like traffic sources. Plots are shown for a range of load values below the critical load ($\lambda_c = 0.39$ for

[*] *Department of Electronic Engineering*
[†] *Mathematics Research Centre*

this network). The series show that the network remains in a steady state for all load values below $\lambda \approx= 0.30$. Above this point average host sizes rise, indicating the onset of congestion. (b) Time series of average host and router queues for a network with *LRD* traffic sources. For comparison with fig. 6(a), plots are shown for the same values of the load. These time series are quite different, especially for host queue lengths. Fluctuations in average queue lengths are generally much greater, and the steady state in average host queue does not persist loads $\lambda > 0.1$. For the load $\lambda = 0.3$, evidence of congestion can be seen in both host and router queue lengths, in contrast with the steady state behaviour seen in the case of Poisson sources.

Figure 7. Comparison of average lifetime and throughput versus load at different source destination separations. The differences between *LRD* and Poisson-like traffic sources seen in figs 2 and 3 can be seen for all separations. The variation in throughput between different separations is caused by the differing numbers of paths for each length (indicated on each plot) and the preference for shorter path lengths.

Figure 8. Throughput for 9 individual source-destination pairs with separations of 8, 16 and 24 steps. Comparing these plots with the averaged data in fig. 7.

Figure 9. R/S plots and probability distribution plots are shown for time series of the end-to-end delay suffered by packets travelling distances of 1 and 24 steps. The simulated network simulated consists of 32×32 nodes and 164 hosts. Figures 9($a - e$) show data for loads $\lambda = 0.2, 0.29, 0.3, 0.39$ and 0.45 respectively. The plots show evidence of *LRD* only when $\lambda = 0.29$, just before the network's pre-congestion phase.

Figure 10. R/S plots are shown for time series of (a) average host, and (b) average router queue lengths measured at each time tick of the run. High values of H are measured at all loads, indicating the presence of *LRD* within the network.

Figure 11. As figure 9, except that *LRD* sources are used instead of Poisson sources. Slightly increased H values are seen, but the evidence for *LRD* $\lambda = 0.29$ seen for Poisson sources is removed, probably due to the earlier onset of congestion in the case of *LRD* sources. Probability distributions of delays are much wider, again due to the earlier onset of congestion.

Figure 12. As figure 10, except that *LRD* sources are used instead of Poisson sources. Values of $H \approx 1$, showing that the *LRD* of the sources has fed through to the queue behaviour of both hosts and routers.

Figure 13. Plots showing the behaviour of four network parameters when control is applied to a network with Poisson sources. Average lifetime, throughput, average host queue length and average router queue length are plotted against (a) carried load and (b) offered load. The network modelled is the same as before - 32×32 nodes with 164 hosts. Control has little effect until the network becomes congested.

Figure 14. As figure 13, with *LRD* sources instead of Poisson sources. Behaviour is similar except that congestion begins at a lower load for *LRD* sources, so that control takes effect at lower values of load.

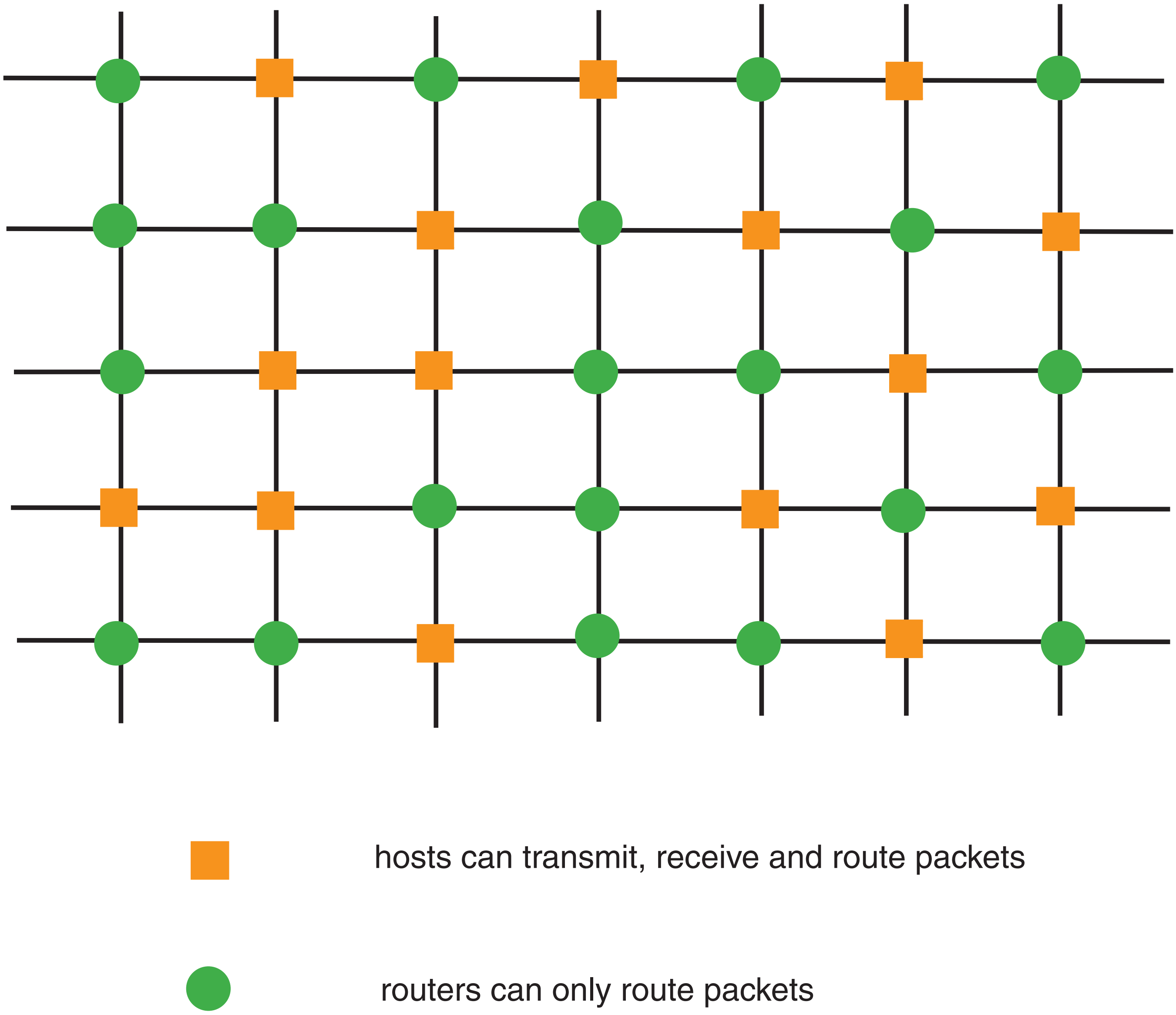

hosts can transmit, receive and route packets
routers can only route packets

Figure 1

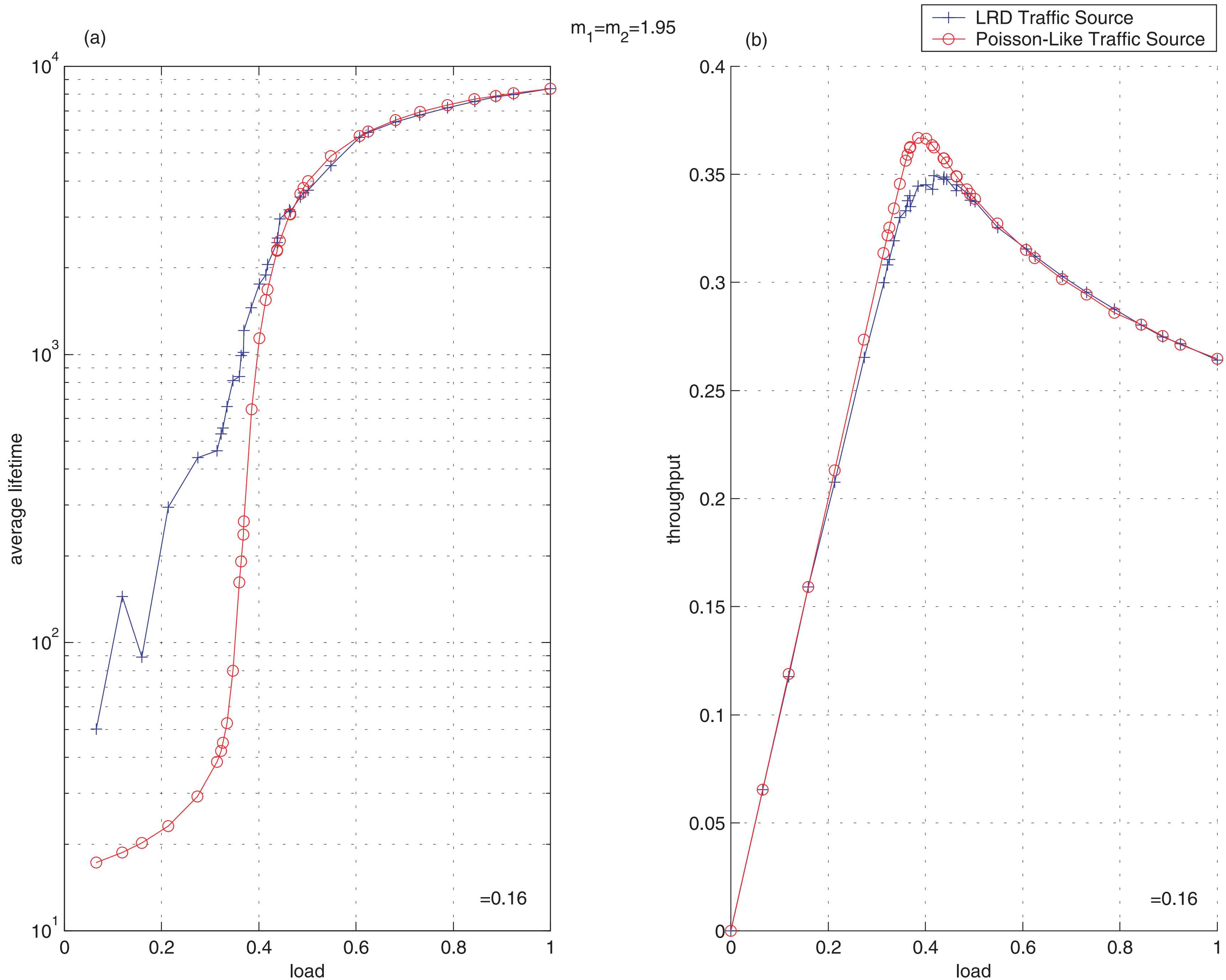

(a)
m1=m2=1.95
average lifetime
load
=0.16
(b)
LRD Traffic Source
Poisson-Like Traffic Source
throughput
load
=0.16

Figure 2

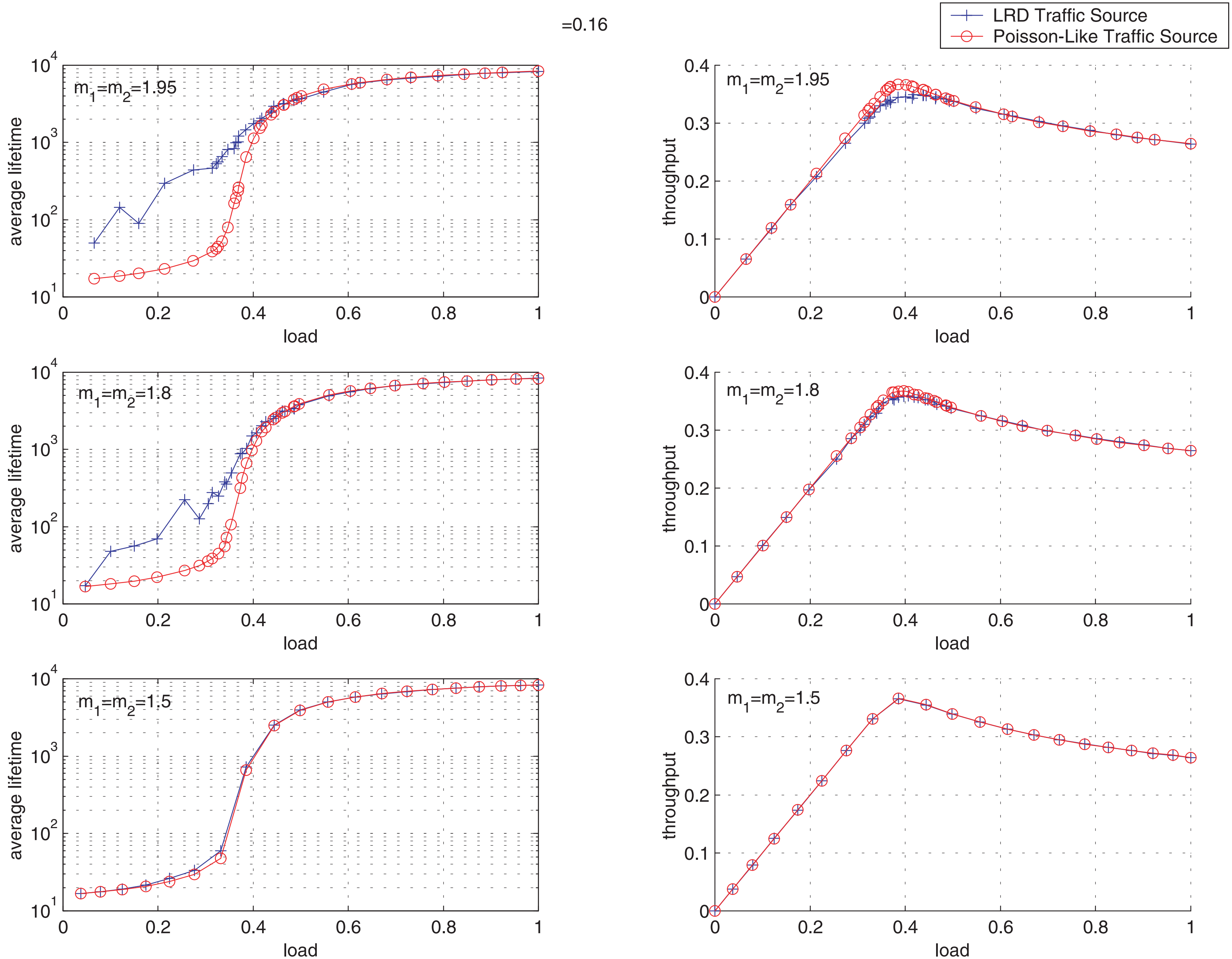

=0.16
LRD Traffic Source
Poisson-Like Traffic Source
$m_1=m_2=1.95$
$m_1=m_2=1.8$
$m_1=m_2=1.5$
average lifetime
throughput
load

Figure 3

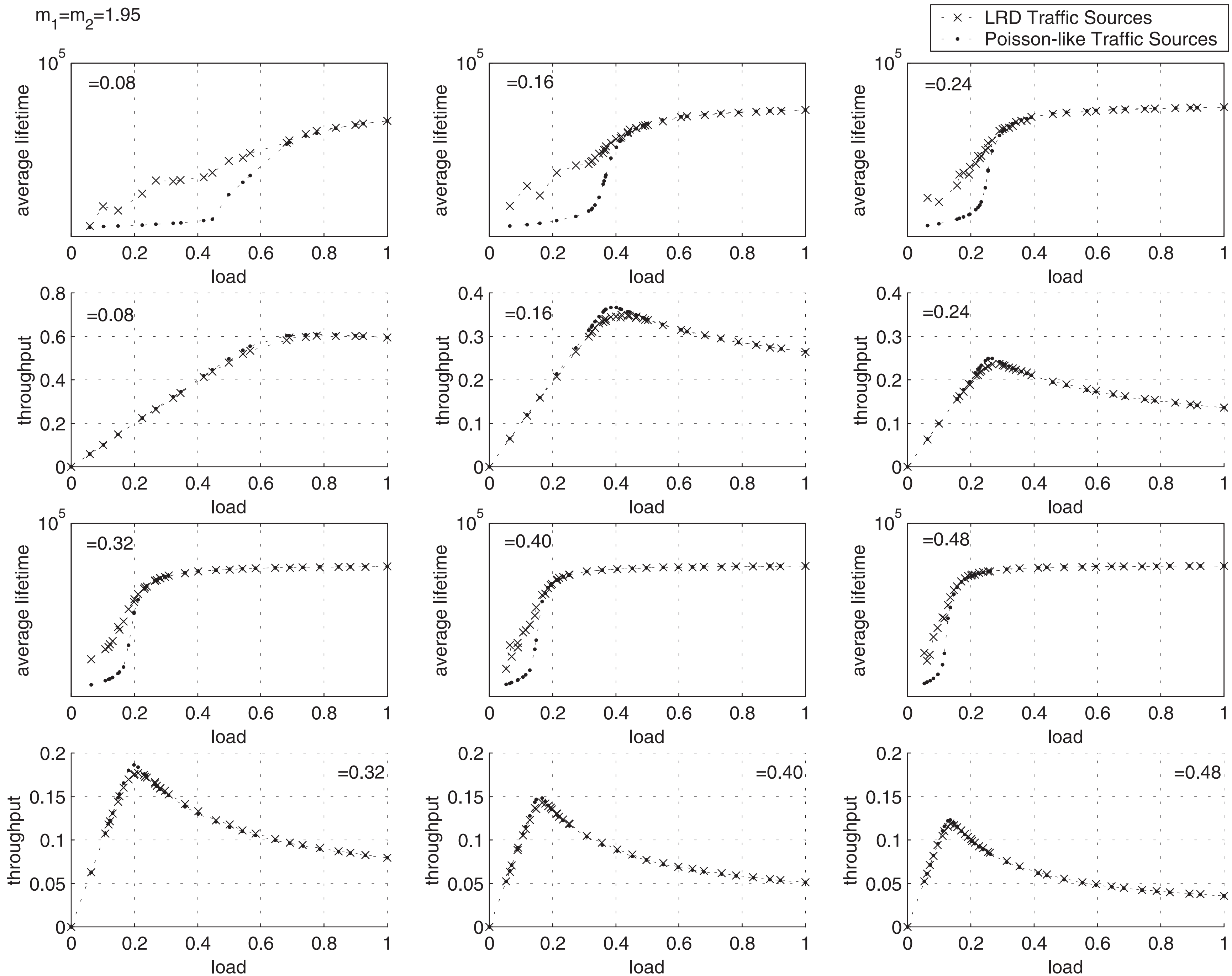
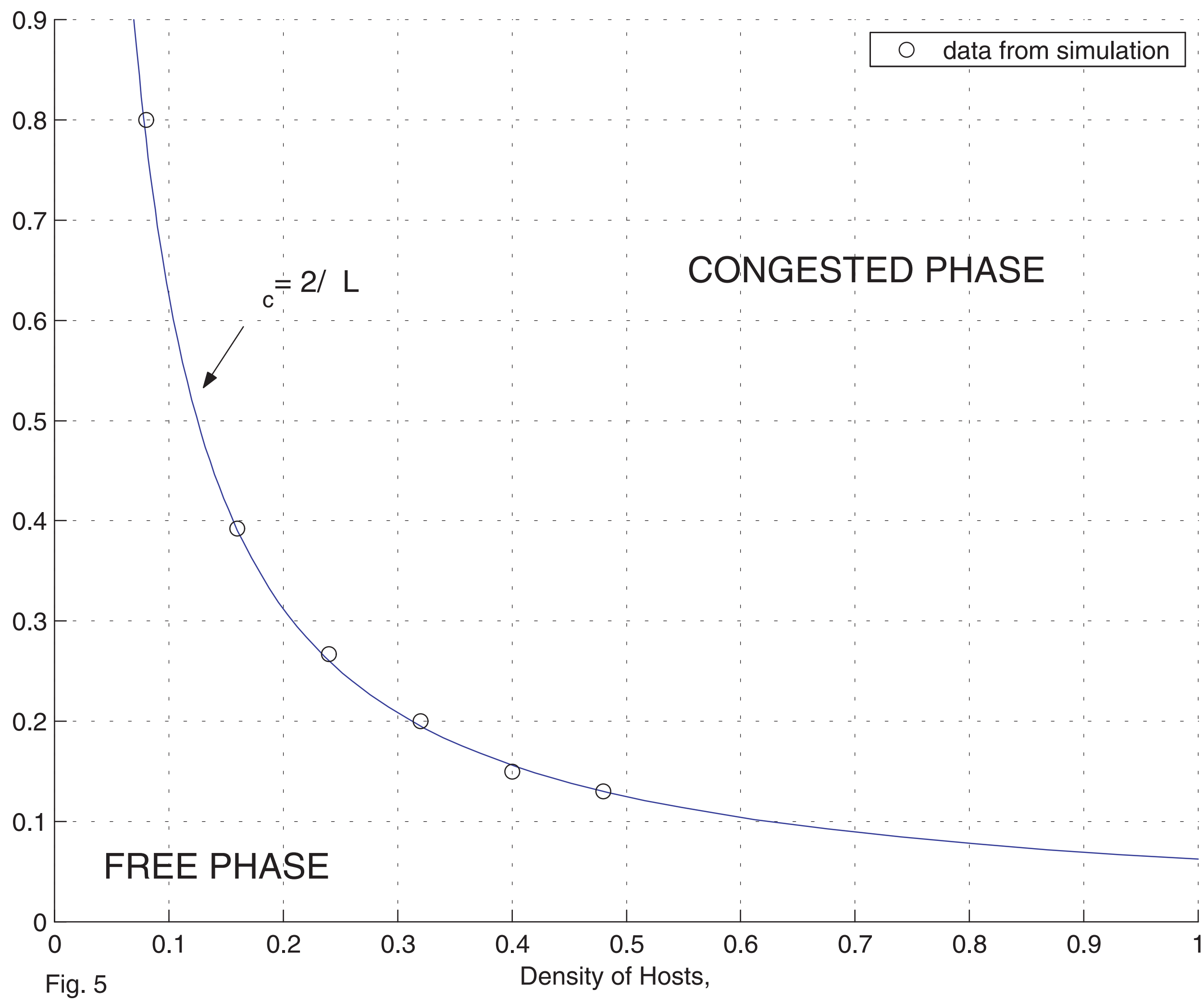

m1=m2=1.95
LRD Traffic Sources
Poisson-like Traffic Sources
=0.08
=0.16
=0.24
=0.32
=0.40
=0.48
average lifetime
throughput
load
10^5

Figure 4



data from simulation
CONGESTED PHASE
c= 2/ L
FREE PHASE
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Density of Hosts,
Fig. 5

Figure 5

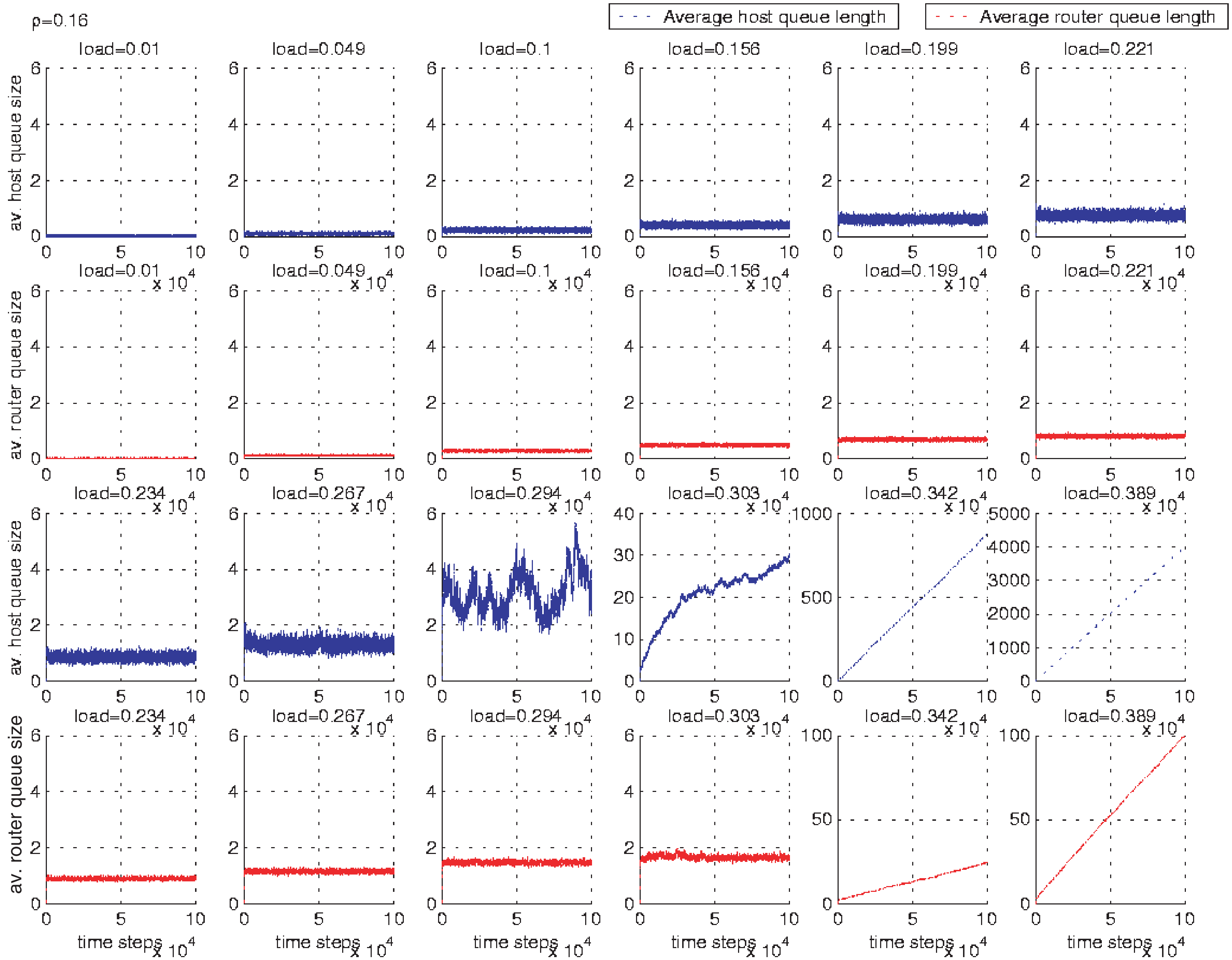
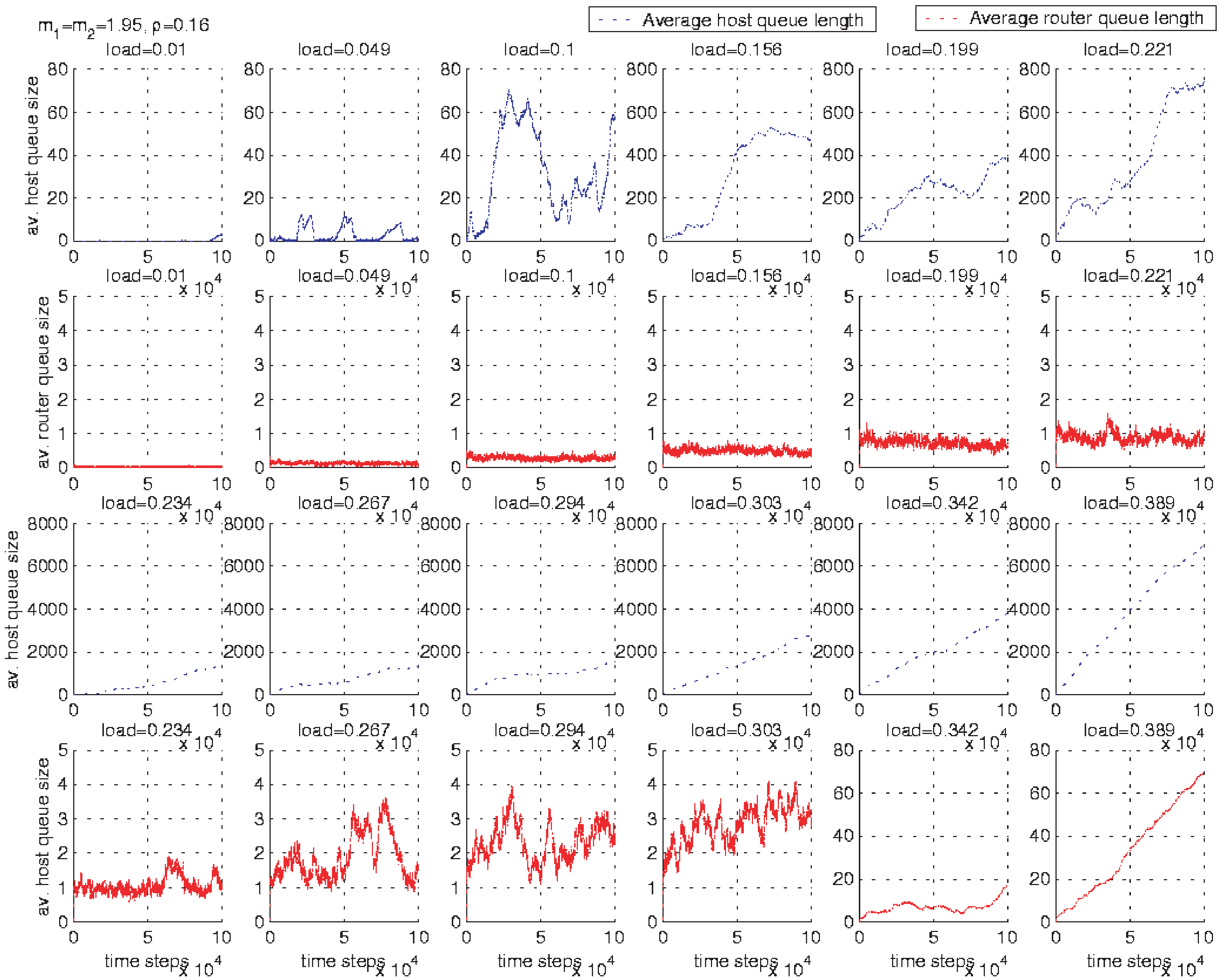
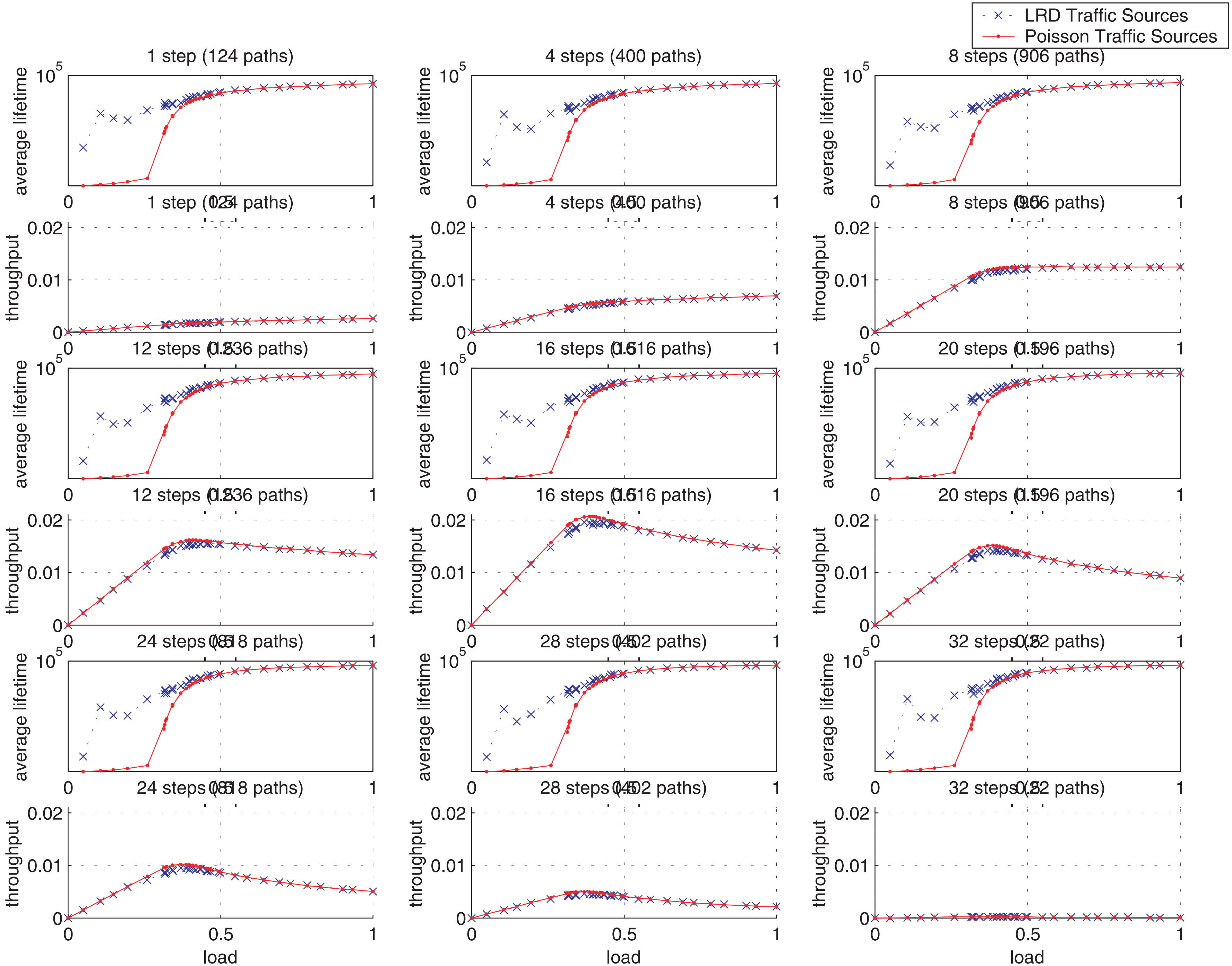

ρ=0.16
Average host queue length
Average router queue length
load=0.01
load=0.049
load=0.1
load=0.156
load=0.199
load=0.221
load=0.234
load=0.267
load=0.294
load=0.303
load=0.342
load=0.389
av. host queue size
av. router queue size
time steps
x 10^4

Figure 6(a)



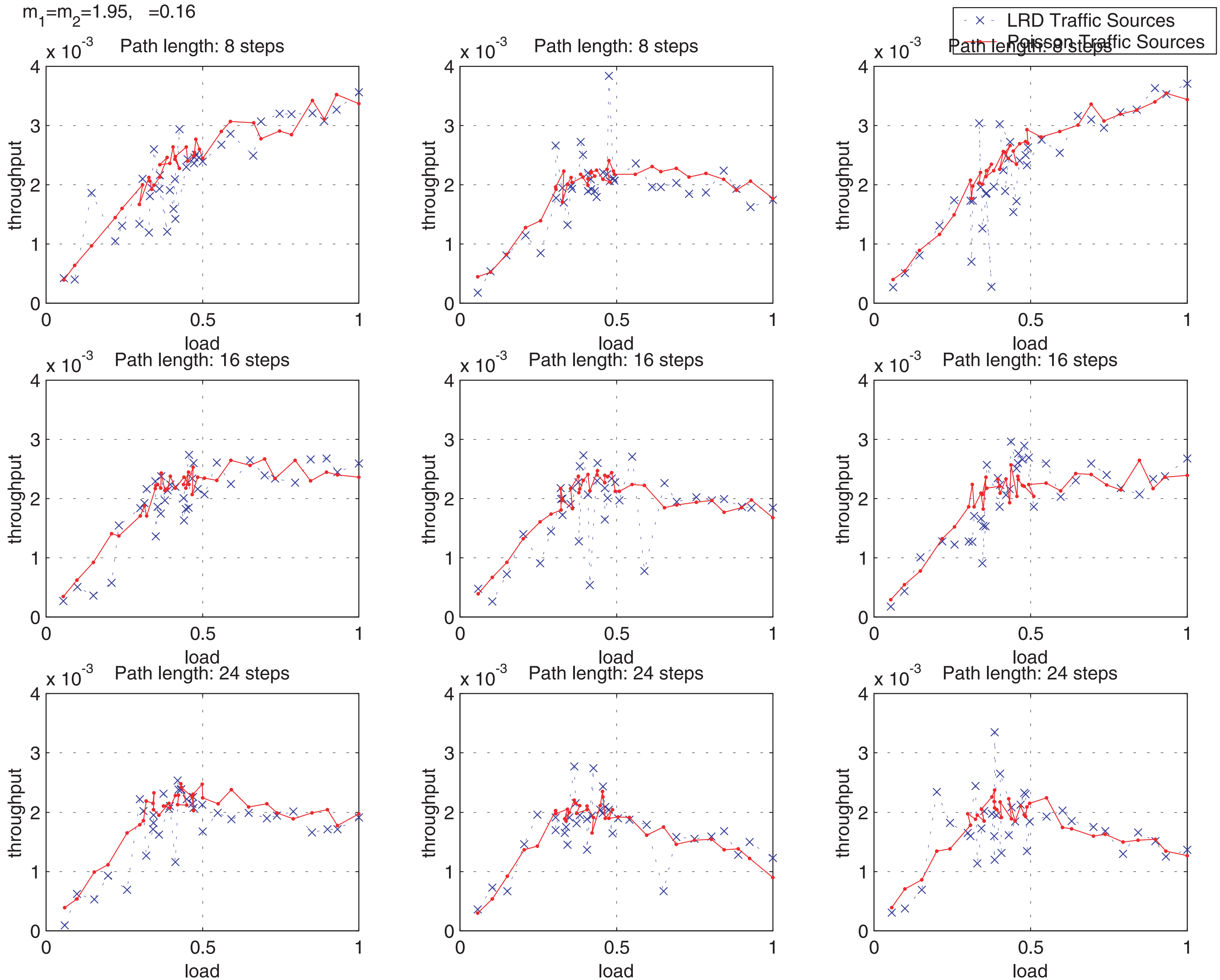
m1=m2=1.95, ρ=0.16
Average host queue length
Average router queue length
load=0.01
load=0.049
load=0.1
load=0.156
load=0.199
load=0.221
load=0.234
load=0.267
load=0.294
load=0.303
load=0.342
load=0.389
av. host queue size
av. router queue size
time steps
x 10^4

Figure 6 (b)



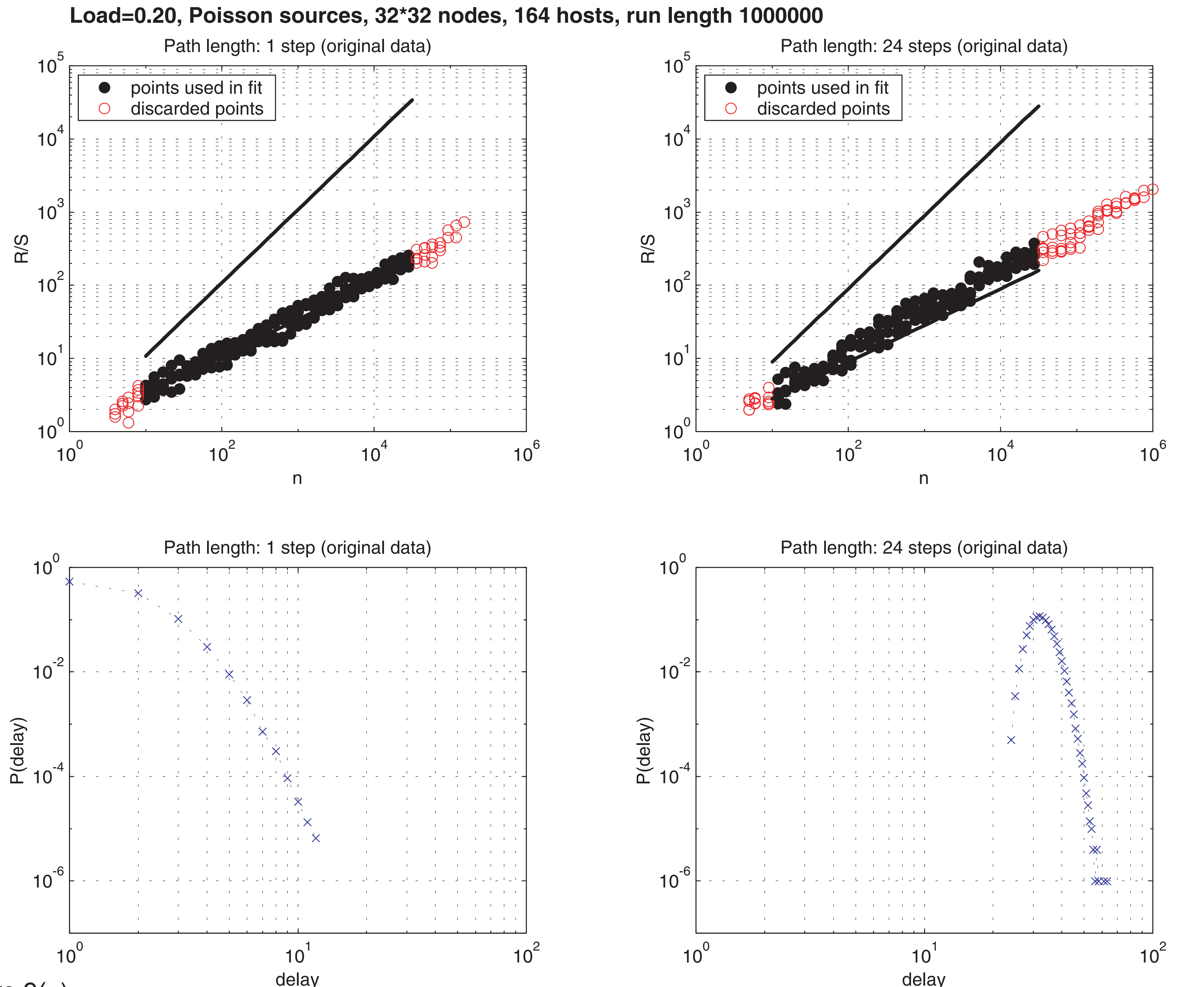
LRD Traffic Sources
Poisson Traffic Sources
1 step (124 paths)
4 steps (400 paths)
8 steps (906 paths)
12 steps (1236 paths)
16 steps (1516 paths)
20 steps (1596 paths)
24 steps (818 paths)
28 steps (402 paths)
32 steps (52 paths)
average lifetime
throughput
load

Figure 7



m1=m2=1.95, =0.16
LRD Traffic Sources
Poisson Traffic Sources
Path length: 8 steps
Path length: 16 steps
Path length: 24 steps
x 10-3
throughput
load

Figure 8



Load=0.20, Poisson sources, 32*32 nodes, 164 hosts, run length 1000000
Path length: 1 step (original data)
Path length: 24 steps (original data)
points used in fit
discarded points
R/S
n
Path length: 1 step (original data)
Path length: 24 steps (original data)
P(delay)
delay

Figure 9(a)

Load=0.29, Poisson sources, 32*32 nodes, 164 hosts, run length 1000000

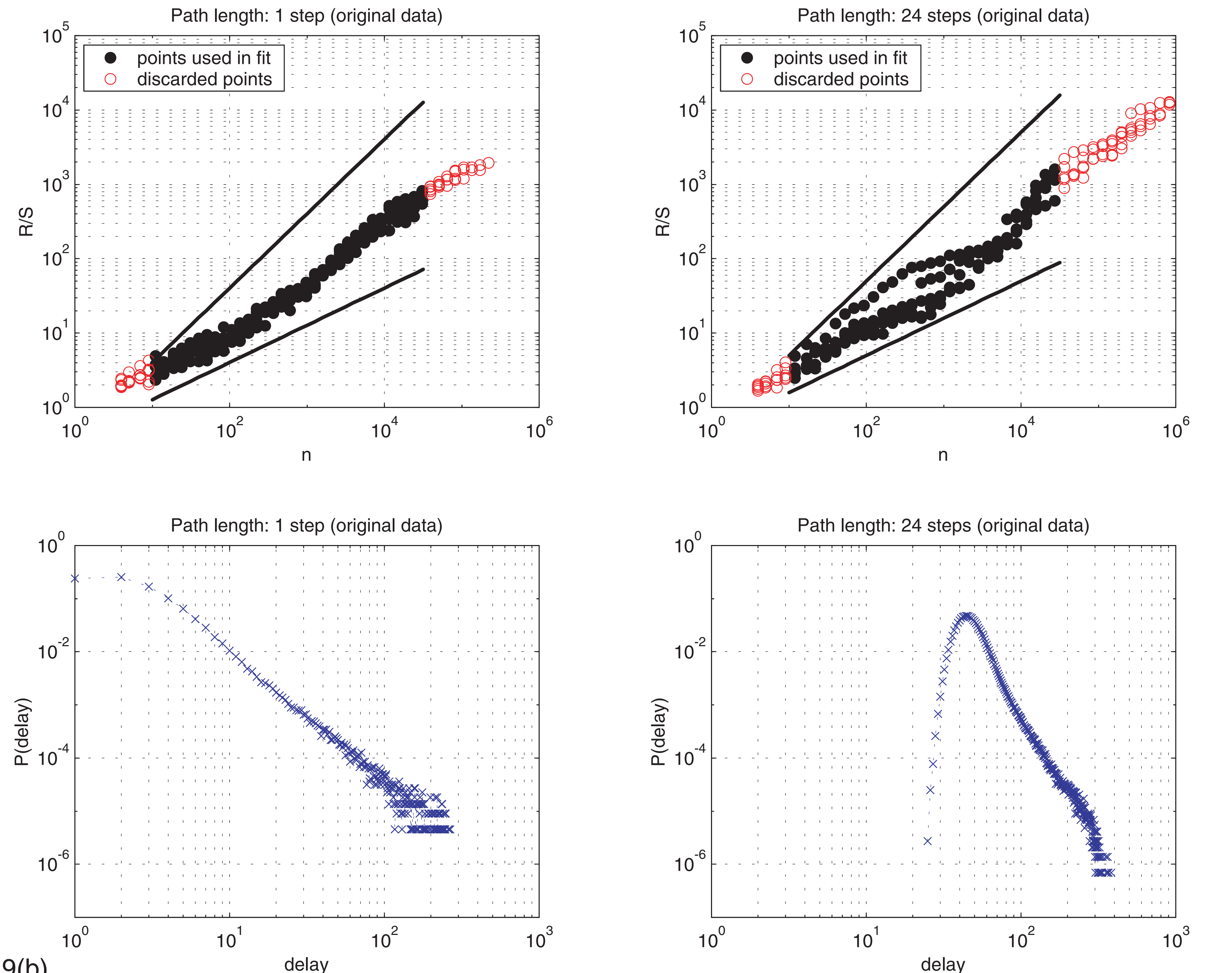
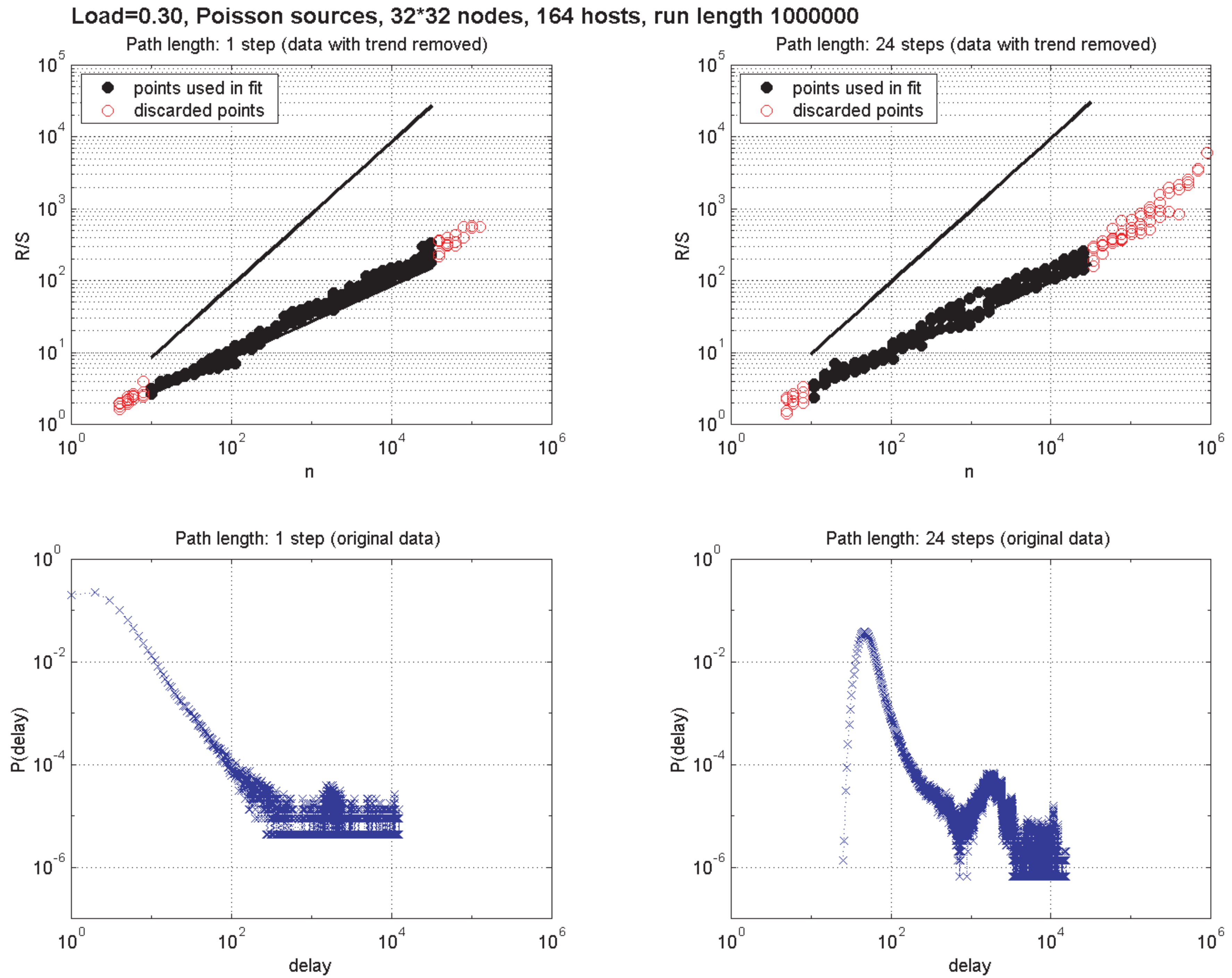


Figure 9(b)



Load=0.30, Poisson sources, 32*32 nodes, 164 hosts, run length 1000000
Path length: 1 step (data with trend removed)
Path length: 24 steps (data with trend removed)
points used in fit
discarded points
R/S
n
Path length: 1 step (original data)
Path length: 24 steps (original data)
P(delay)
delay

Figure 9(c)

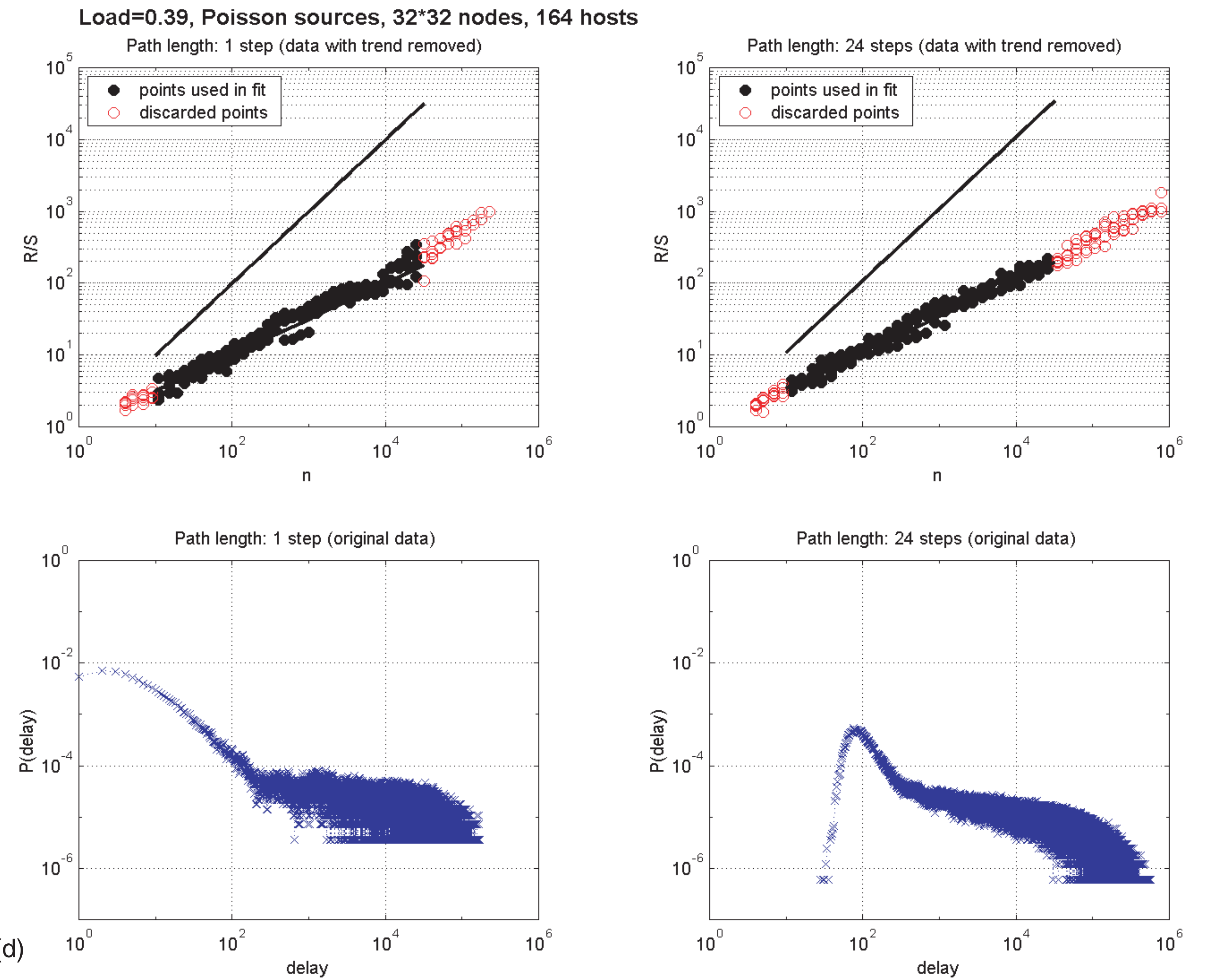
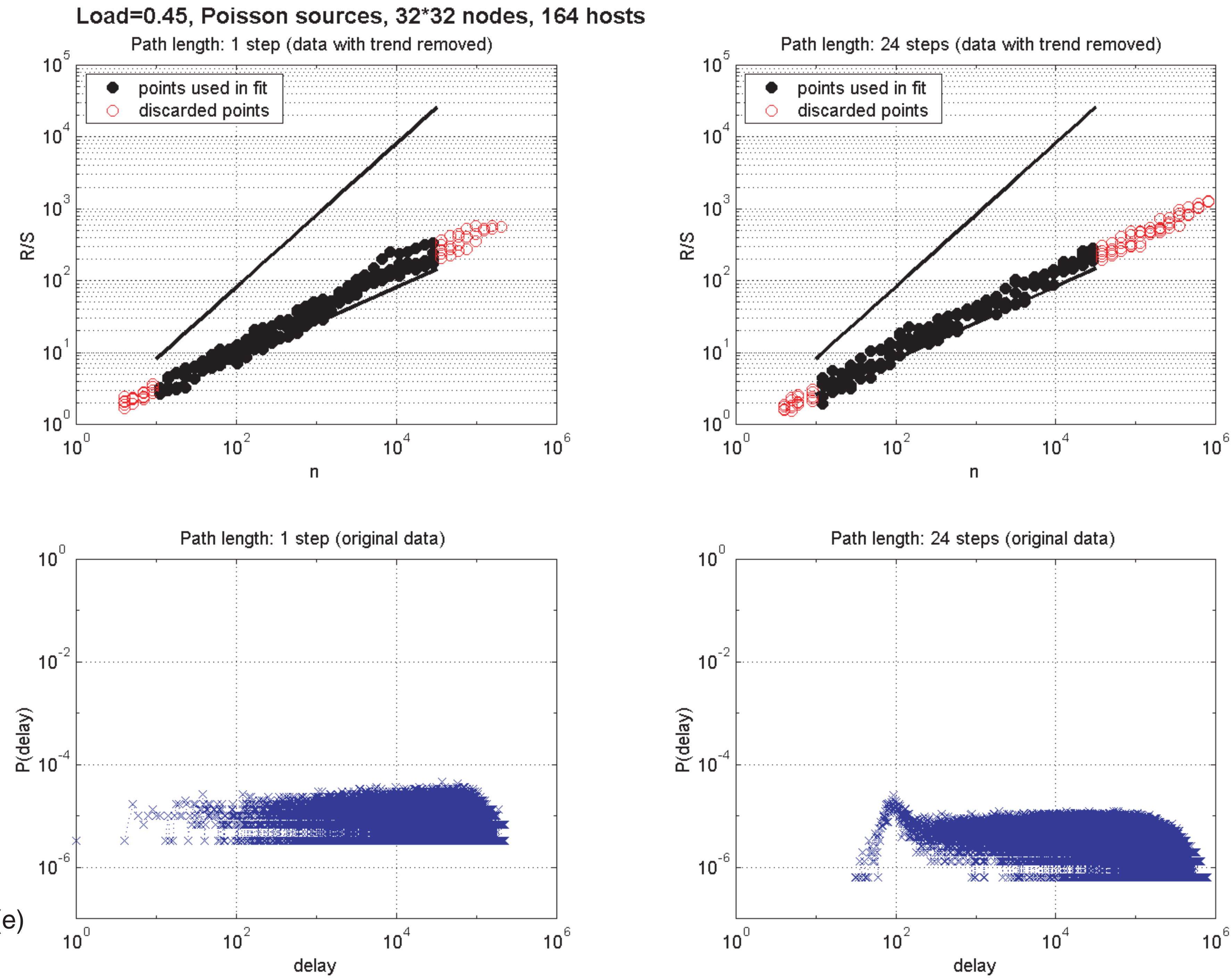
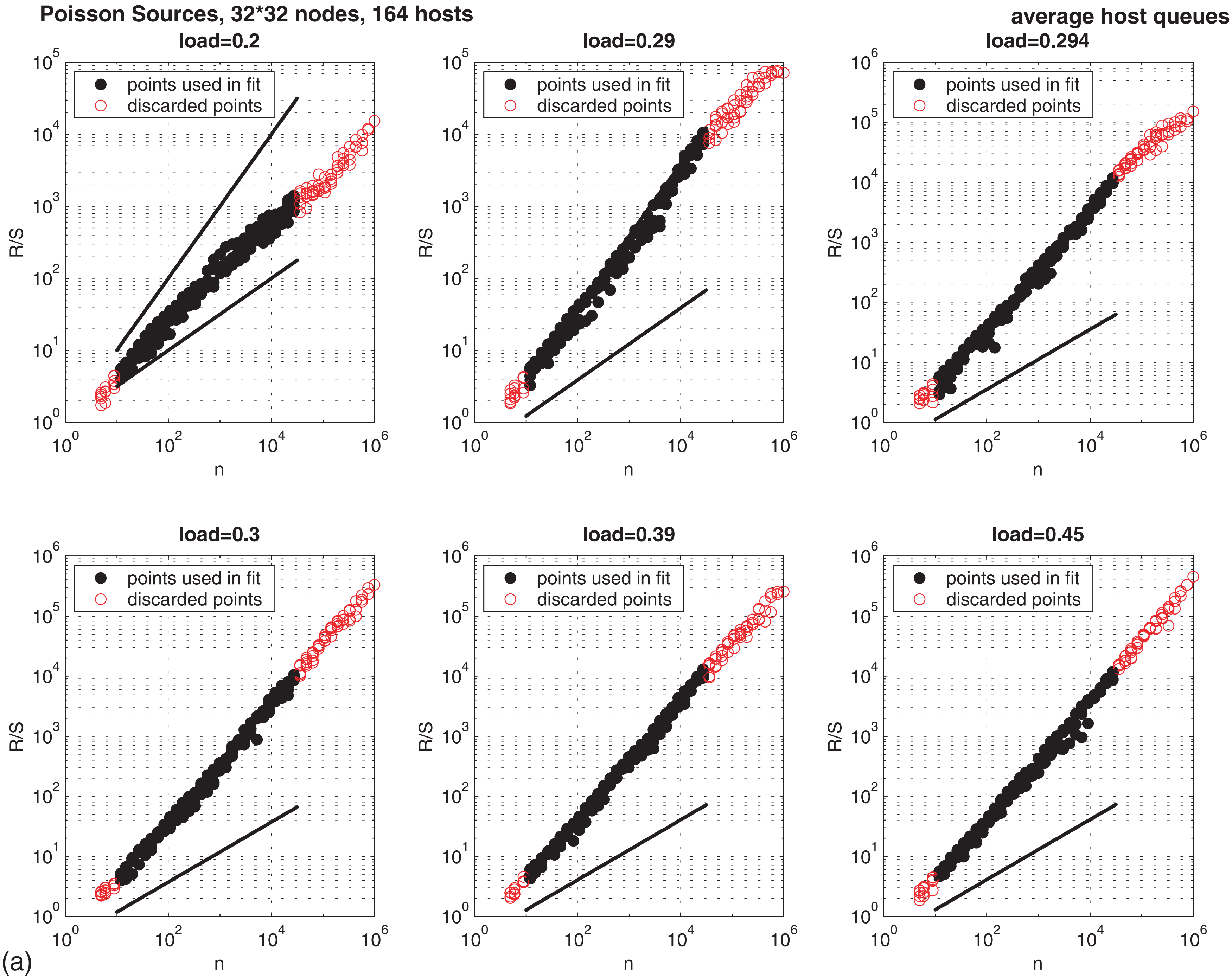

Load=0.39, Poisson sources, 32*32 nodes, 164 hosts
Path length: 1 step (data with trend removed)
Path length: 24 steps (data with trend removed)
points used in fit
discarded points
R/S
n
Path length: 1 step (original data)
Path length: 24 steps (original data)
P(delay)
delay

Figure 9(d)



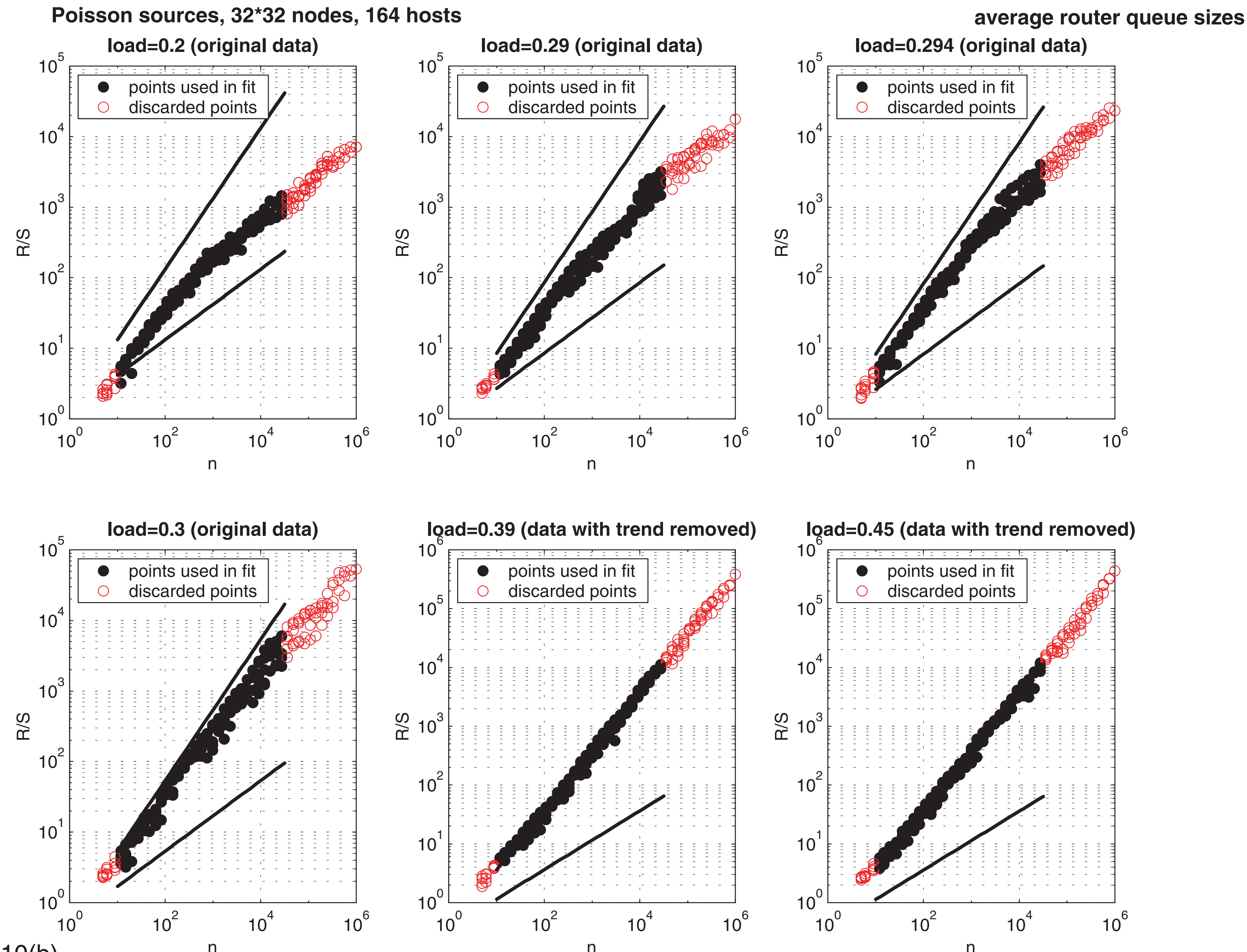
Load=0.45, Poisson sources, 32*32 nodes, 164 hosts
Path length: 1 step (data with trend removed)
points used in fit
discarded points
R/S
n
Path length: 24 steps (data with trend removed)
points used in fit
discarded points
R/S
n
Path length: 1 step (original data)
P(delay)
delay
Path length: 24 steps (original data)
P(delay)
delay

Figure 9(e)



Poisson Sources, 32*32 nodes, 164 hosts
average host queues
load=0.2
load=0.29
load=0.294
load=0.3
load=0.39
load=0.45
points used in fit
discarded points
R/S
n

Figure 10(a)



Poisson sources, 32*32 nodes, 164 hosts
average router queue sizes
load=0.2 (original data)
load=0.29 (original data)
load=0.294 (original data)
load=0.3 (original data)
load=0.39 (data with trend removed)
load=0.45 (data with trend removed)
points used in fit
discarded points
R/S
n

Figure 10(b)

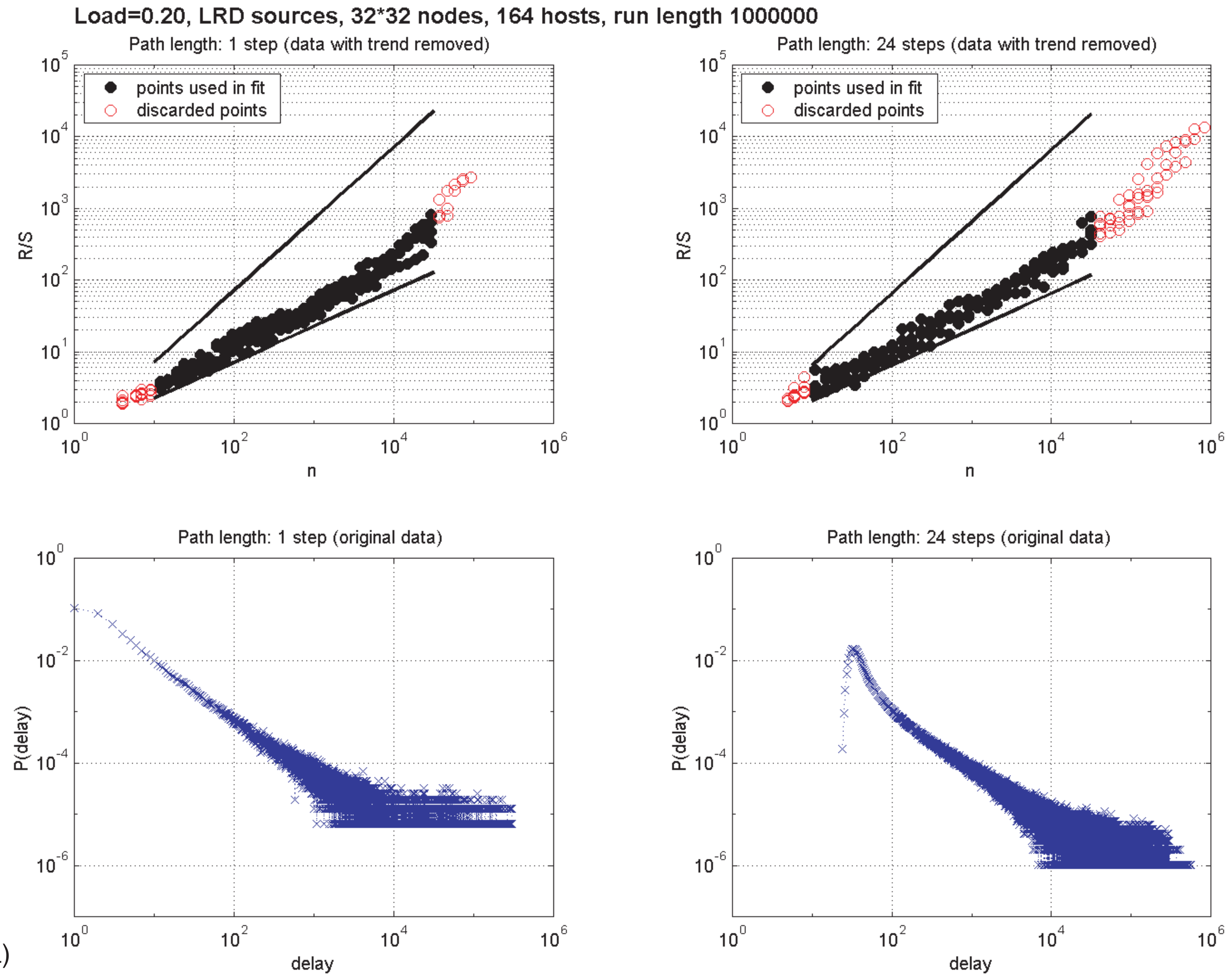

Load=0.20, LRD sources, 32*32 nodes, 164 hosts, run length 1000000
Path length: 1 step (data with trend removed)
Path length: 24 steps (data with trend removed)
points used in fit
discarded points
R/S
n
Path length: 1 step (original data)
Path length: 24 steps (original data)
P(delay)
delay

Figure 11(a)

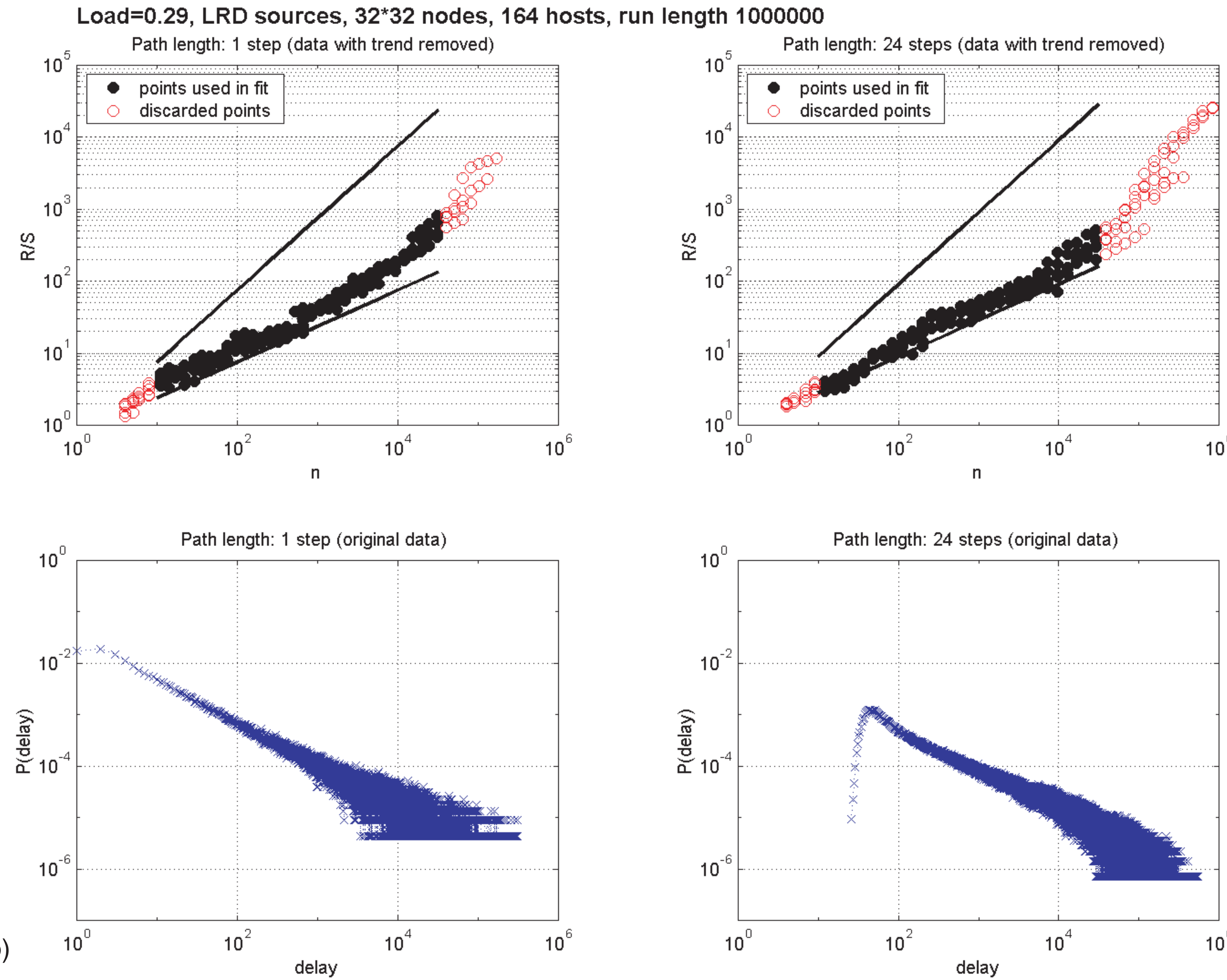

Load=0.29, LRD sources, 32*32 nodes, 164 hosts, run length 1000000
Path length: 1 step (data with trend removed)
Path length: 24 steps (data with trend removed)
points used in fit
discarded points
R/S
n
Path length: 1 step (original data)
Path length: 24 steps (original data)
P(delay)
delay

Figure 11(b)

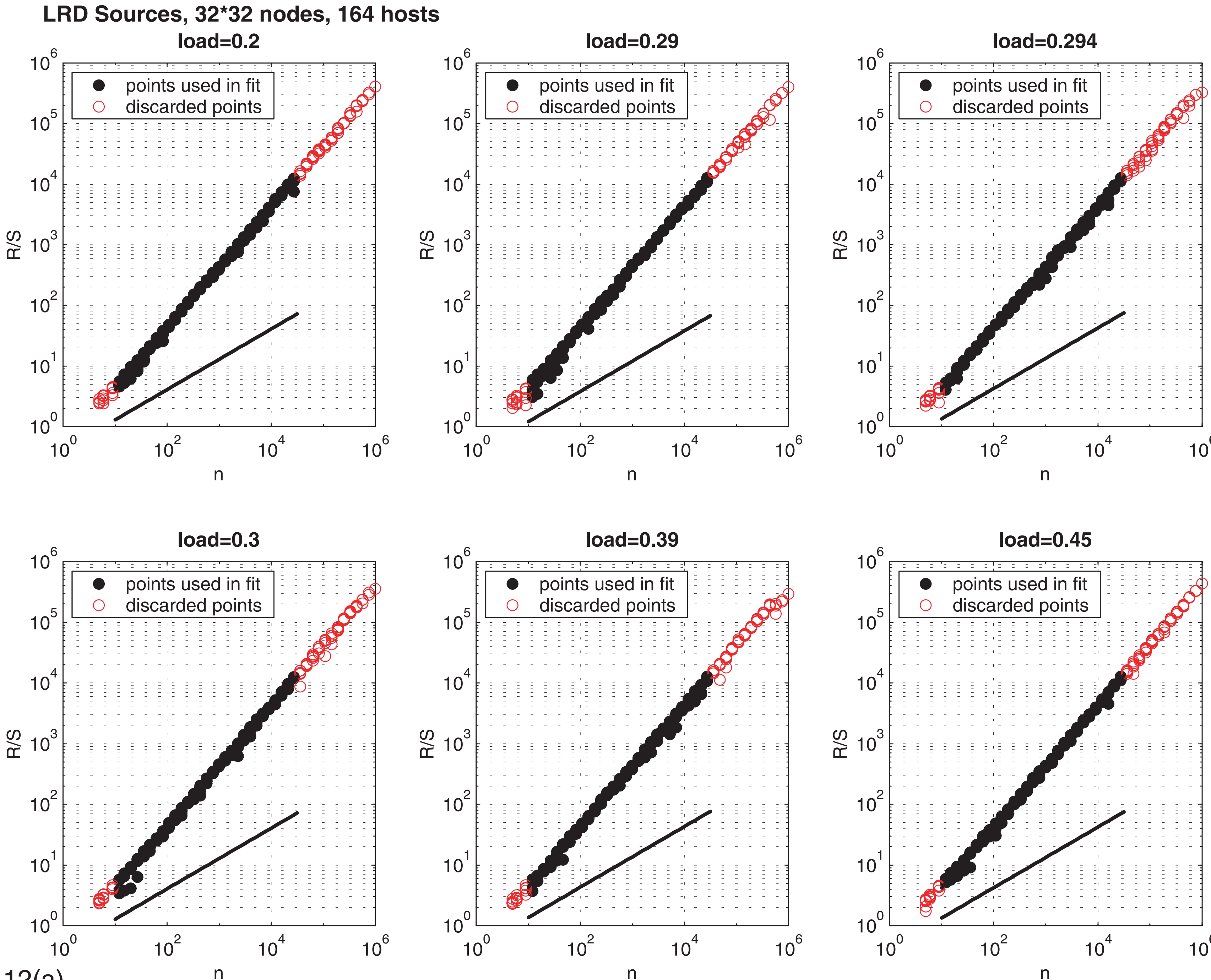
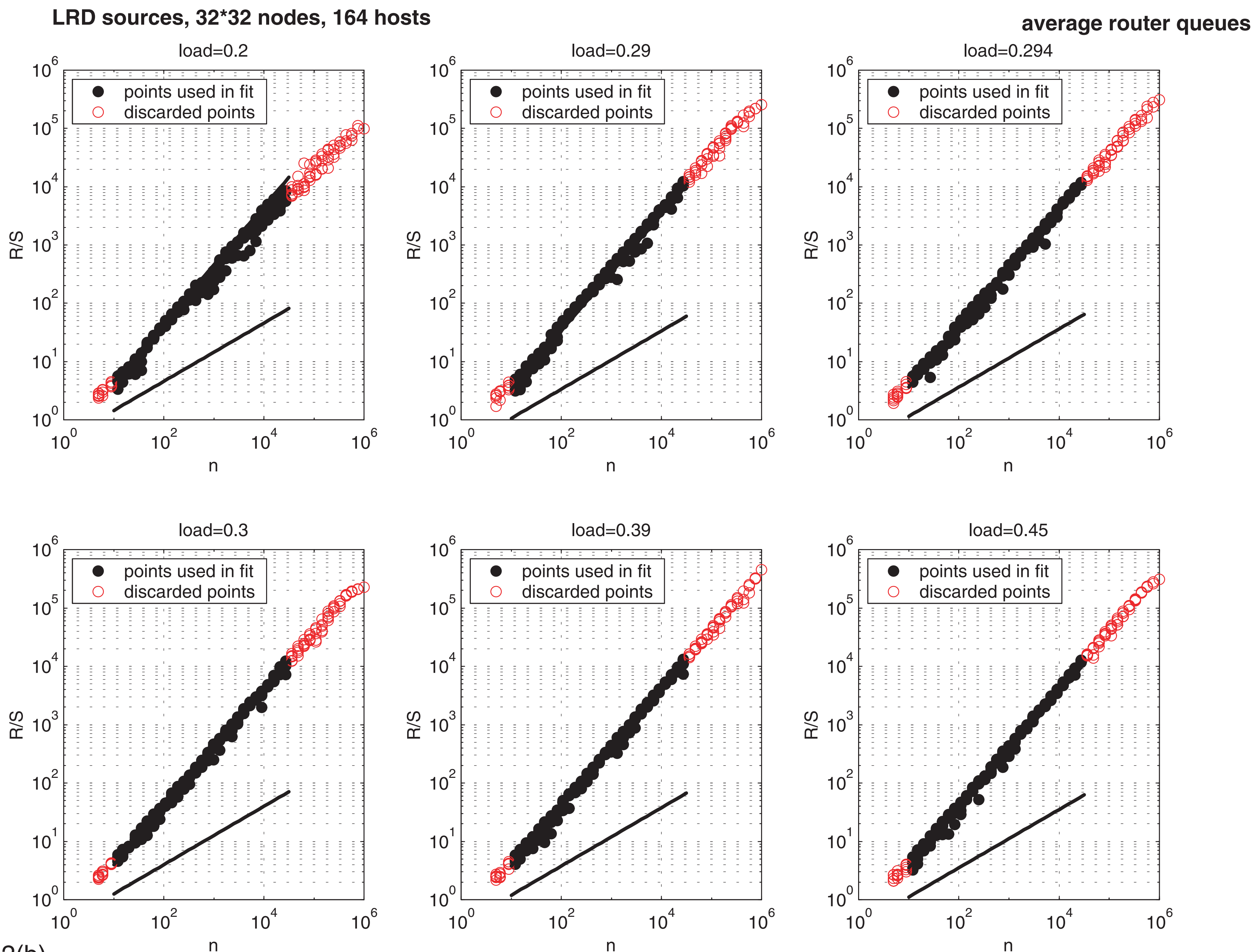
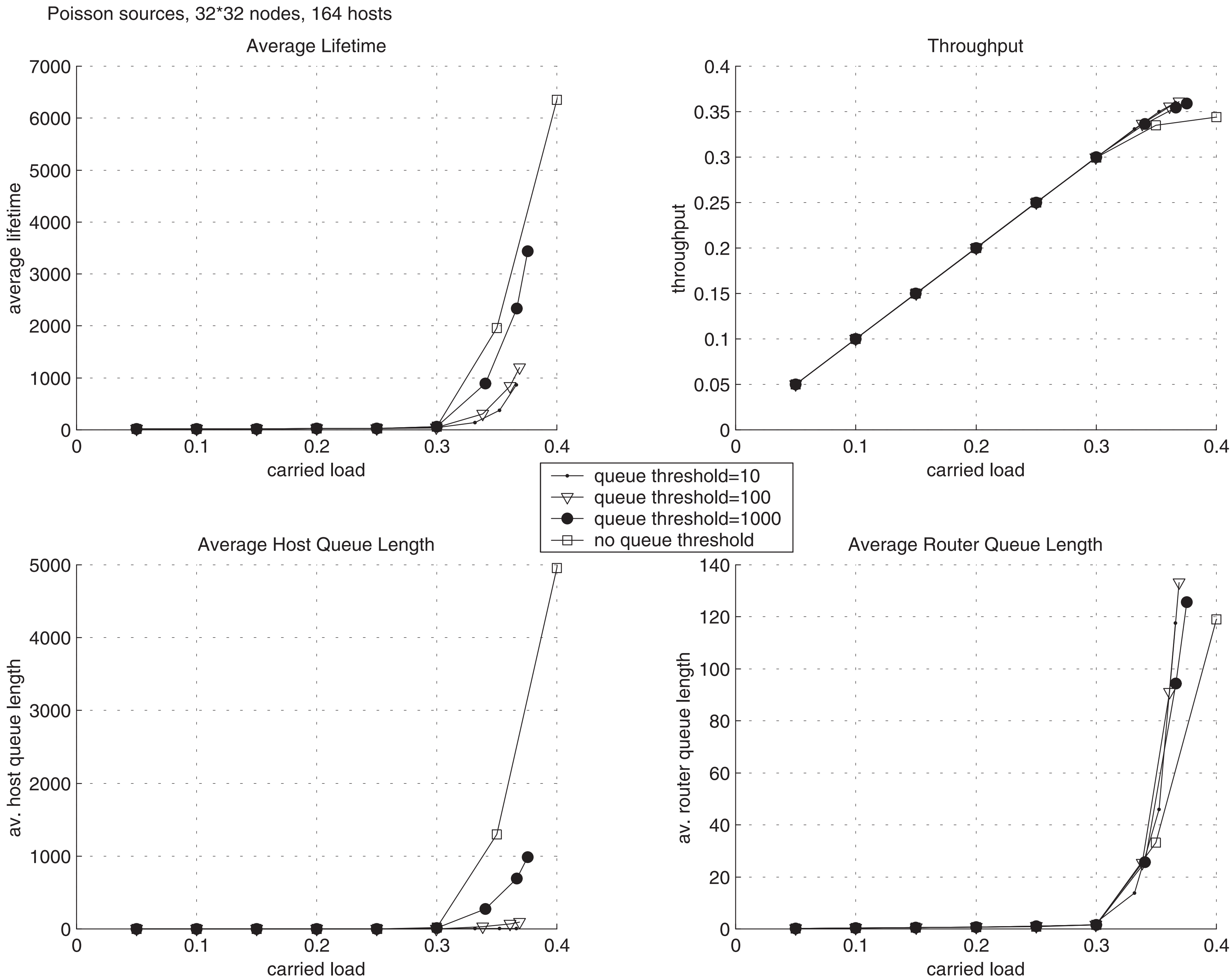

LRD Sources, 32*32 nodes, 164 hosts
load=0.2
load=0.29
load=0.294
load=0.3
load=0.39
load=0.45
points used in fit
discarded points
R/S
n

Figure 12(a)



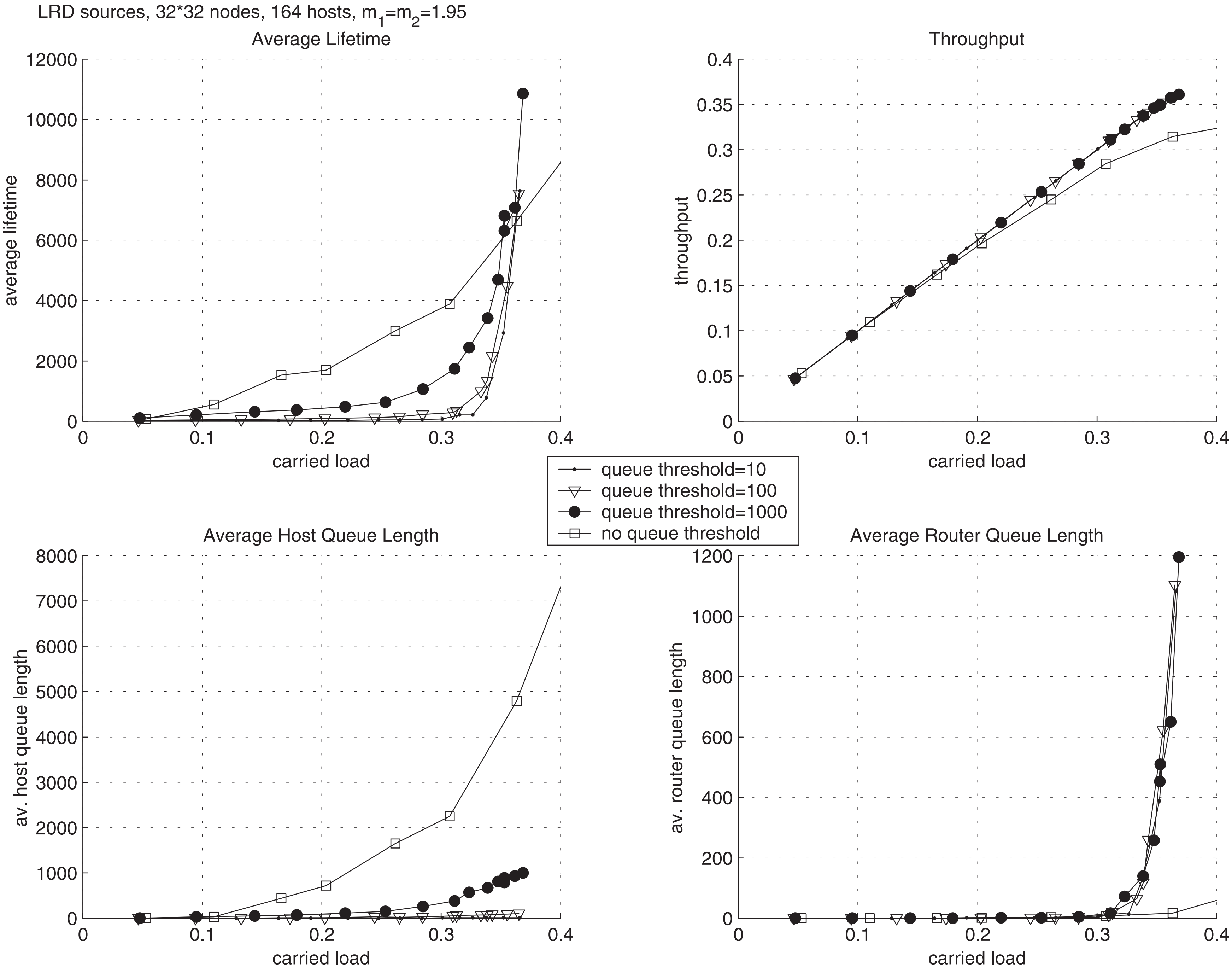
LRD sources, 32*32 nodes, 164 hosts
average router queues
load=0.2
load=0.29
load=0.294
load=0.3
load=0.39
load=0.45
points used in fit
discarded points
R/S
n

Figure 12(b)



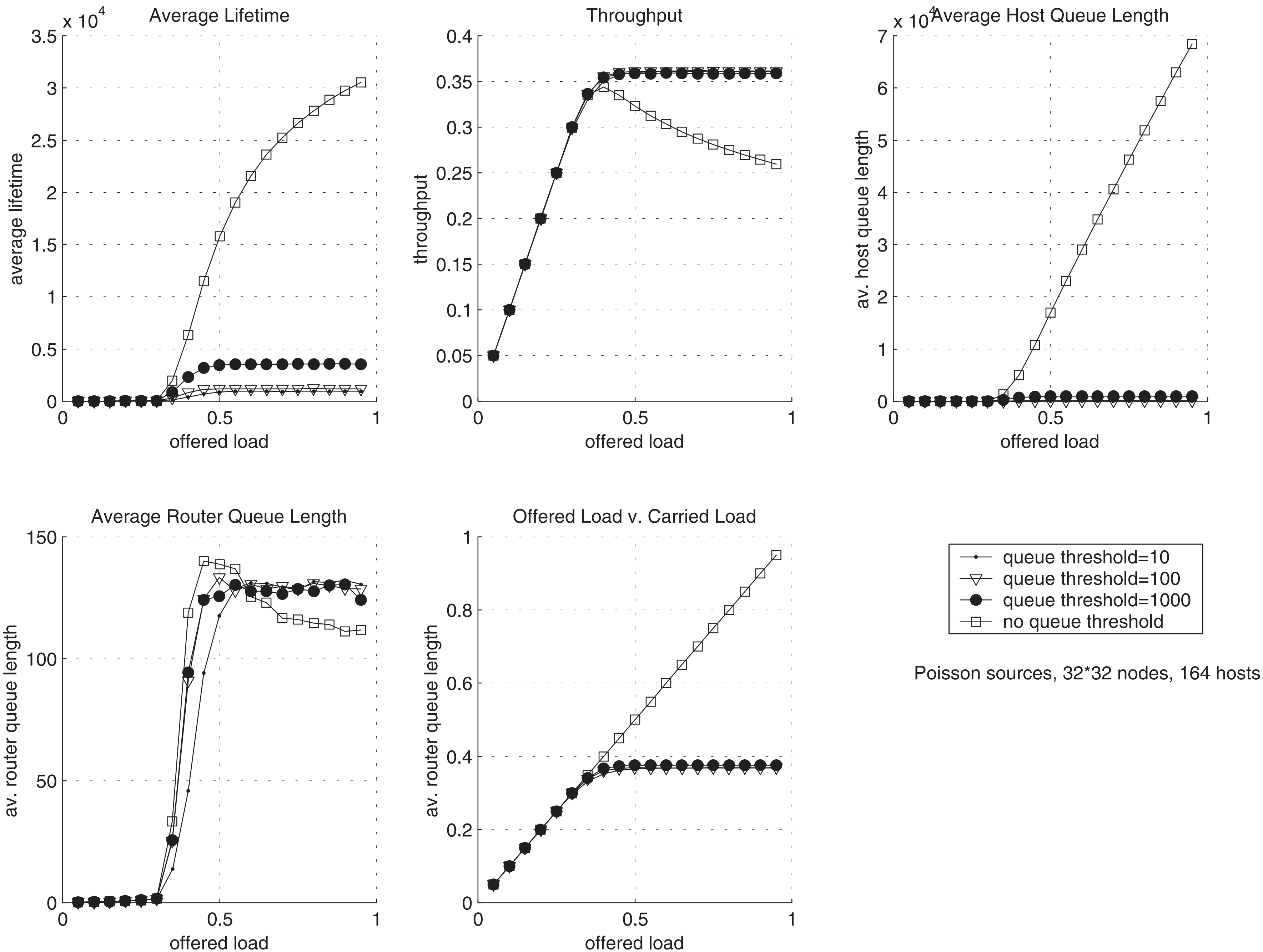
Poisson sources, 32*32 nodes, 164 hosts
Average Lifetime
average lifetime
carried load
Throughput
throughput
carried load
queue threshold=10
queue threshold=100
queue threshold=1000
no queue threshold
Average Host Queue Length
av. host queue length
carried load
Average Router Queue Length
av. router queue length
carried load

Figure 13(a)



LRD sources, 32*32 nodes, 164 hosts, m_1=m_2=1.95
Average Lifetime
average lifetime
12000
10000
8000
6000
4000
2000
0
0
0.1
0.2
0.3
0.4
carried load
Throughput
throughput
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
carried load
queue threshold=10
queue threshold=100
queue threshold=1000
no queue threshold
Average Host Queue Length
av. host queue length
8000
7000
6000
5000
4000
3000
2000
1000
0
carried load
Average Router Queue Length
av. router queue length
1200
1000
800
600
400
200
0
carried load

Figure 13(b)



Average Lifetime
x 10^4
average lifetime
offered load
Throughput
throughput
offered load
Average Host Queue Length
x 10^4
av. host queue length
offered load
Average Router Queue Length
av. router queue length
offered load
Offered Load v. Carried Load
av. router queue length
offered load
queue threshold=10
queue threshold=100
queue threshold=1000
no queue threshold
Poisson sources, 32*32 nodes, 164 hosts

Figure 14(a)

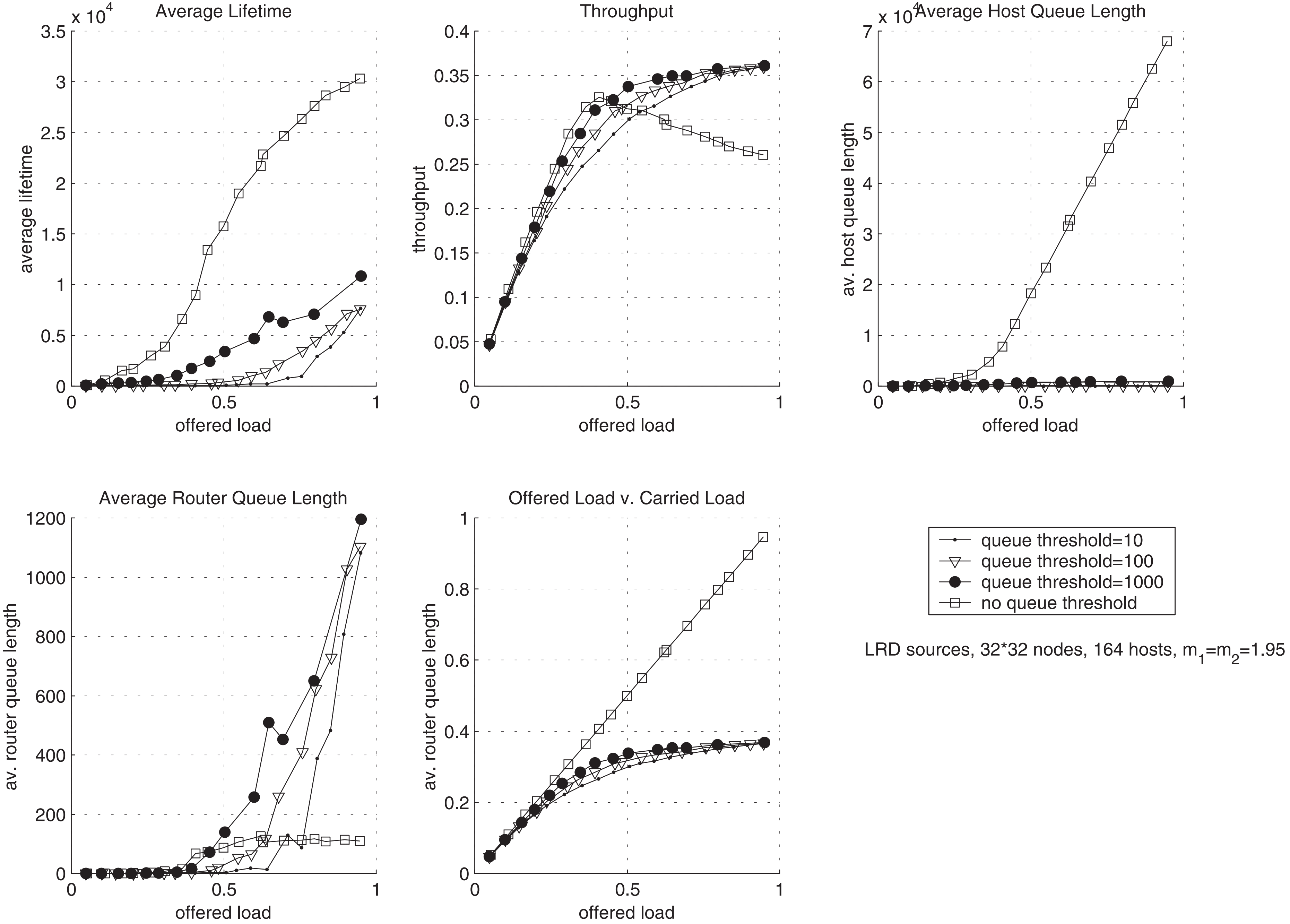

Average Lifetime
x 10^4
average lifetime
offered load
Throughput
throughput
offered load
Average Host Queue Length
x 10^4
av. host queue length
offered load
Average Router Queue Length
av. router queue length
offered load
Offered Load v. Carried Load
av. router queue length
offered load
queue threshold=10
queue threshold=100
queue threshold=1000
no queue threshold
LRD sources, 32*32 nodes, 164 hosts, m1=m2=1.95

Figure 14(b)