MANMADE

DIAGNOSING VULNERABILITY, EMERGENT PHENOMENA,
and VOLATILITY in MANMADE NETWORKS

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Spectral and topological analysis of weighted networks

F. Bono, E. Gutierrez, K. Poljansek

presented by

David Arrowsmith

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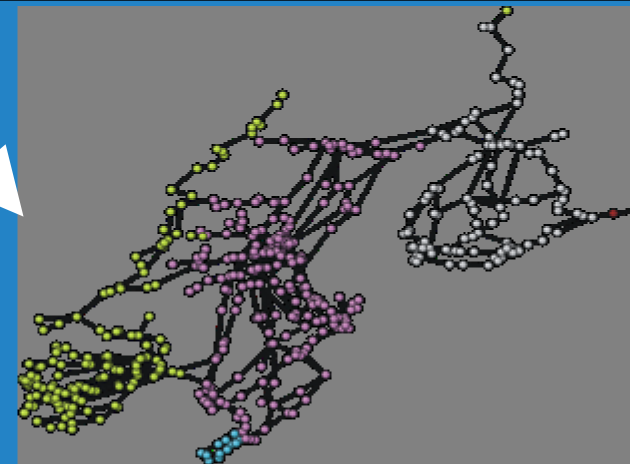
Collaborators

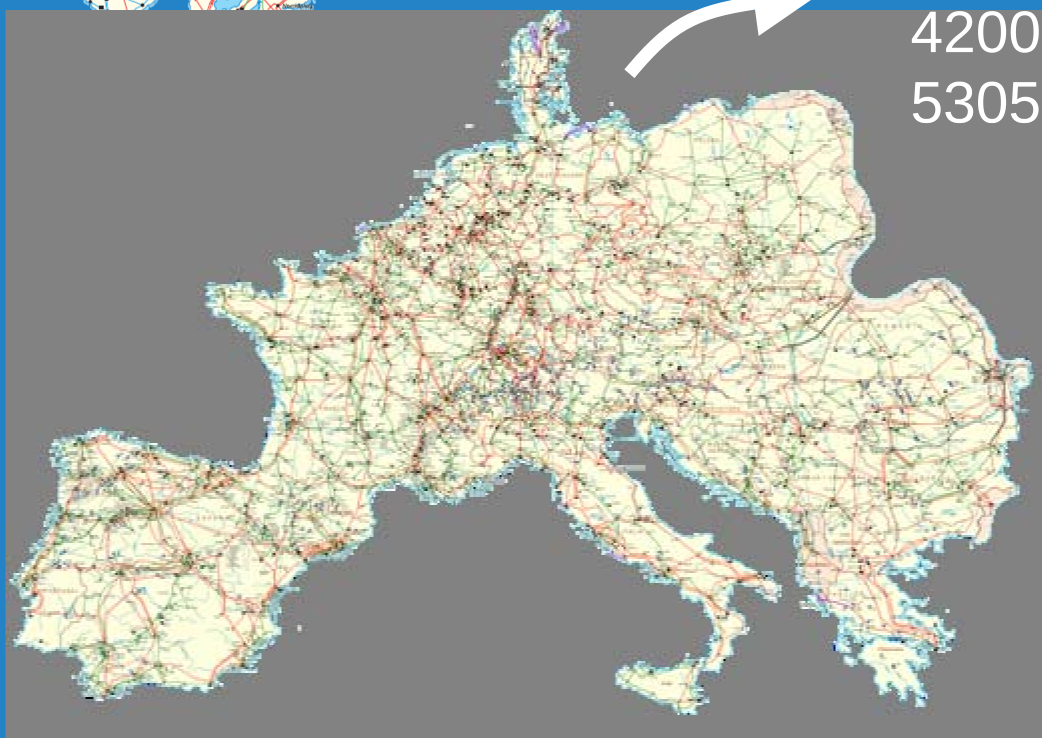
- Collegium Budapest
- EU Joint Research Centre, ISPRA
- Macedonian Academy of Sciences and Arts
- Queen Mary University of London
- Università Carlo Cattaneo
- Stakeholders
  - National Emergency Supply Agency, FINGRID
  - EDF-UK

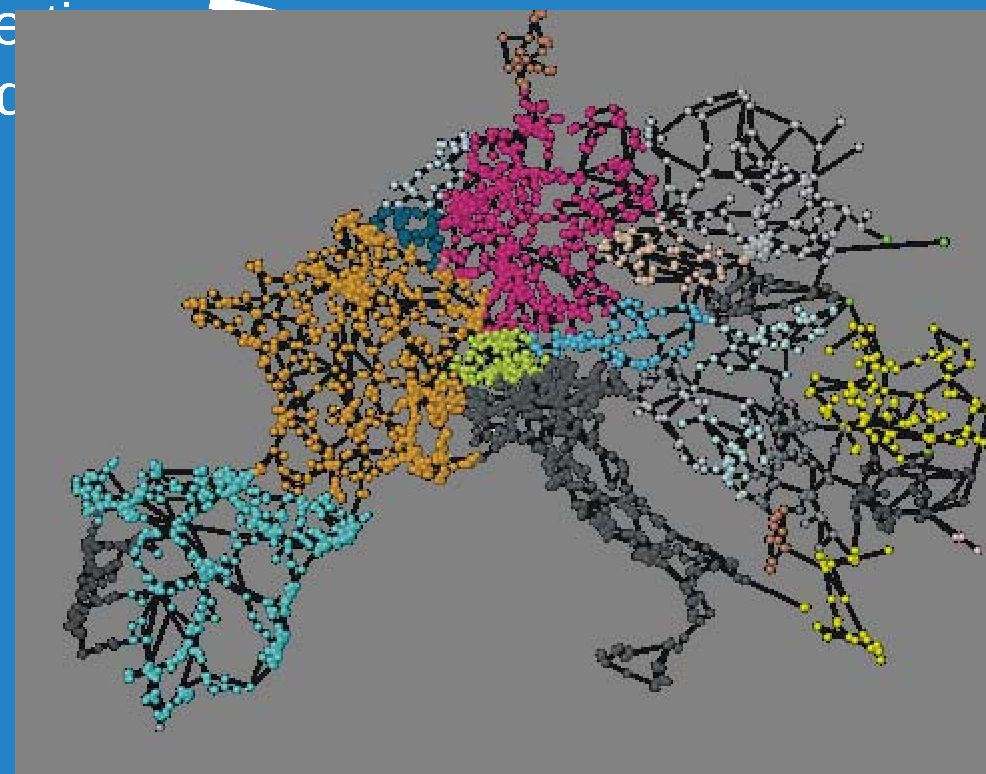
Electricity networks

NORDEL:
526 vertices
638 edges



UCTE:
4200 ve
5305 ed





Vulnerability

The more critical the component the more severe is the damage to the system when it is lost

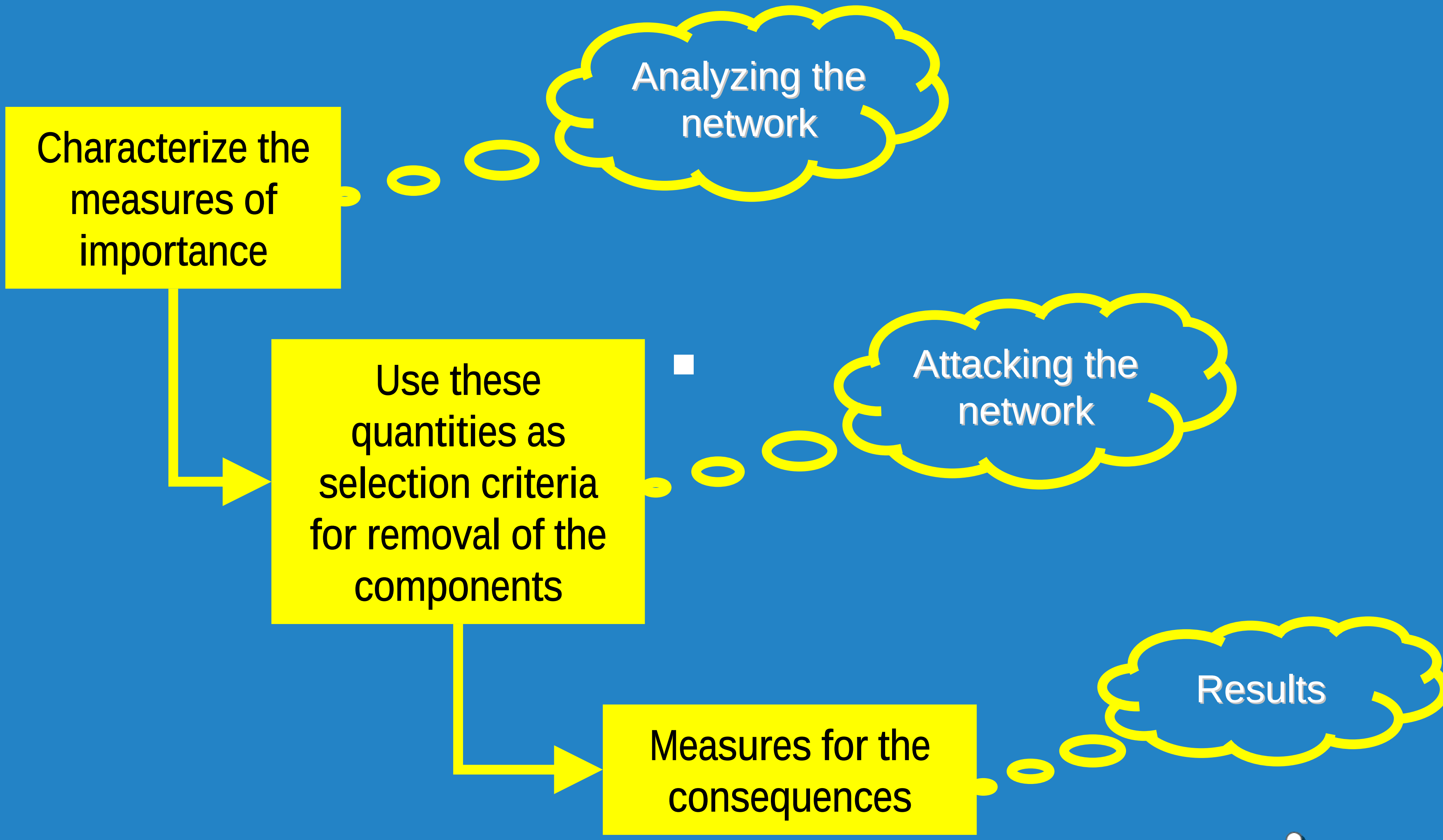
Criticality of the component ↔ Vulnerability of the system

Critical component = WEAK + IMPORTANT

WEAK: How susceptible to malfunction?

IMPORTANT: The consequences are great

How to tackle this issue?



Network graphs and incidence matrix

- Graph G consists of a set of *N* nodes $V=\{v_1, v_2, v_3, \ldots, v_N\}$ and a set of *M* edges $E=\{e_1, e_2, e_3, \ldots e_M\}$
- An edge connects two nodes $e=(v_i, v_j)$ – there can be multi-edges
- The *incidence* between the edges and the nodes is recorded via an *adjacency matrix* $A=(a_{ij})$
- If edge j is joined to node i, then $a_{ij}=1$, otherwise 0
- The *degree* of node i is the # of `1’ ’s in row i of *A*
- The indicators $a_{ij}=1$ or 0 can be changed to *weights* w_{ij} where if $a_{ij}=0$, then $w_{ij}=0$
- The *Laplacian matrix* $L=(l_{ij})$ is an N x N matrix with

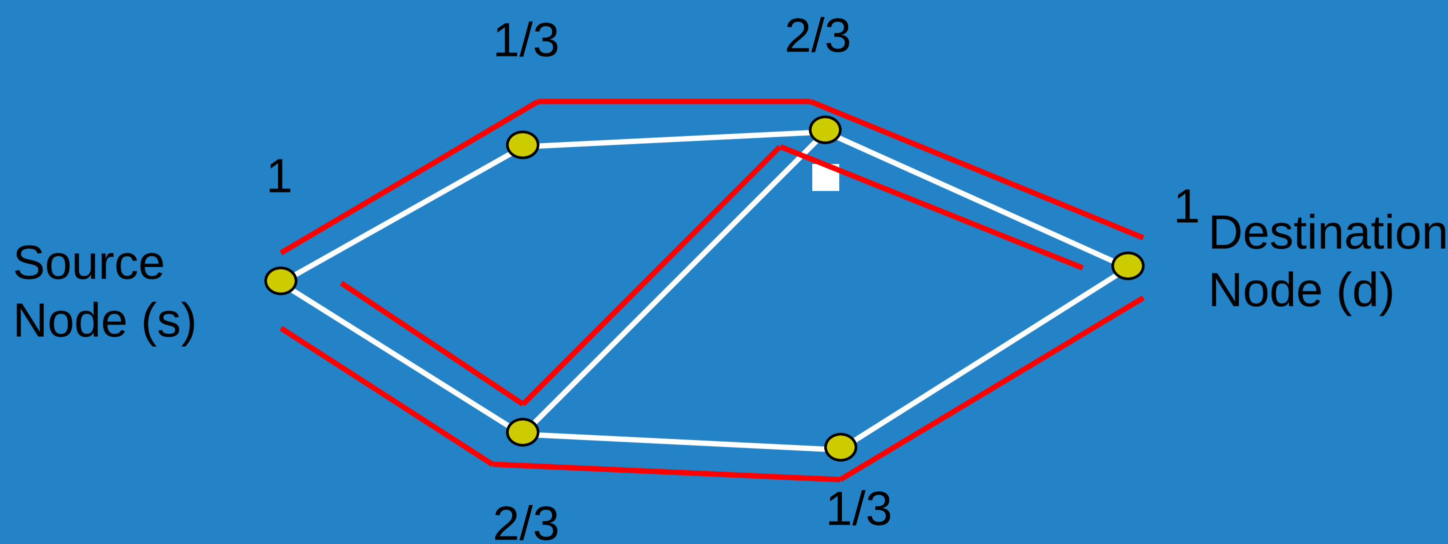
$L_{ij}=\deg(i)$ for $i=j$, $L_{ij}= -1$ for $i\neq j$ and v_i is adjacent by an edge to v_j, otherwise 0.

Relevance of the Laplacian matrix

- The *algebraic connectivity* of a graph G is the second-smallest eigenvalue of the Laplacian Matrix of G
- This eigenvalue is greater than 0 if and only if G is a connected graph
- This is a corollary to the fact that the number of times 0 appears as an eigenvalue in the Laplacian is the number of connected components in the graph
- The magnitude of this value reflects how well connected the overall graph is, and has implications for properties such as synchronizability and clustering

Betweenness centrality

- *Betweenness centrality* measures the importance of nodes in terms of the frequency of their appearance on shortest paths



- *Betweenness* of node i for the pair *sd* = relative number of shortest paths between s and d which visit node I
- *Betweenness* of node i for graph requires calculation for all pairs of vertices

Analysis of weighted undirected networks

Graph -> Network ▶ $G = (V, E)$

Vertices (substations)

Edges (electrical lines)

Adjacency matrix $A(G)$

defines which vertices are connected

$$a_{ij} = \begin{cases} 1, \text{if } v_i \text{ and } v_j \text{ are neighbours,} \\ 0, \text{otherwise} \end{cases}$$

Weights matrix $W(G)$

defines the weights on the edges

$$w_{ij} = \begin{cases} w_{ij}, \text{if } a_{ij} = 1, \\ 0, \text{ if } a_{ij} = 0 \end{cases}$$

Weights = Capacity of the electrical line (kV)

[220kV – 2500kV]

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Measures of importance

Inverse “distance” from the given node to every other node in network - efficiency

$$C_i = \sum_j \frac{1}{d_{ij}}$$

shortest paths in the graphs

d_{ij}

Degree

Efficiency / Closeness

Betweenness

$$B_i = \sum_v \sum_u \frac{n_{uv}(i)}{n_{uv}}$$

The proportion of all shortest paths in the network that run through a given node

Centrality indices

Modal index

$$D_i = \sum_j a_{ij}$$

or

$$D_i = \sum_j w_{ij}$$

Spectral decomposition of the Laplacian matrix

$$L = D - W \rightarrow \lambda_i \phi_i = L \phi_i$$

$$\Gamma = L'\Phi \rightarrow M_i = \sum_j |\gamma_{ij}|$$

Sum of modal contributions (load in each mode) for each vertex

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Weighted vs. unweighted node ranking

of Nordel network

Spearman ranking correlation coefficients for
Efficiency, Betweenness and Spectral analysis

NORDEL	Eff-W	Eff-UnW	BC-W	BC-UnW	SA-W	SA-UnW
Eff-W	1.00	0.89	0.34	0.32	0.22	0.58
Eff-UnW	0.89	1.00	0.42	0.44	0.55	0.72
BC-W	0.34	0.42	1.00	0.96	0.51	0.61
BC-UnW	0.32	0.44	0.96	1.00	0.57	0.62
SA-W	0.22	0.55	0.51	0.57	1.00	0.76
SA-UnW	0.58	0.72	0.61	0.62	0.76	1.00

WEIGHTED ⟷ UNWEIGHTED

BETWEENNESS ⟷ EFFICIENCY

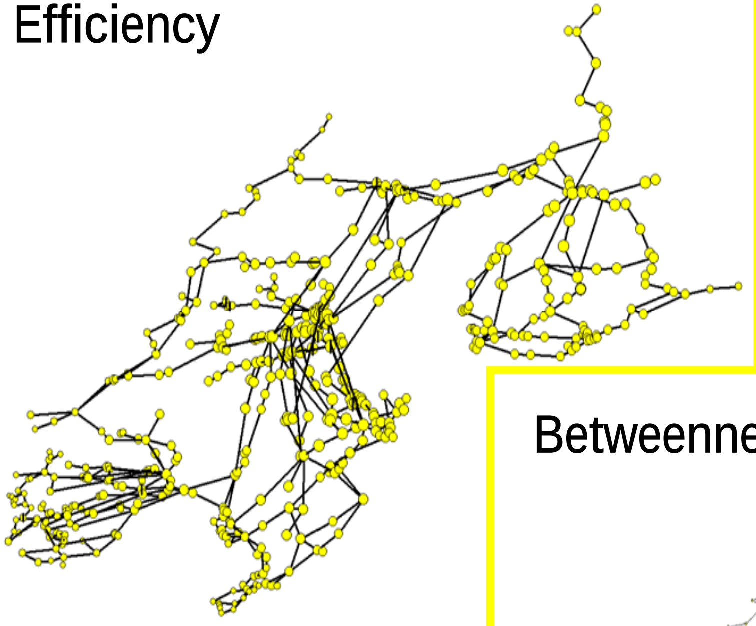
SPECTRAL ANALYSIS

Ranking

Size of the vertices
Width of the edges → the relative value of the importance

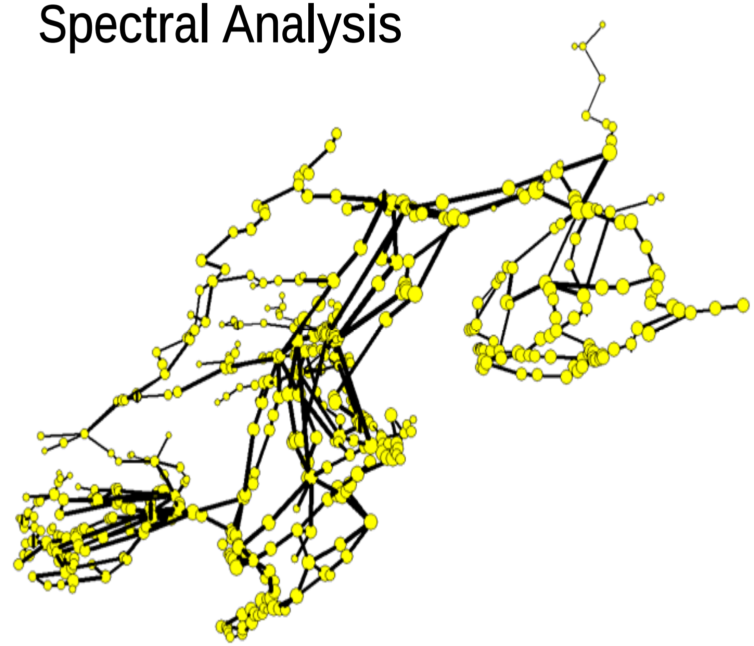
NORDEL

Efficiency



Betweenness

Spectral Analysis



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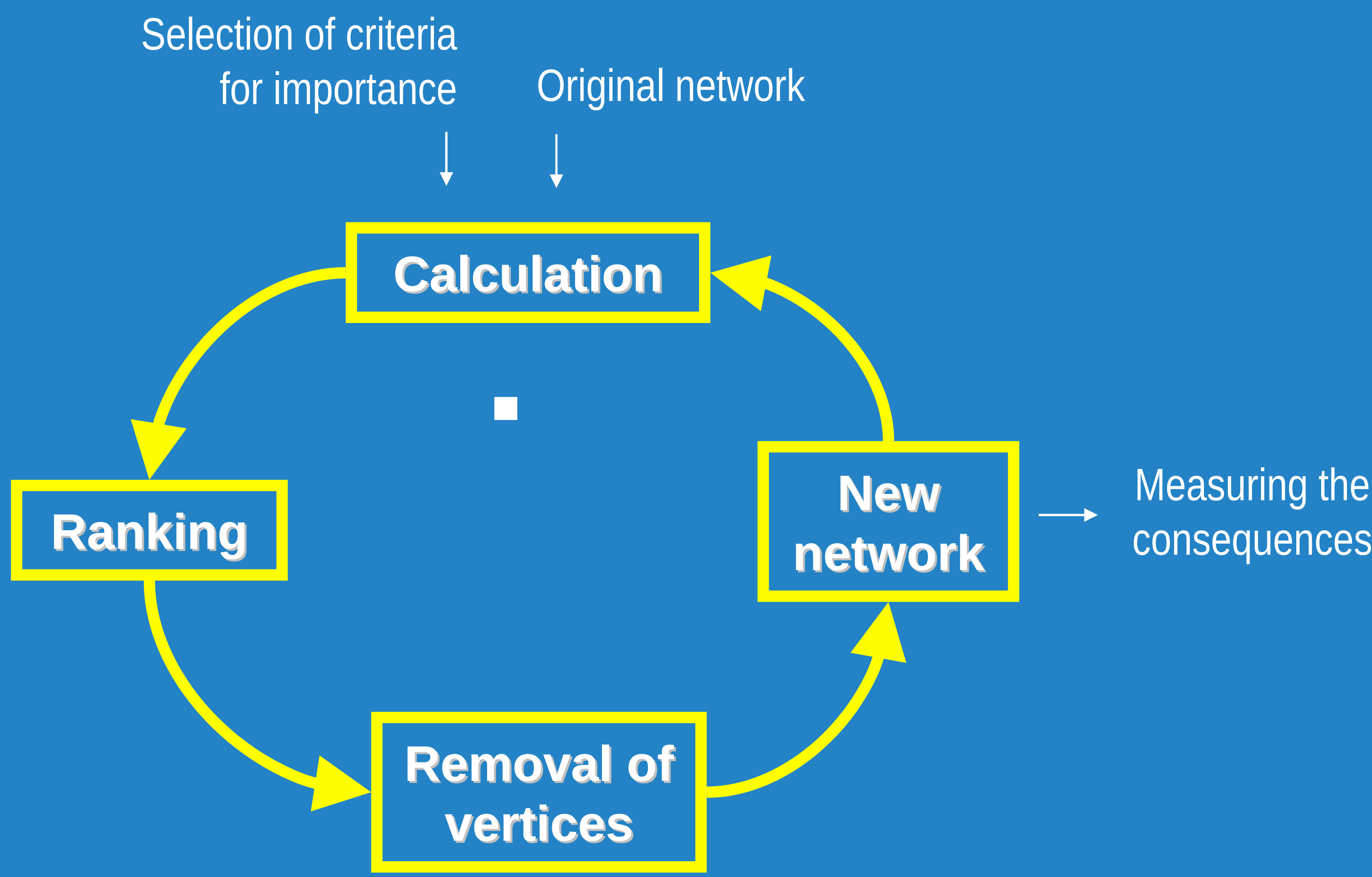
Algorithm of attacking the network



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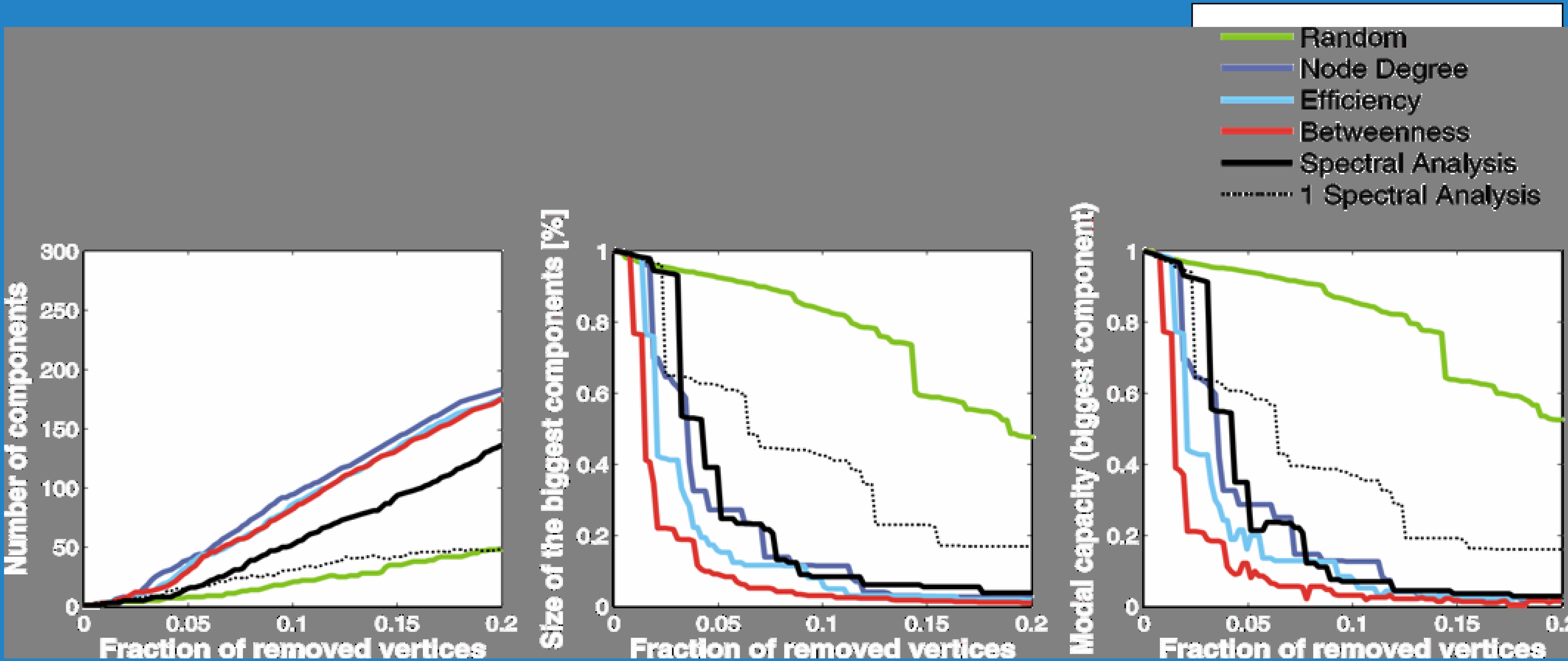
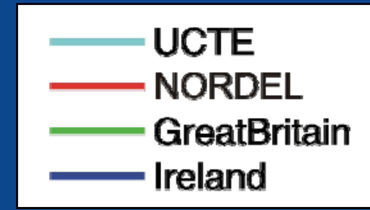
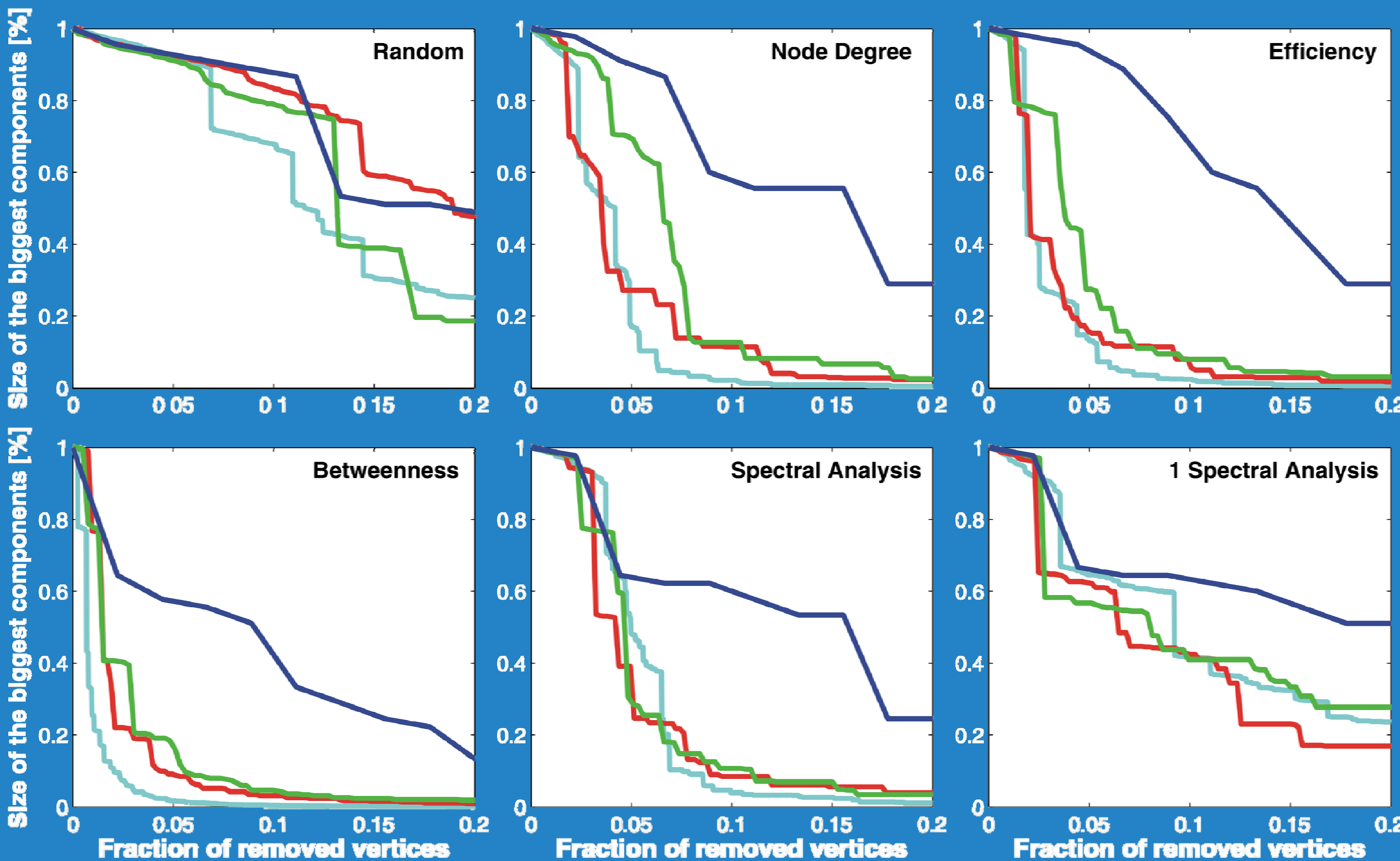
Measuring the consequences



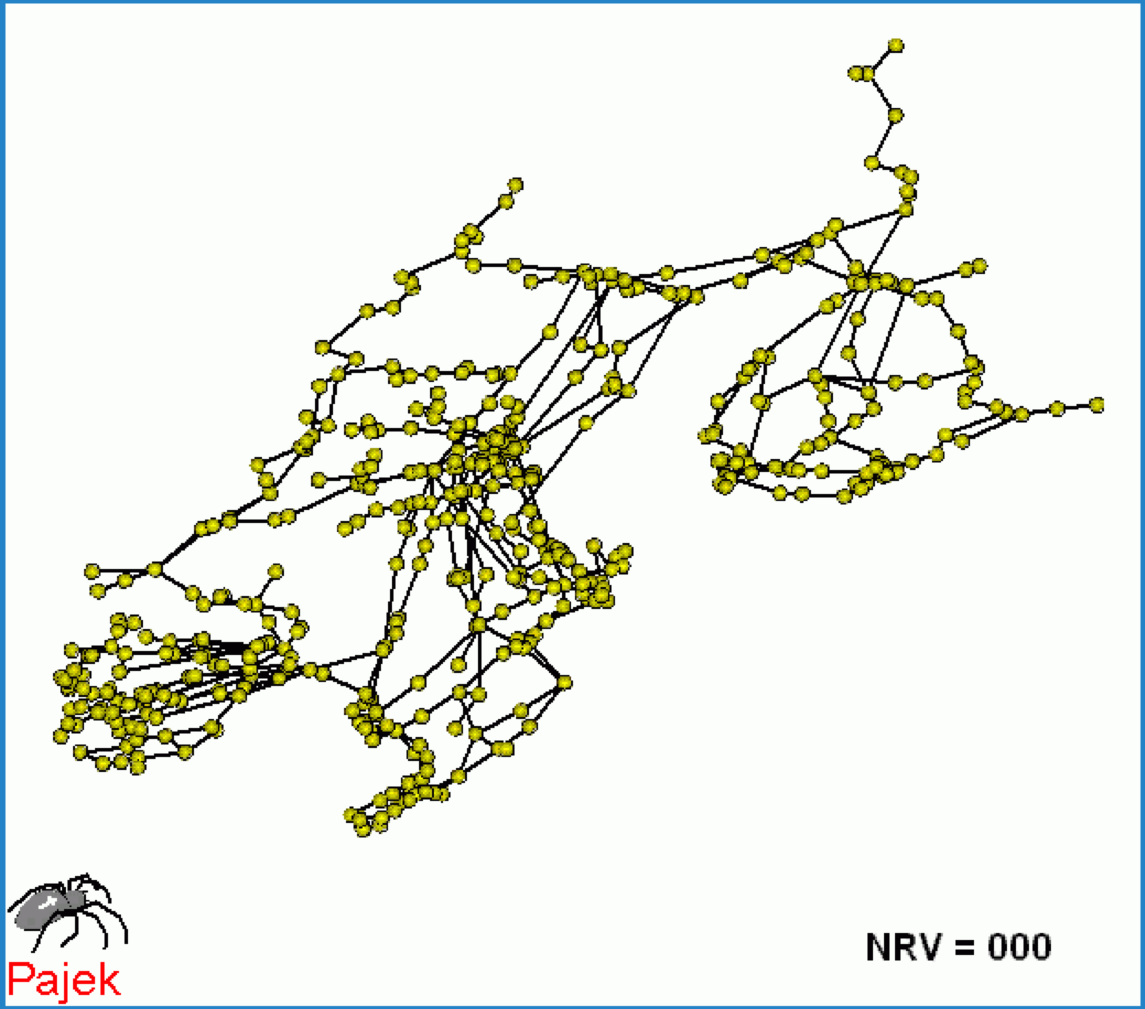
NORDEL

Measuring the consequences





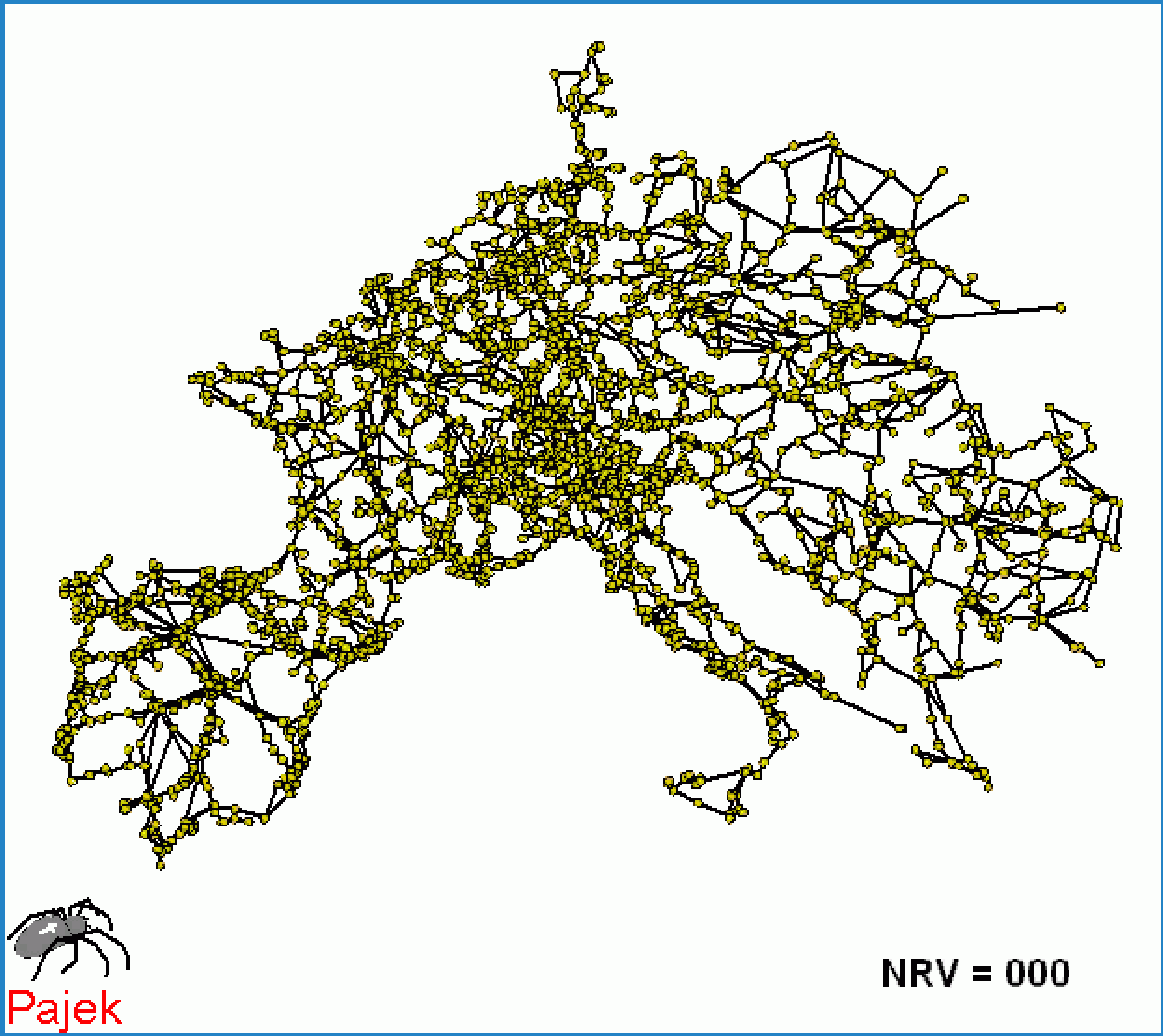
BREAK-UP OF THE NETWORK



Betweenness centrality
NORDEL

NRV – number of removed vertices

BREAK-UP OF THE NETWORK



Betweenness centrality UCTE

NRV – number of removed vertices

FASTEST DECAY OF NETWORK

SIMULATION OF THE NETWORK ATTACK

•Removal of random nodes according to a ranking
•Decay is broadly comparable for the different types of status BC, Eff, SA
•Fastest first level fragmentation is by using the measure –

BETWEENNESS CENTRALITY

•other higher level fragmentation needs to be considered

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Conclusions

- We can build models to attack the network and consider disconnections
- Conclusions are not simple in that major disconnections of themselves do not necessarily produce disruption
- So far we have used topological and graph theoretical conditions to rank nodes
- We can also consider the relative "attack efficiencies" of the various EU countries
- Clear conclusions can be drawn within the mathematical framework
- The other aspects of "importance" of a node may be political, geographical, physical
- This is what we are now trying to deal with through stakeholders

such as NESA in Finland

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