

# MODELLING INTERNET PACKET TRAFFIC CONGESTION

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MATHEMATICAL SCIENCES & ELECTRONIC ENGINEERING

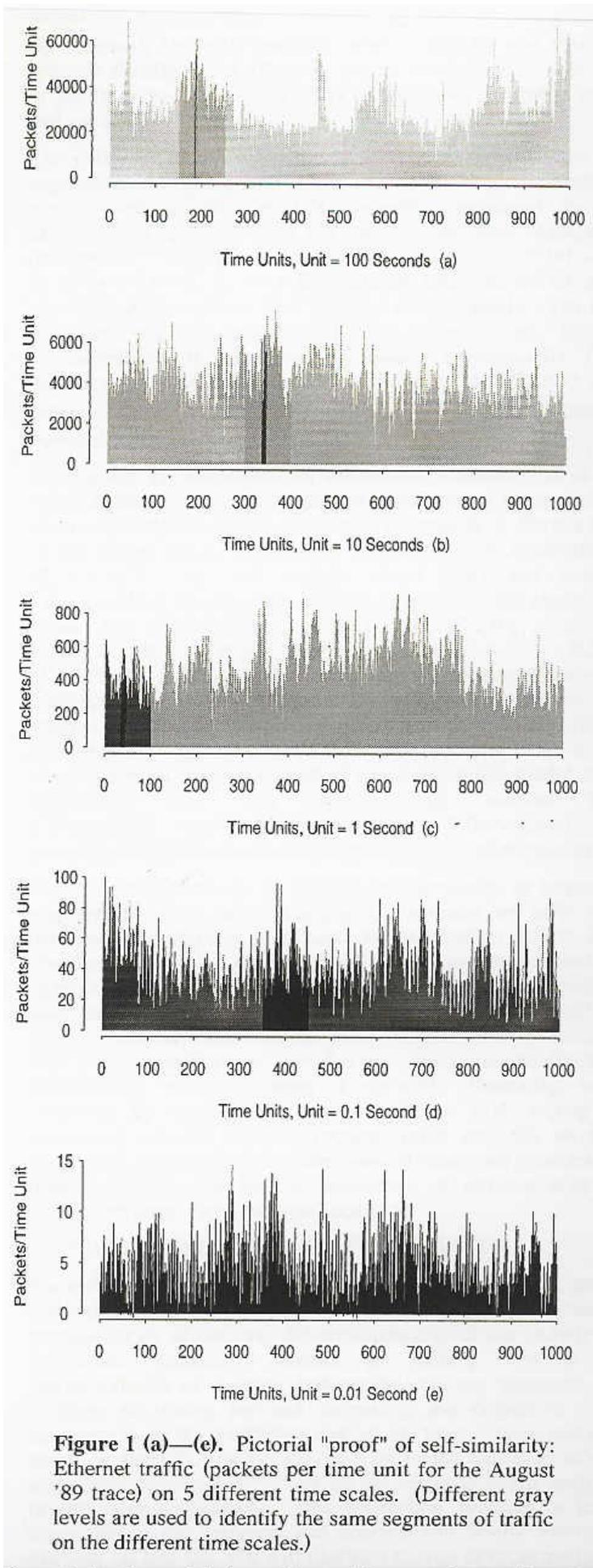
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[www.maths.qmul.ac.uk/~arrow/IEEEBangkok.pdf](http://www.maths.qmul.ac.uk/~arrow/IEEEBangkok.pdf)

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IEEE/May 2003



**Figure 1 (a)–(e).** Pictorial "proof" of self-similarity: Ethernet traffic (packets per time unit for the August '89 trace) on 5 different time scales. (Different gray levels are used to identify the same segments of traffic on the different time scales.)

*Ethernet*

*packet rates*

*on different*

*time scales*

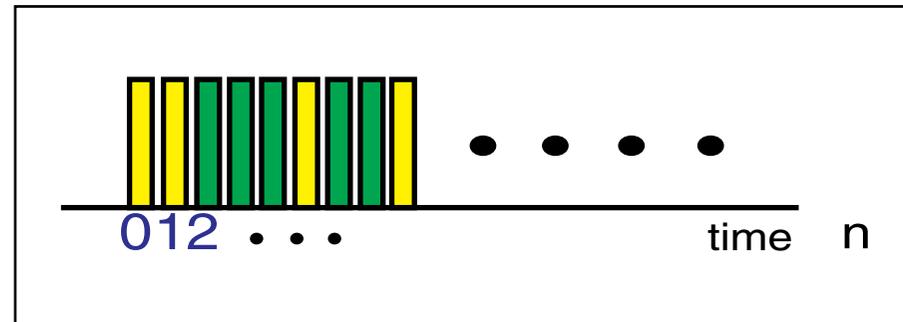
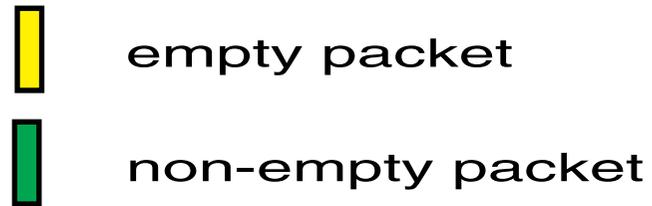
*Leland et al*

*1995*

*"Pictorial proof*

*of self-similarity"*

# COMMUNICATIONS - PACKET TRAFFIC



1994 - "On the self-similar nature of Ethernet traffic", Leland *et al*, *Comp Comm Rev*

- "bursty activity" of packet rates over many time-scales
- conventional "Poisson" models for over optimistic on queue performance
- "Poisson-like" view of aggregated sources suggests smoother traffic

# AUTOCORRELATION - nature of traffic streams

STATIONARY BINARY TIME SERIES:

$$X_t \in \{0, 1\}, \quad t = 1, 2, 3, \dots$$

BATCHED TIME SERIES- size  $N$ :

$$X_t^{(N)} = \frac{1}{N} \sum_{j=tN+1}^{(t+1)N} X_j$$

AUTO-CORRELATION FORMULA for  $X_t$  :

$$c(k) = \frac{E(X_t X_{t+k}) - E(X_t)E(X_{t+k})}{\sqrt{(\text{Var}(X_t)\text{Var}(X_{t+k}))}}$$

AUTO-CORRELATION FOR **LRD** TRAFFIC:

**POWER  
LAW DECAY**

$$c(k) \sim k^{-\beta}, \quad \beta \in (0, 1)$$

AUTO-CORRELATION FOR **SRD** TRAFFIC:

**EXP  
DECAY**

$$c(k) \sim \alpha^{-k}, \quad \alpha > 1$$

SELF-SIMILARITY FOR **LRD** TRAFFIC :

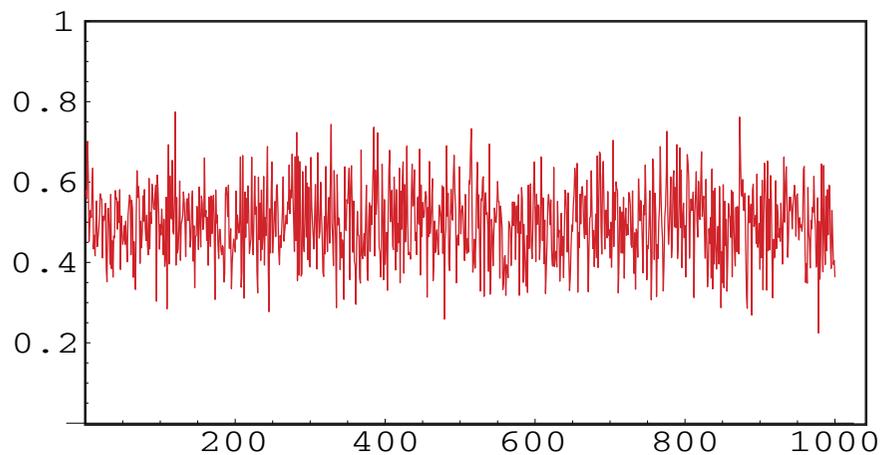
2nd ORDER statistics of  $X_t$  and  $X_t^{(N)}$  are the same

SELF-SIMILARITY HURST PARAMETER  $H$  :

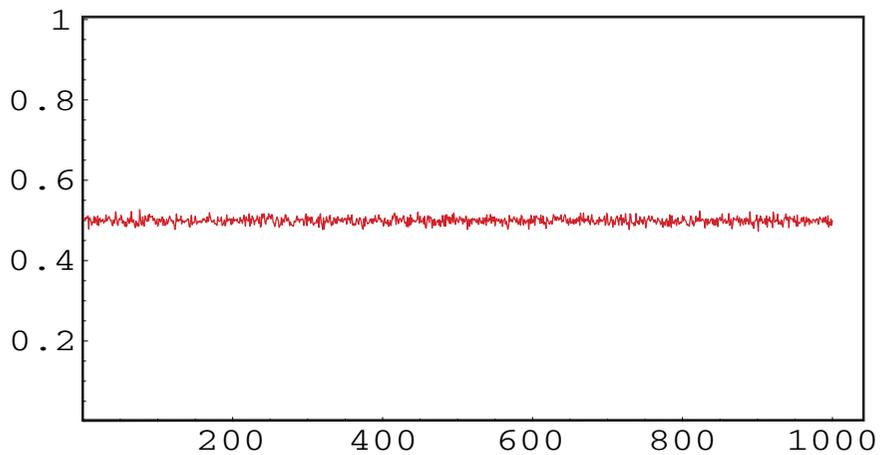
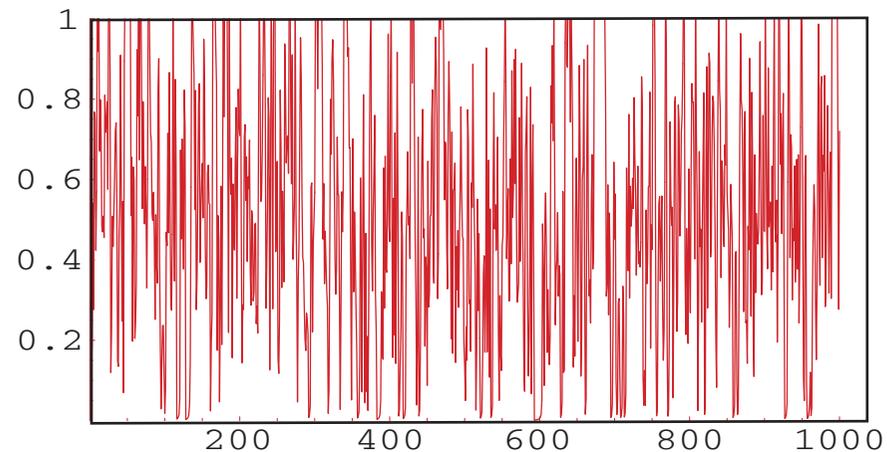
$$H = 1 - \beta / 2$$

# *SRD and LRD OUTPUT*

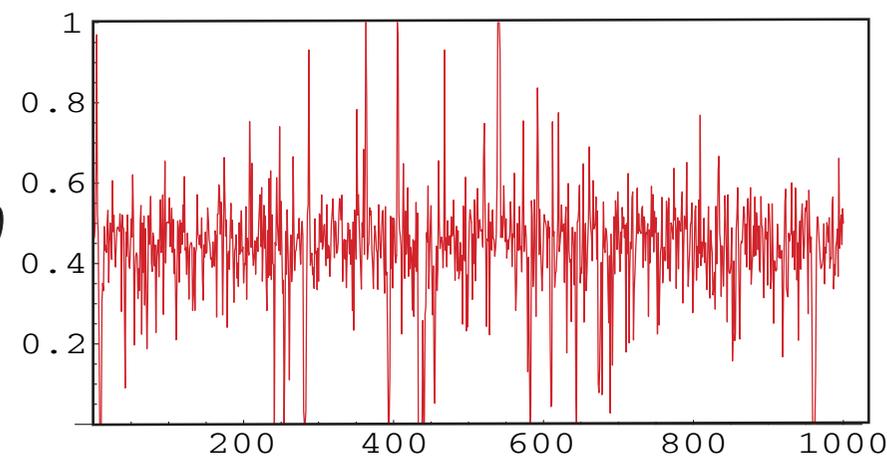
□  $N = \text{BATCH SIZE of AVERAGED DATA}$



$N=100$



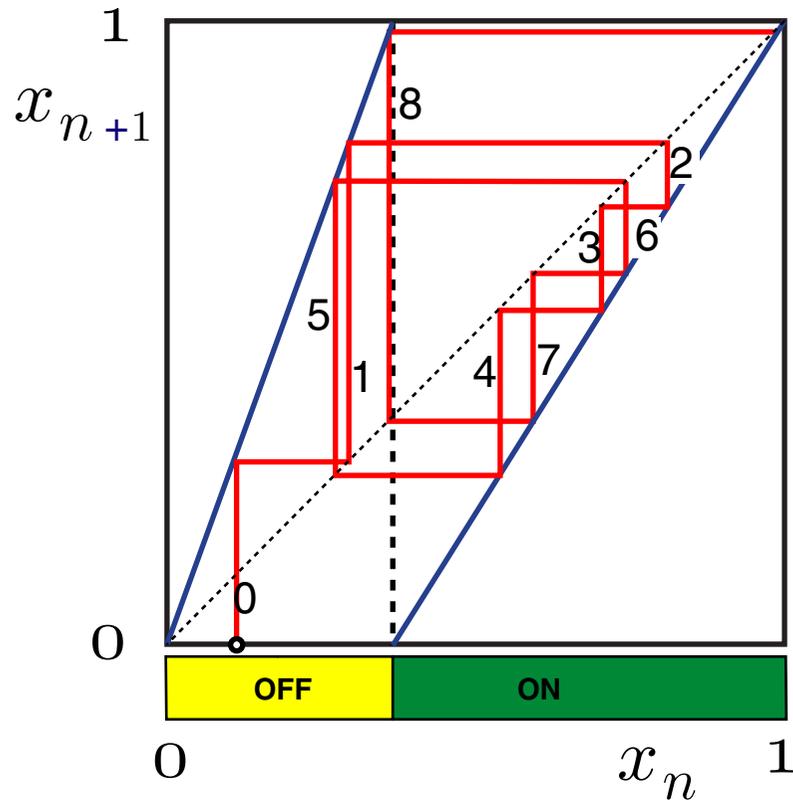
$N=10000$



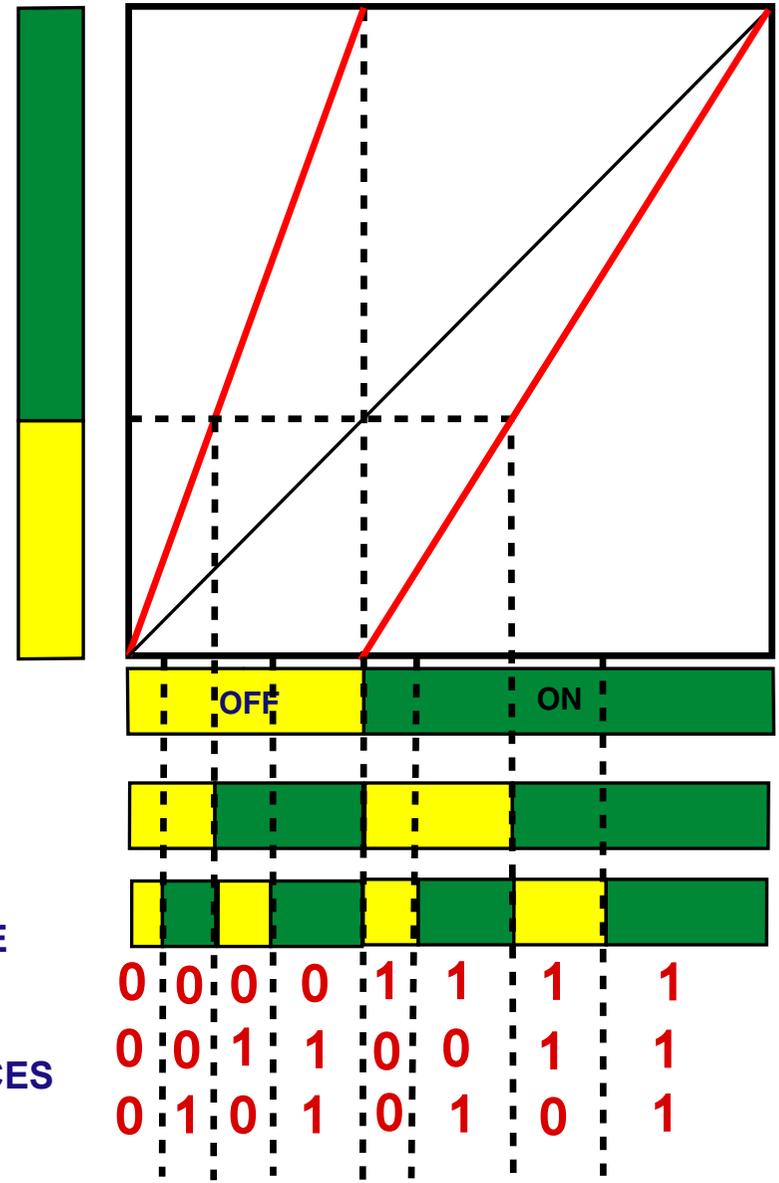
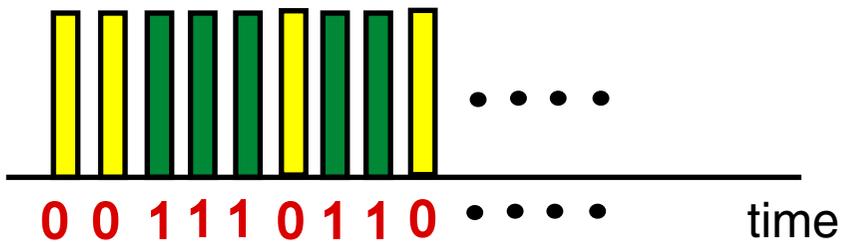
*Short range dependent*

*Long range dependent*

# CHAOTIC MAPS *DIGITAL OUTPUT and CODING*



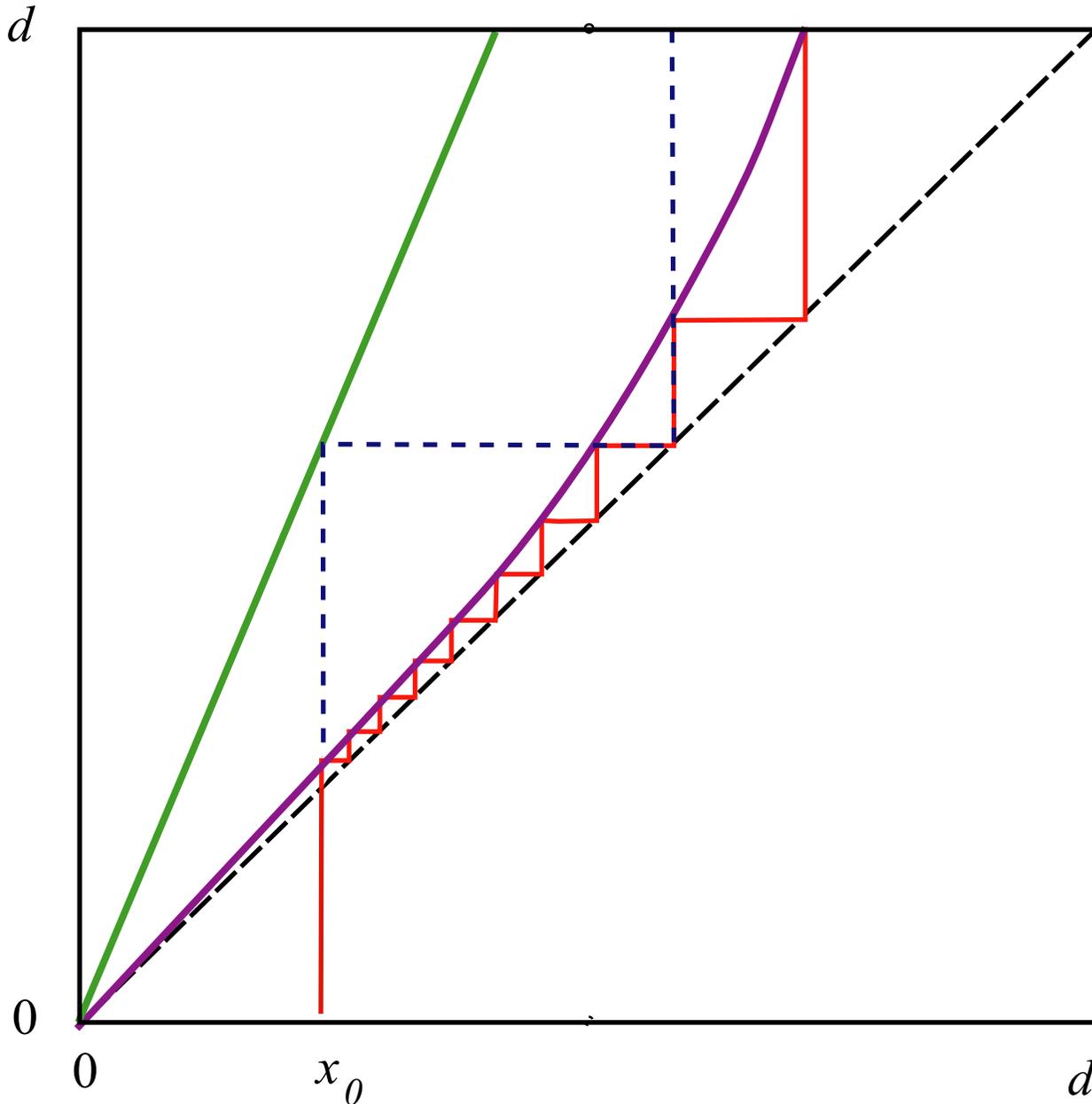
DIGITAL OUTPUT



**ALL**  
POSSIBLE  
DIGITAL  
OUTPUT  
SEQUENCES



# INTERMITTENCY and LINEAR MAPS



intermittency map  $f(x) = x + ax^m$

tangency with  $y=x$  at  $x=0$

implies intermittency at  $x=0$

and small iterative changes  
in the values of  $x$

PROBABILITY of ESCAPE  
to the region  $x > d$  in more than  $n$   
iterations of the map  $f$   
is given by

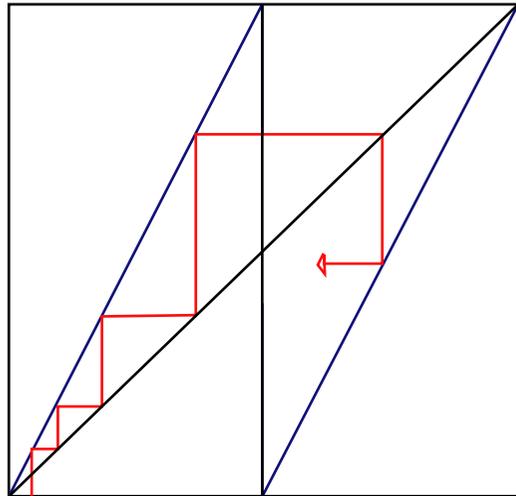
INTERMITTENCY (POWER LAW DECAY)

— Prob  $(n) \sim n^{-(2-m)/(m-1)}$

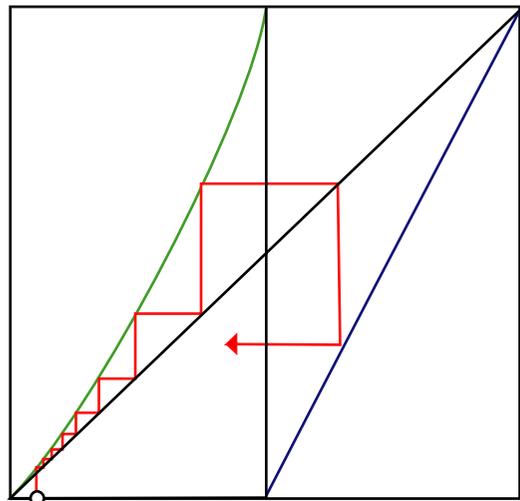
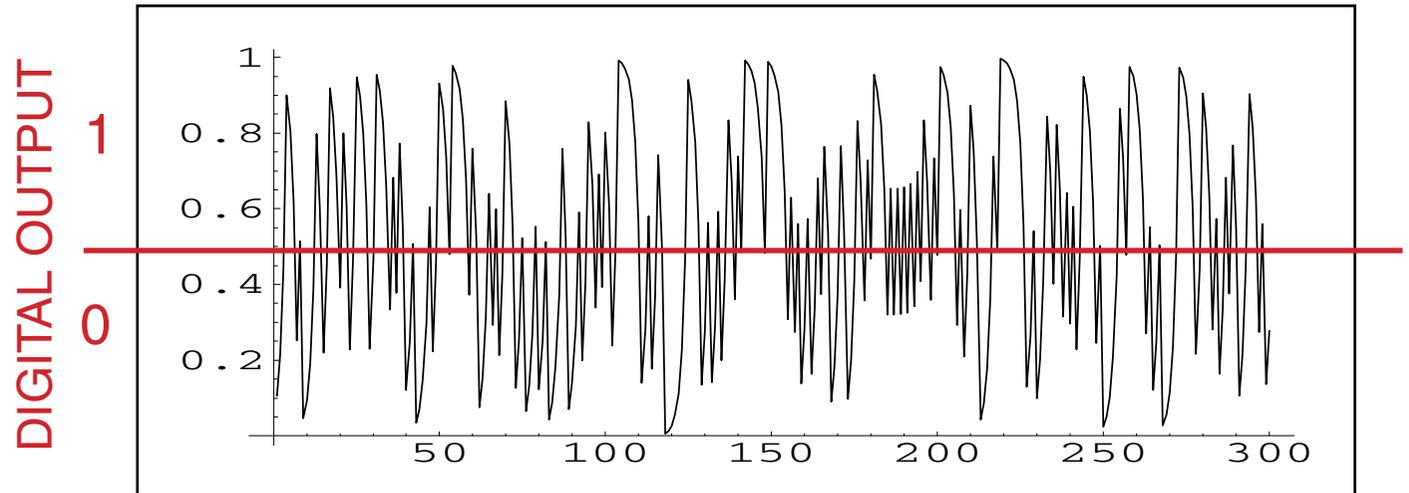
LINEAR (EXPONENTIAL DECAY)

— Prob  $(n) \sim 2^{-n}$

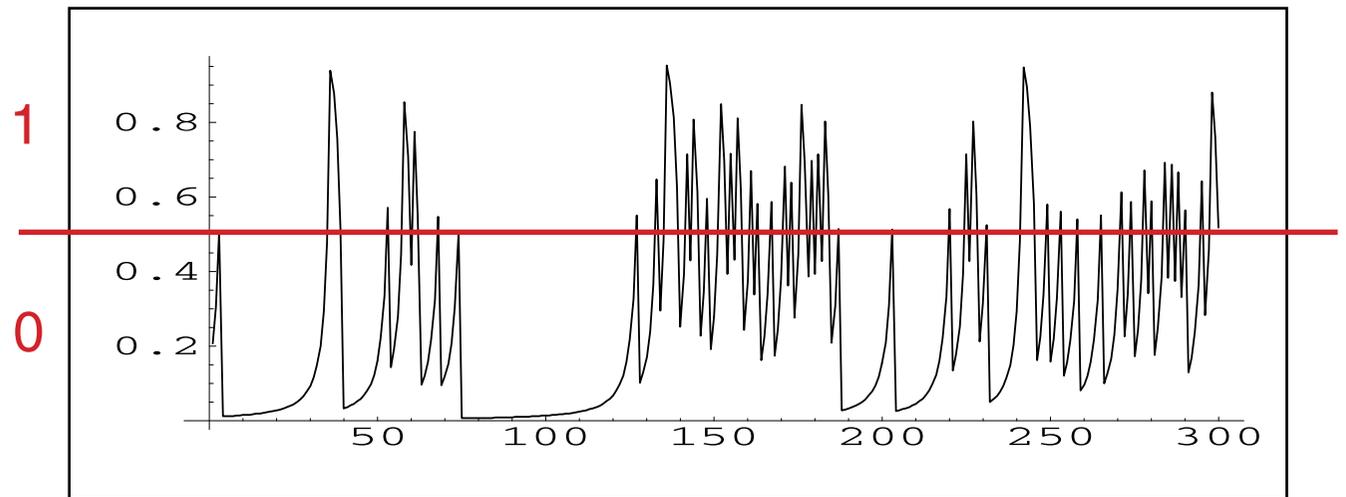
# OUTPUT COMPARISON OF MAPS



**BURST**



**OFF** | **BURST**



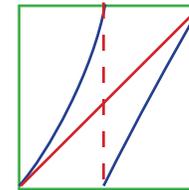
# DYNAMICS - STATISTICS INTERFACE

- INTERMITTENCY IN DYNAMICS
- □ - CORRELATION IN STATISTICS
- □ □ - LONG RANGE DEPENDENCE IN APPLICATIONS

CONJECTURES ON AUTO-CORRELATION (Erramilli *et al* ('95), Giovanardi *et al* ('98,'00))

□

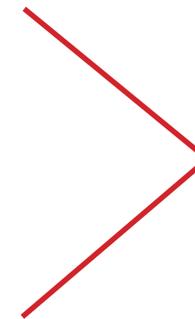
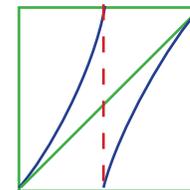
- SINGLE INTERMITTENCY MAPS (WANG, SCHUSTER)



□

□

- DOUBLE INTERMITTENCY MAPS (BARENCO and A.)



ERRAMILLI MAPS

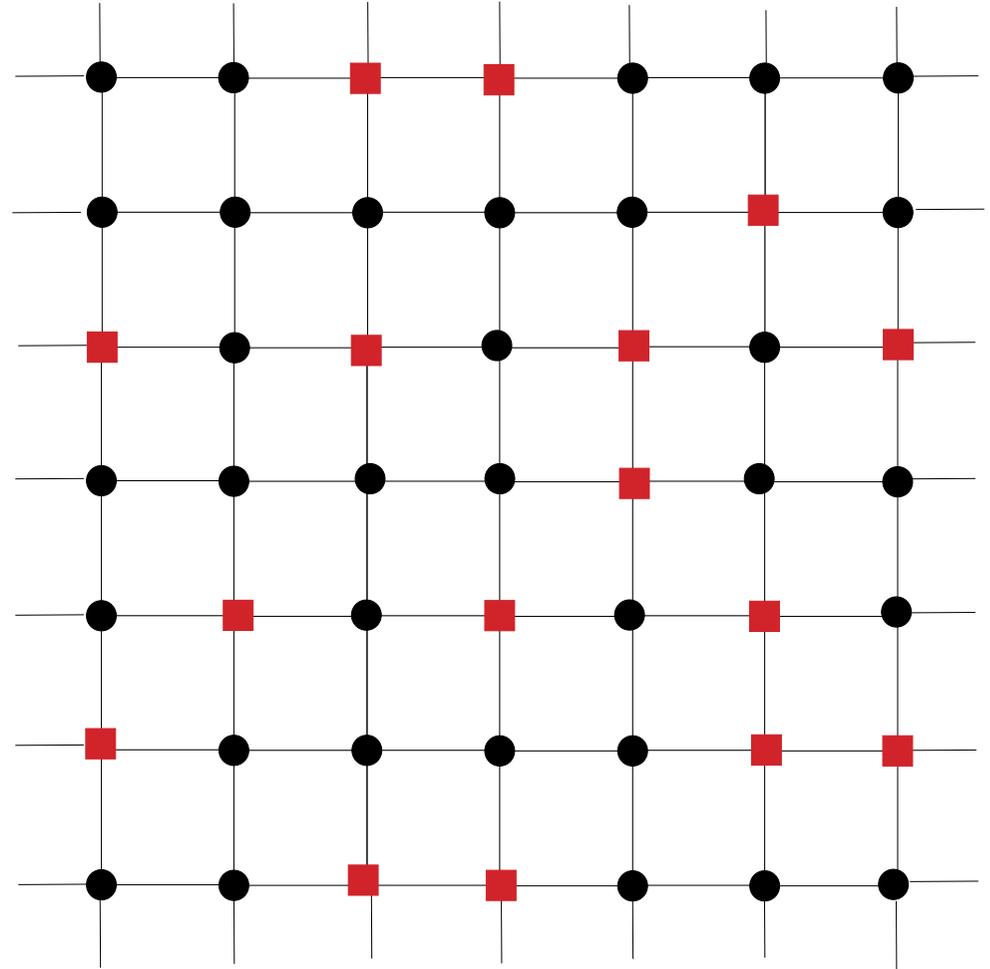
$$f_i(x) = \begin{cases} x + ax^{m_1}, & 0 < x < d \\ x - b(1 - x)^{m_2} & d \leq x < 1 \end{cases}$$

auto-correlation/intermittency property :

$$c(k) \sim k^{-\gamma} \quad \gamma = (2-m)/(m-1), \quad m = \text{Max}\{m_i\}$$

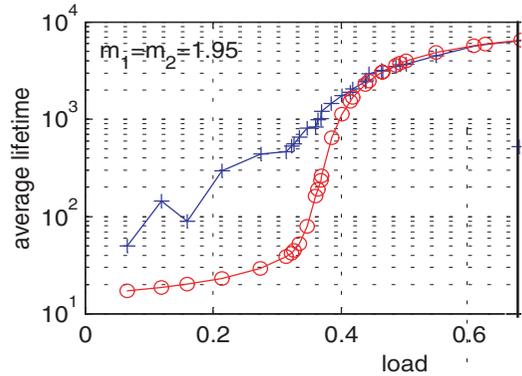
# RECTANGULAR GRID NETWORK MODEL

- hosts ■ can source, transfer and receive packets
- random host destination
- routers ● can transfer packets
- every node has a buffer for queueing packets
- packets at head of queue move one step closer to destination for each time step

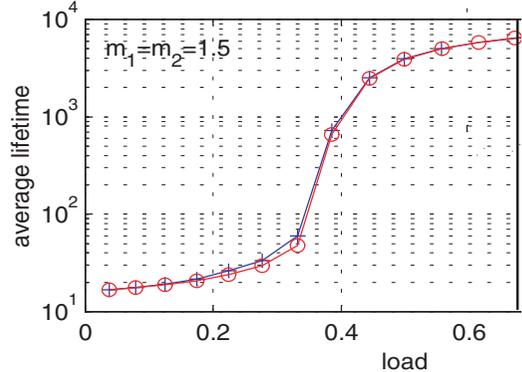
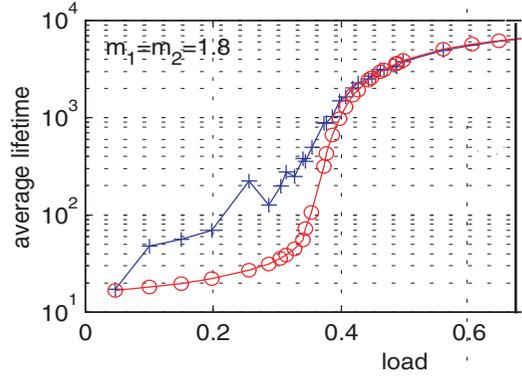


# SRD $\rightarrow$ LRD

via intermittency



+ LRD Traffic Source  
o Poisson-Like Traffic Source

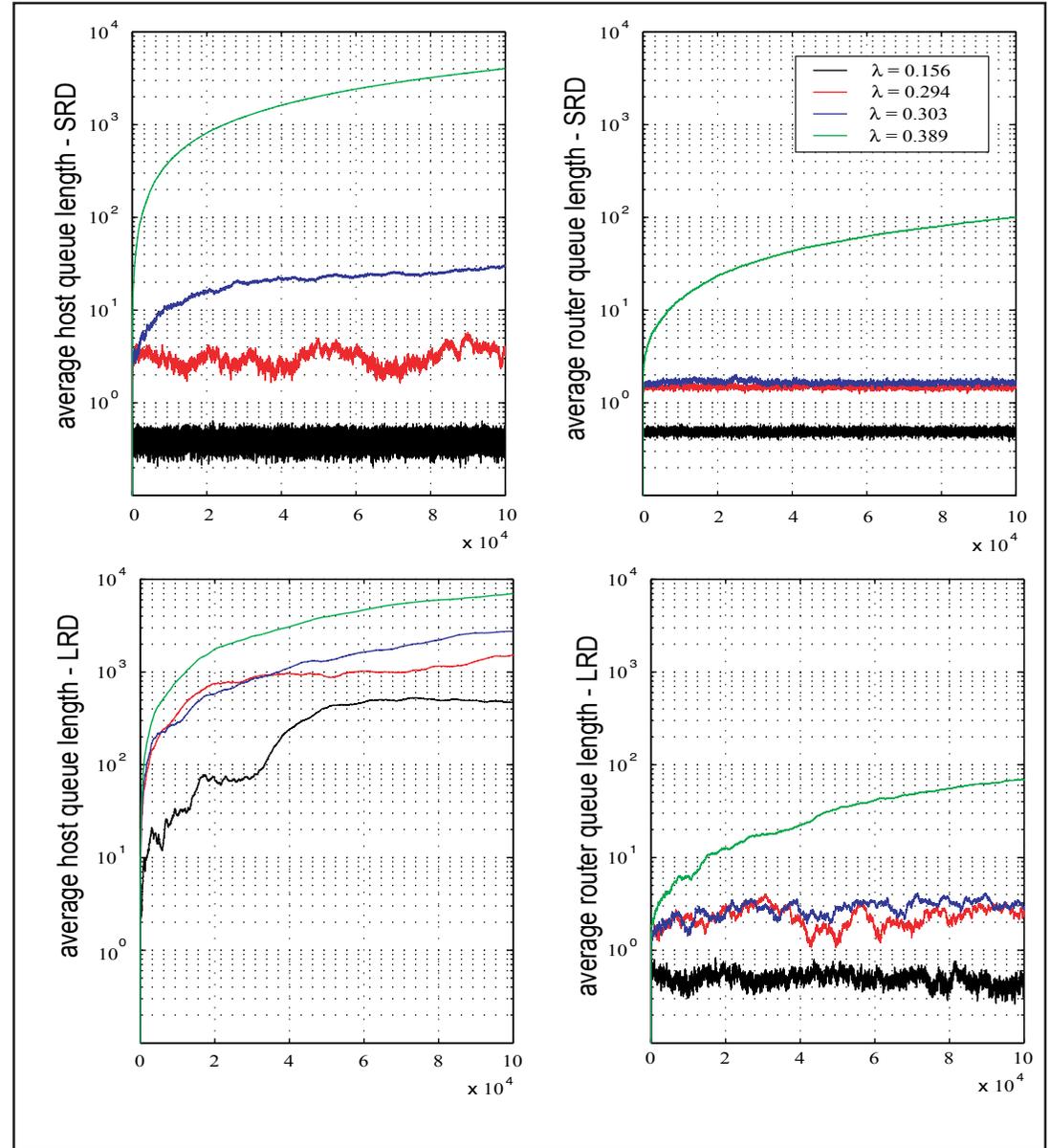


**LRD/POISSON LOADS**

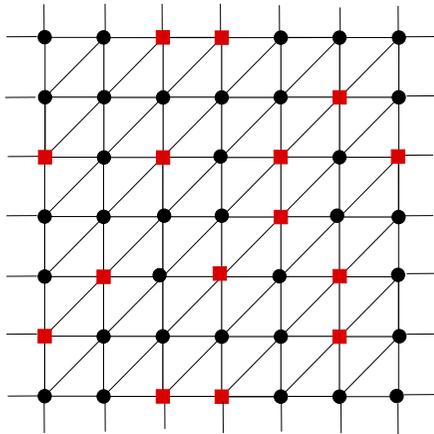
- below critical  $\lambda = 0.156$
- pre-critical  $\lambda = 0.294$
- post-critical  $\lambda = 0.303$
- heavy congestion  $\lambda = 0.389$

Host density = 16 %

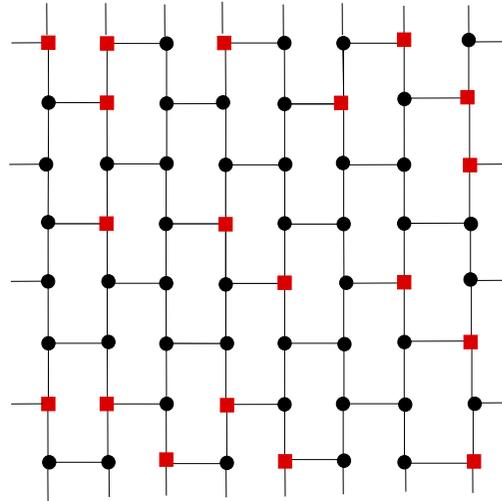
# SRD $\rightarrow$ LRD queue lengths



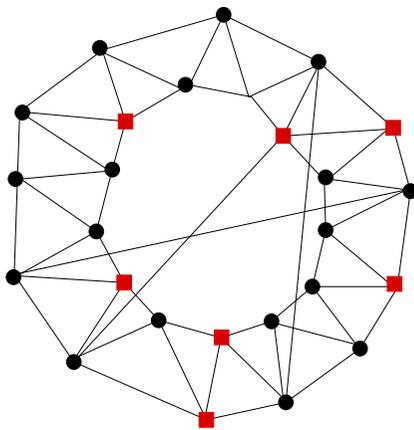
# NETWORK MODELS



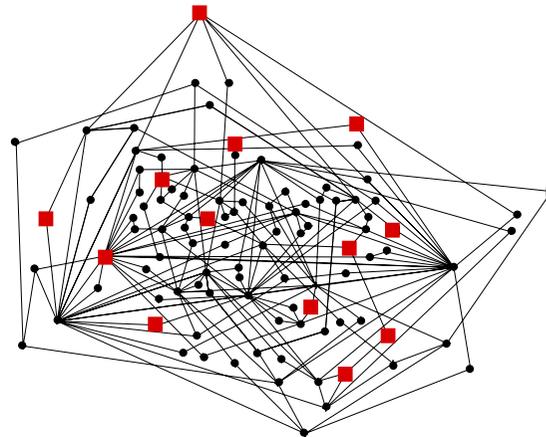
(a)



(b)



(c)



(d)

different types of graph:

**REGULAR:**

(a) triangular

(b) hexagonal,

and

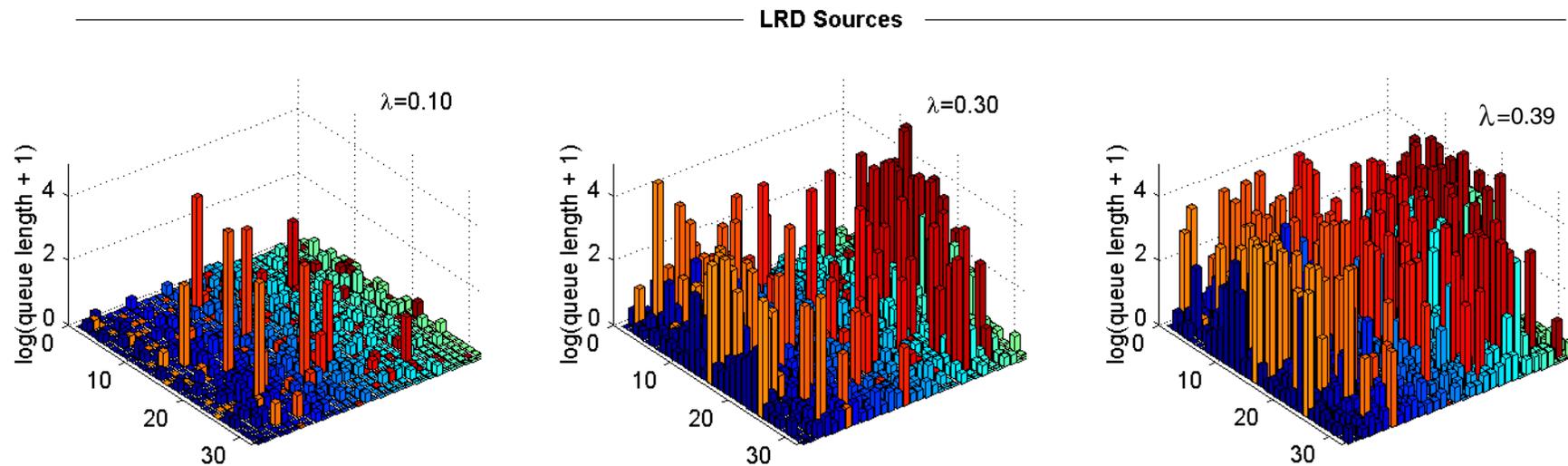
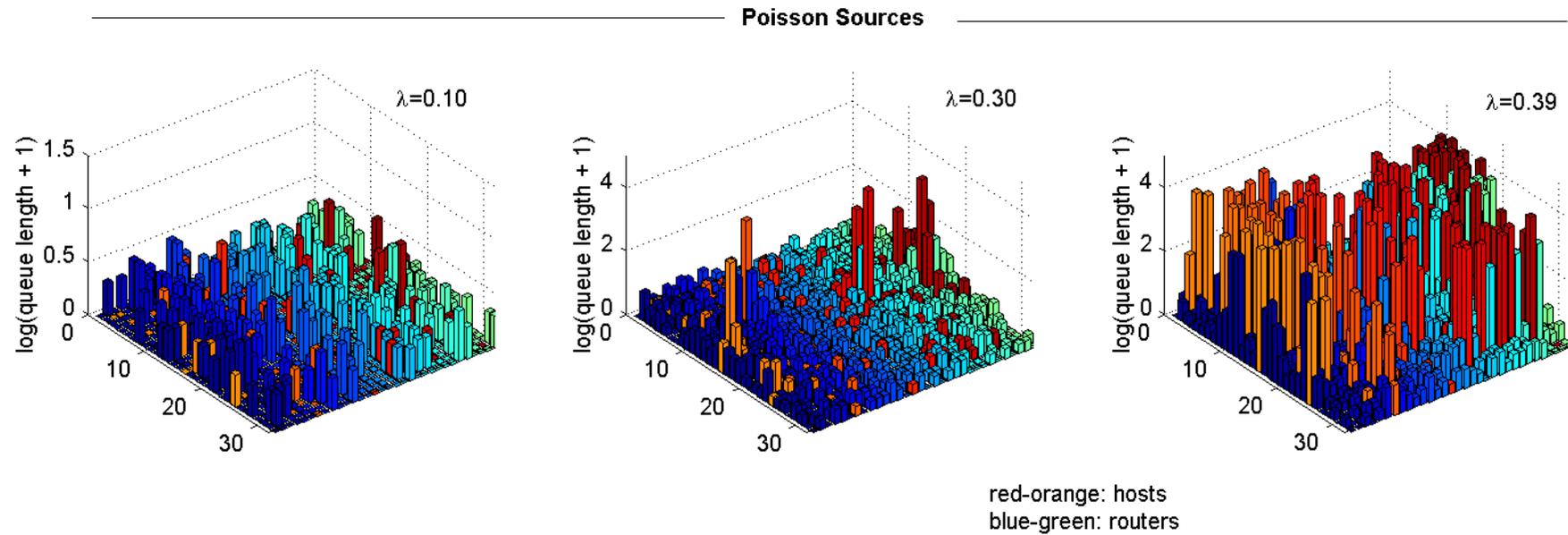
(c) **SMALL-WORLD**

random connections added  
to increase connectivity

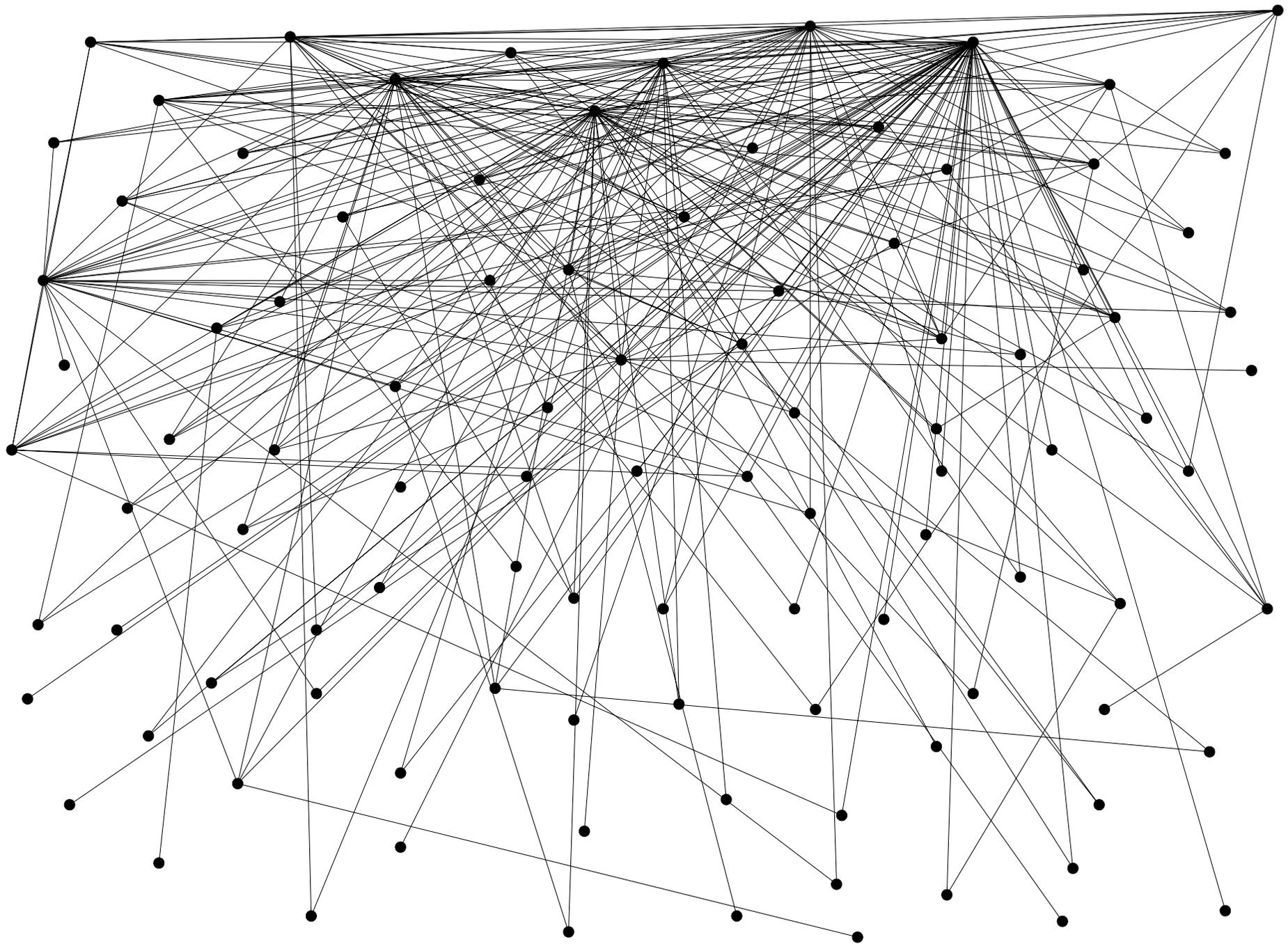
(d) **SCALE-FREE**

Prob(vertex valency =  $n$ ) =  $n^{-\gamma}$   
with exponent  $\gamma \in [2,3]$ .

# Manhattan network queues as load increases : Poisson vs. LRD



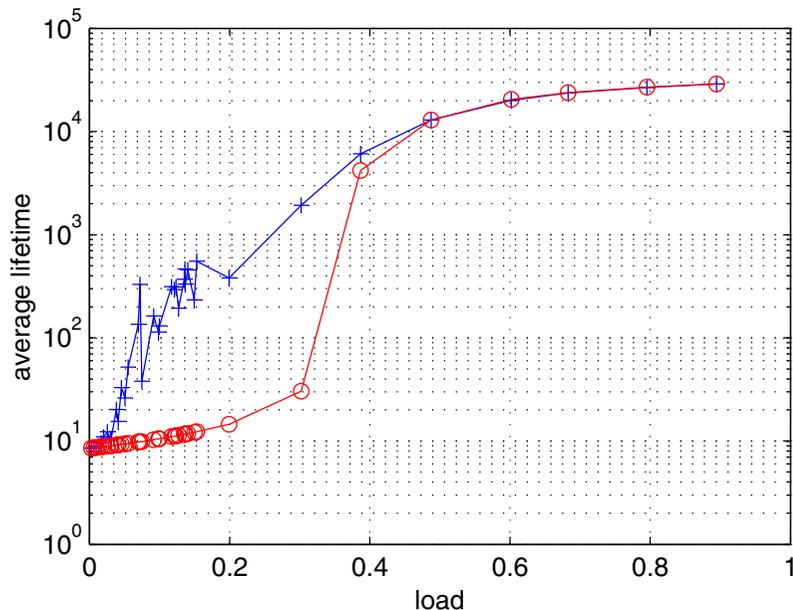
Comparing Queue Lengths for a Manhattan Network with Poisson and LRD sources



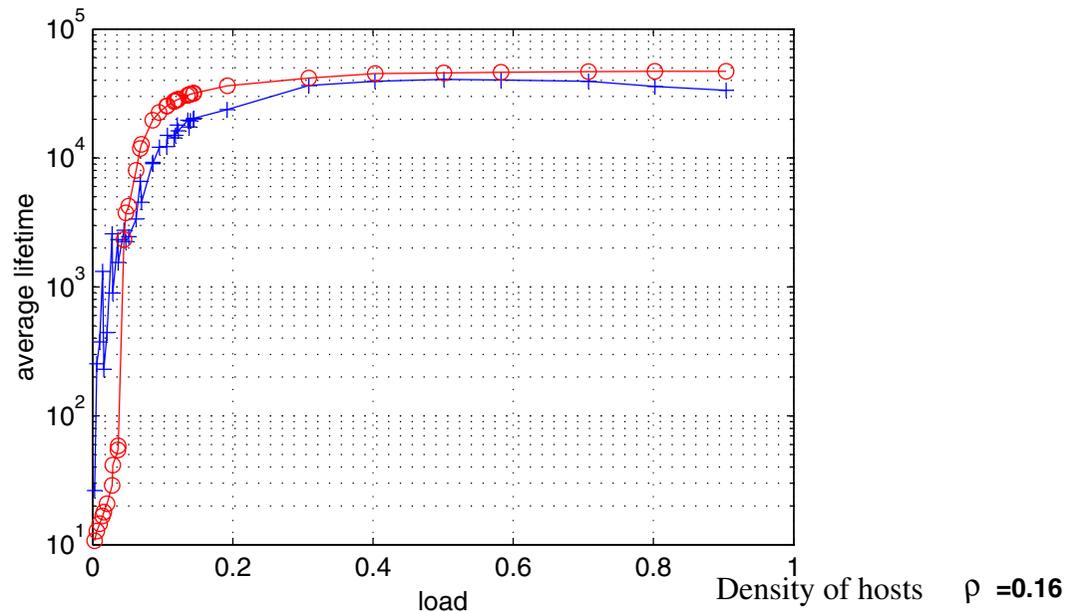
*100 Node Scale-Free IG Network (Zhou, Mondragon: ITC, Berlin 2003)*

# TCP window dynamics

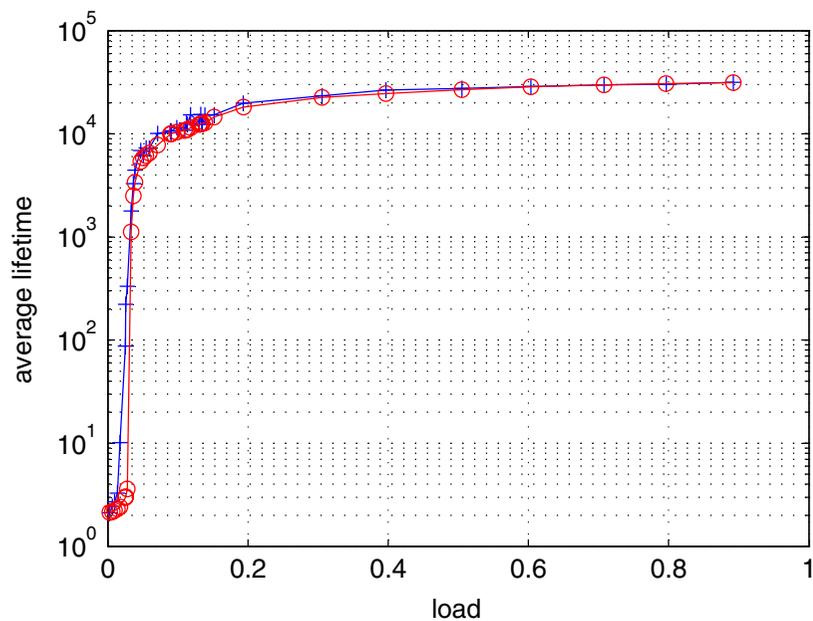
Open-Loop Traffic Sources in a Manhattan Network



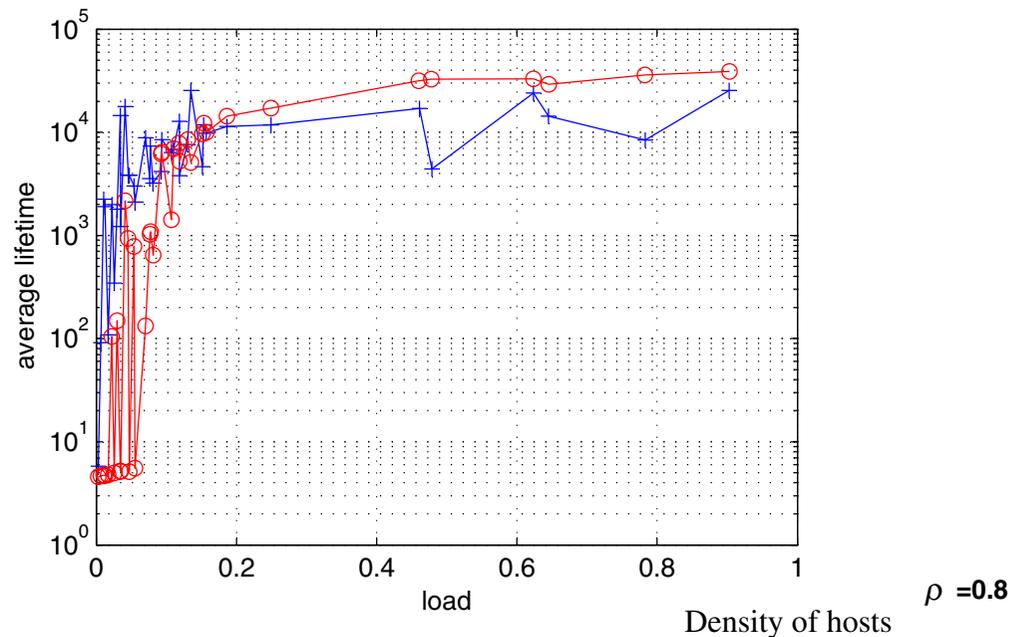
Slow-Start TCP in a Manhattan Network



Open-Loop Traffic Sources in an IG Model SF Network



Slow-Start TCP in an IG Model SF Network



# *ONGOING WORK*

- Erratic onset of congestion is a robust feature of LRD modelling in various networks
- LRD can arise from various sources -  
data streams, network structures, aggregation, TCP
- comparative and hierarchical modelling to study  
relative LRD strengths for each of these features