The statistics of intermittency maps and dynamical modelling of packet traffic networks

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Outline of presentation

1. Overview of modelling procedure

2. Intermittency maps and power-law properties

3. Autocorrelation of binary output from maps

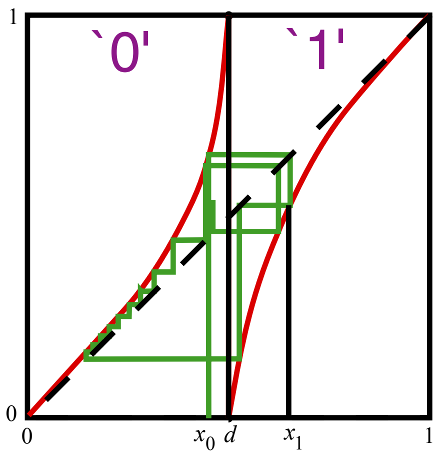
4. Network traffic

5. Some robust behaviour

6. Remarks

DYNAMICAL SYSTEMS

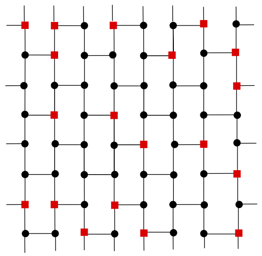
- iteration of simple maps
- `LR' conversion to binary data



Used to model

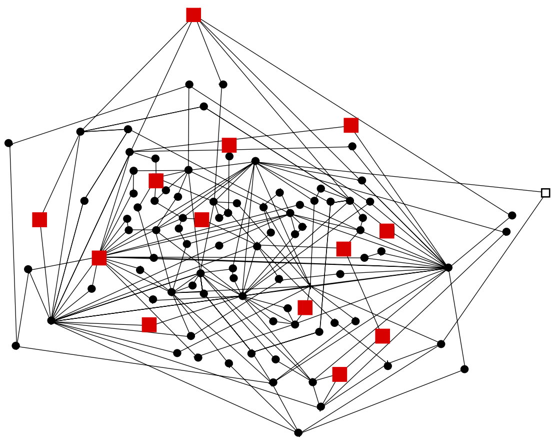
binary data with or without MEMORY

NETWORKS



- regular

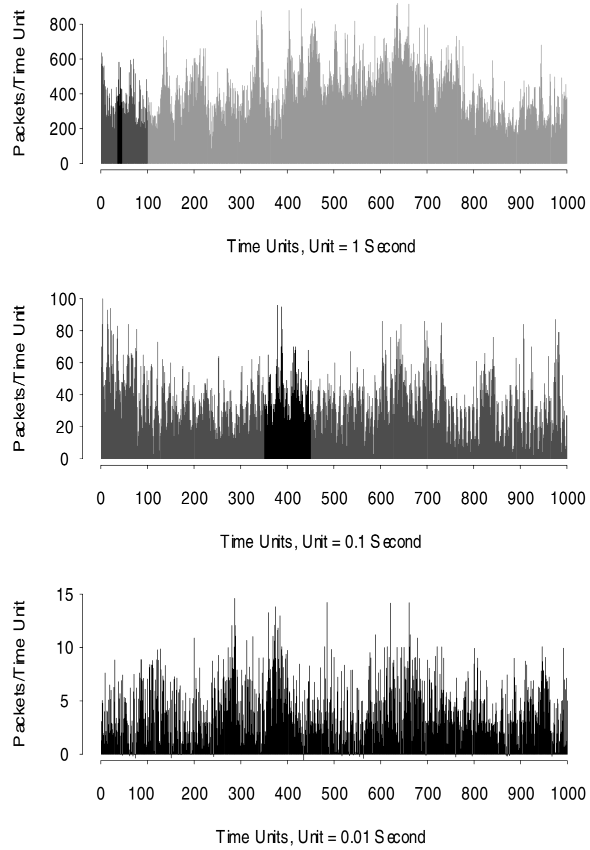
scale-free -



PACKET TRAFFIC on NETWORKS

dynamical systems used to
- emulate ethernet traffic
- model TCP(Reno)

bursty packet rates on different time scales (Leland, 1994)



PACKET TRANSFER PROPERTIES

queue properties, packet lifetime, congestion

DIGITAL OUTPUT FROM INTERMITTENCY MAPS

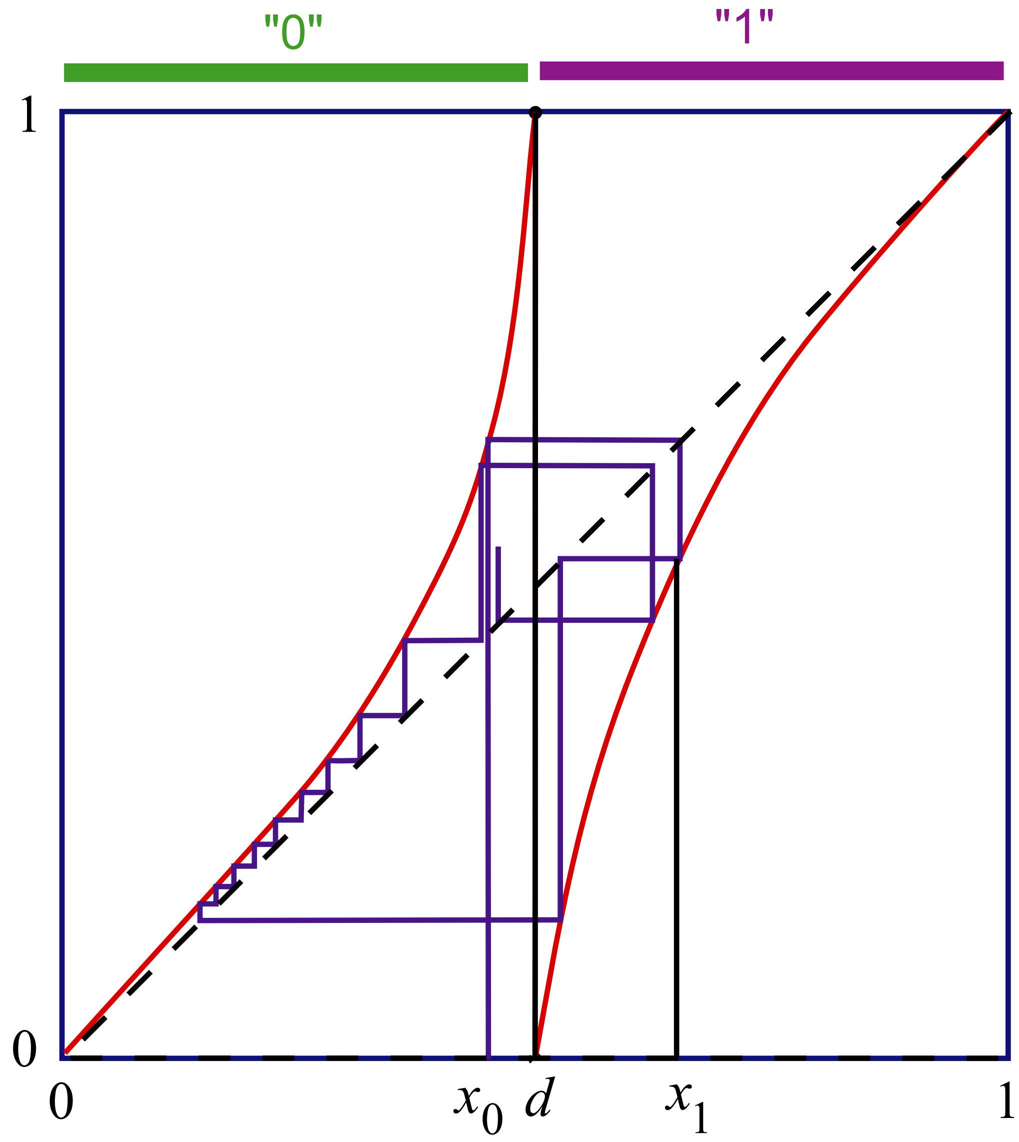
initial point : x_0

iteration : $x_n = f\,(x_{n-1})$

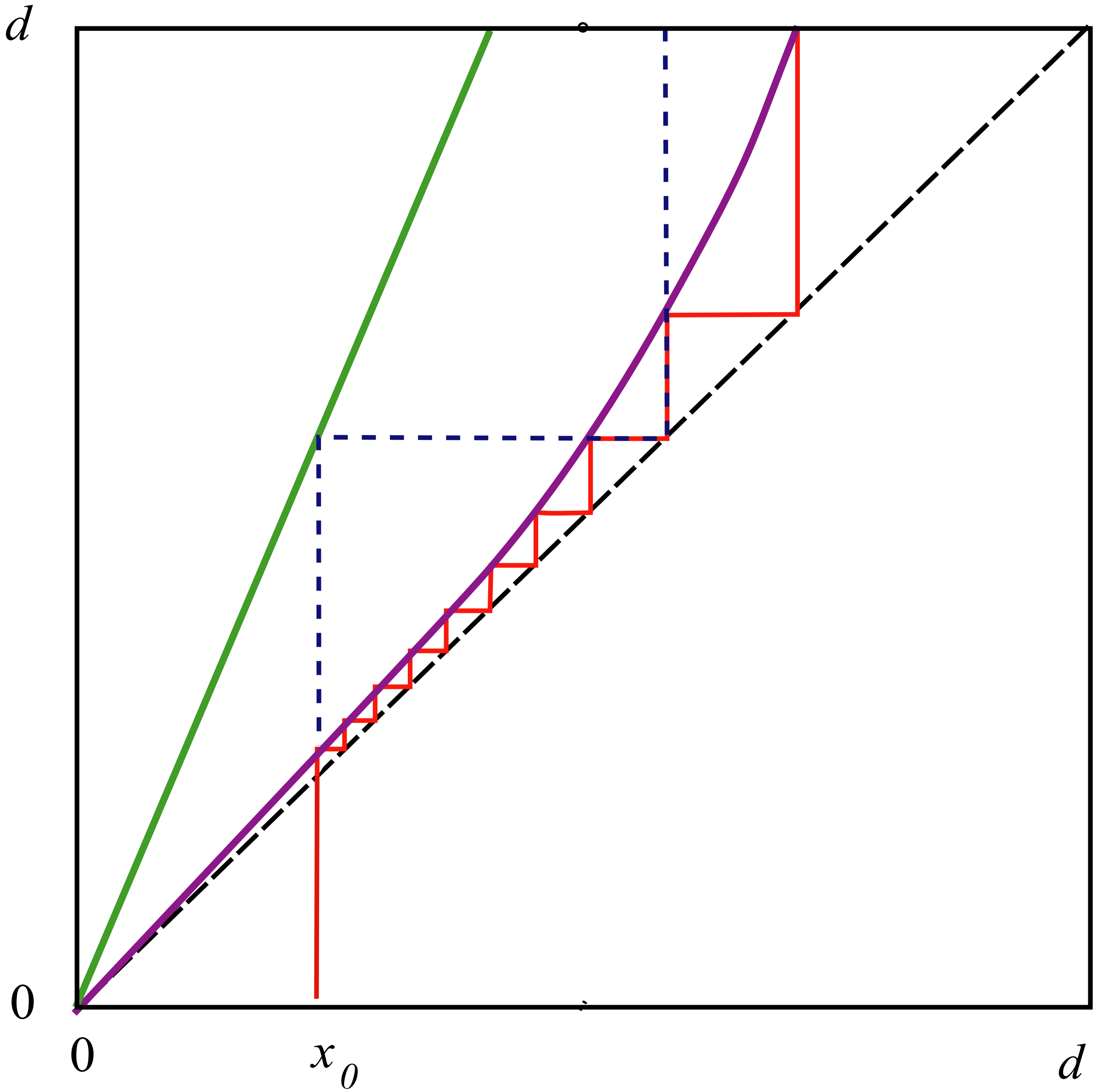
orbit in [0,1] : x_0, x_1, x_2, x_3 ...

binary orbit {0,1} : y_0, y_1, y_2, y_3

double intermittency
induces long uninterrupted
strings of either "0" or "1"
in the sequence y_n



INTERMITTENCY and LINEAR MAPS



intermittency map $f(x) = x+ax^m$

tangency with $y=x$ at $x=0$

implies intermittency at $x=0$

and small iterative changes
in the values of x

PROBABILITY of ESCAPE
to the region $x > d$ in more than n
iterations of the map f
is given by

INTERMITTENCY(POWER LAW DECAY)

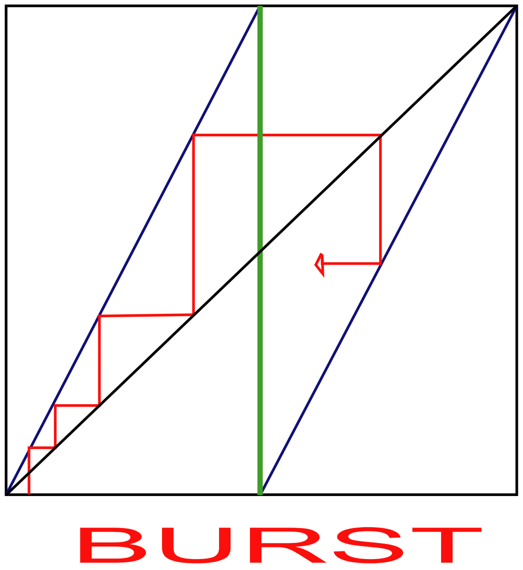
$\text{Prob}(n) \sim n^{-(2-m)/(m-1)}$

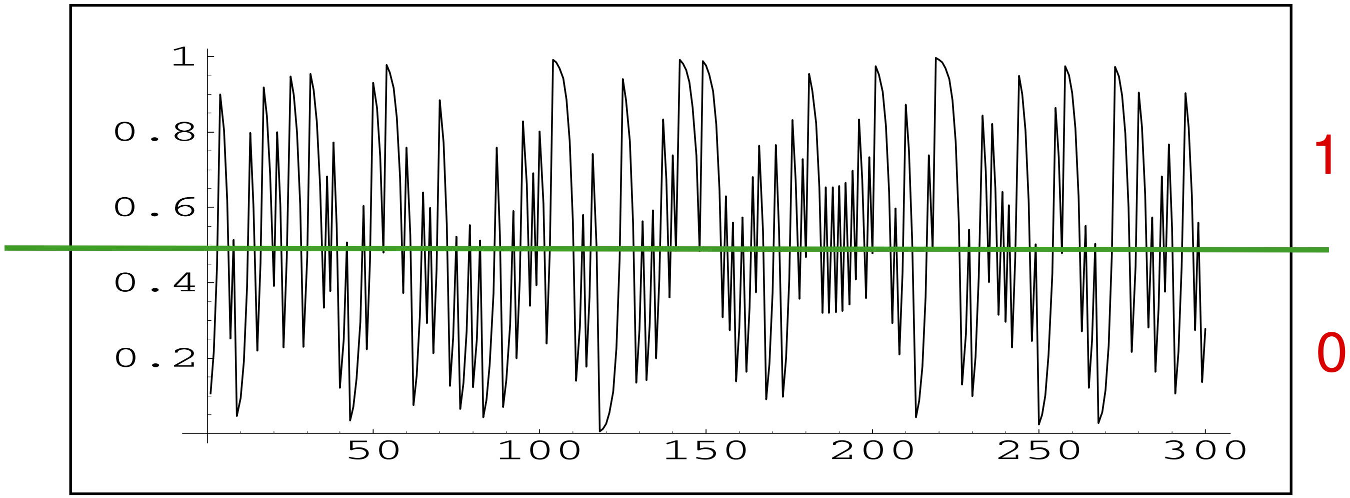
LINEAR(EXPONENTIAL DECAY)

$\text{Prob}(n) \sim 2^{-n}$

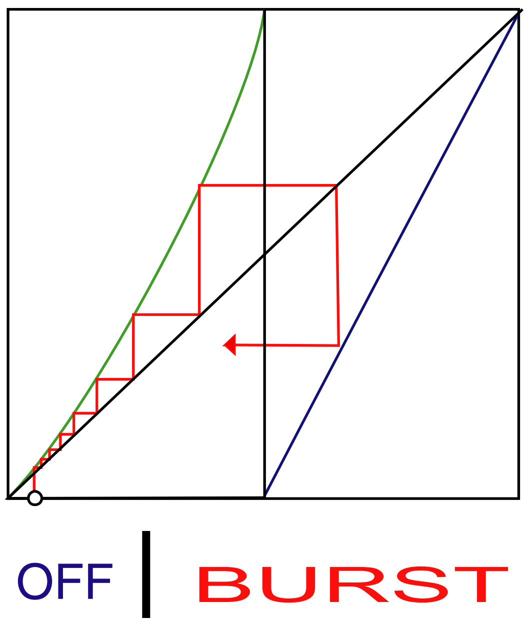
OUTPUT COMPARISON OF MAPS

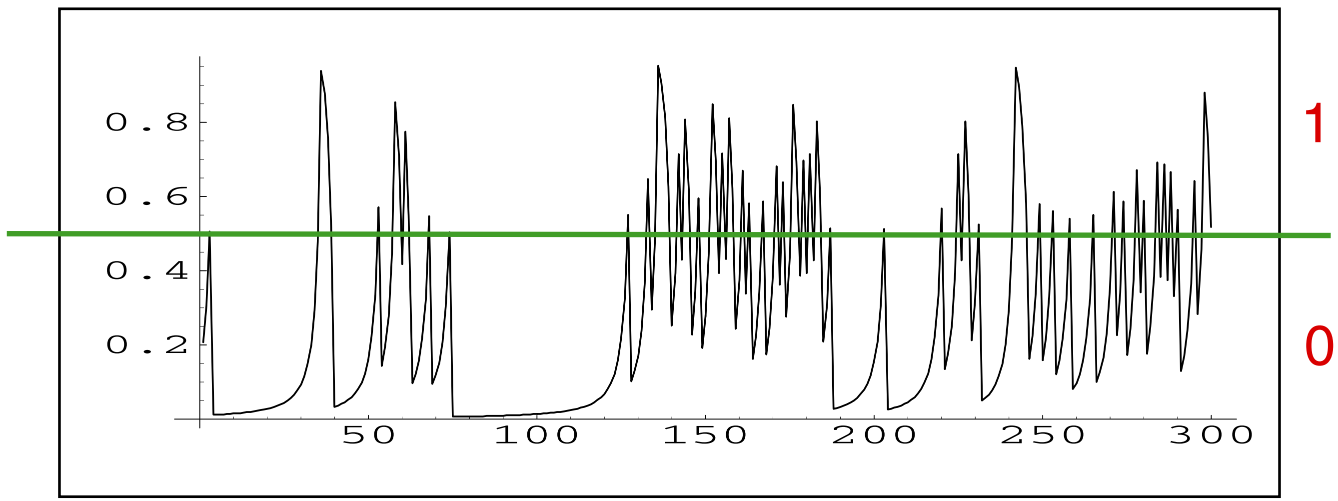
A





B





DYNAMICS-STATISTICS INTERFACE

INTERMITTENCY IN DYNAMICS

- CORRELATION IN STATISTICS

- LONG RANGE DEPENDENCE IN APPLICATIONS

CONJECTURES (Erramilli *et al*, Giovanardi *et al*)

THEORY on AUTOCORRELATION

- SINGLE INTERMITTENCY MAPS (Schuster,Wang)

- DOUBLE INTERMITTENCY MAPS (Barenco and A.)

$$f_i(x) = \begin{cases} x + a x^{m_1}, & 0 < x < d \\ x - b(1 - x)^{m_2} & d \leqslant x < 1 \end{cases}$$

$$c(k) \sim k^{-\gamma}$$

$$\gamma = (2 - m)/(m - 1),\ m = \text{Max}\ m_i$$

AUTOCORRELATION

BINARY TIME SERIES: $X_t \in \{0,1\}, \quad t = 1,2,3,\ldots$

MEAN: $\mu = E(X_t) = E(X_t^2)$

VARIANCE:

$$\mathrm{Var}(X_t) = E(X_t^2) - E(X_t)^2$$

NEW TIME SERIES- averages of batch sizes N:

$$Y_t^{(N)} = \frac{1}{N} \sum_{j=tN+1}^{(t+1)N} X_t$$

AUTOCORRELATION FORMULA :

$$c(k) = \frac{E(X_t X_{t+k}) - E(X_t)E(X_{t+k})}{\sqrt{(\mathrm{Var}(X_t)\mathrm{Var}(X_{t+k}))}} = \frac{E(X_t X_{t+k}) - \mu^2}{\mu(1-\mu)}$$

AUTOCORRELATION FOR LRD TRAFFIC :

POWER LAW DECAY

$$c(k) \sim k^{-\beta}, \quad \beta \in (0,1)$$

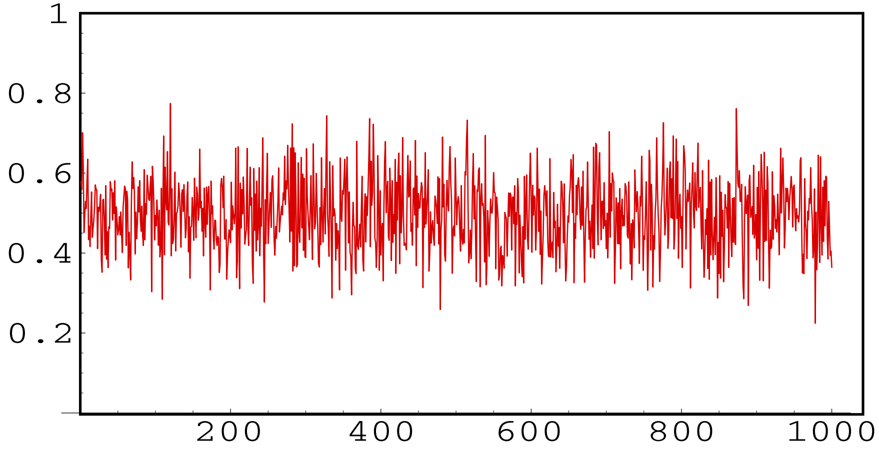
AUTOCORRELATION FOR SRD TRAFFIC :

EXP DECAY

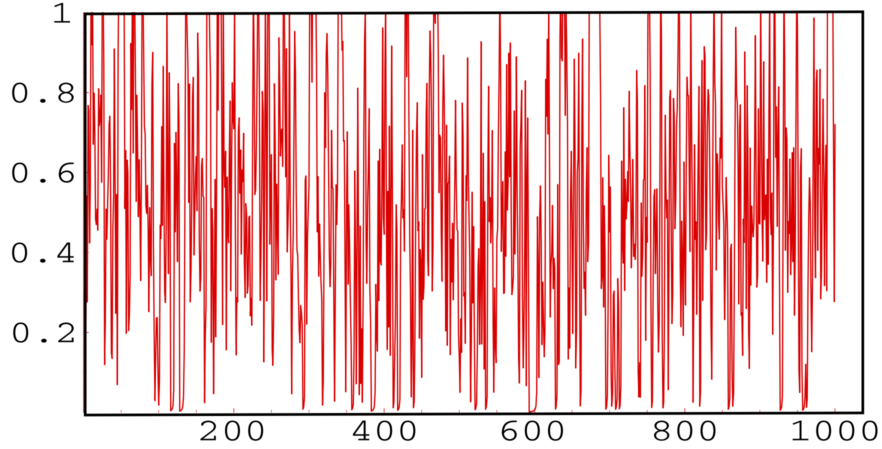
$$c(k) \sim \alpha^{-k}, \quad \alpha > 1$$

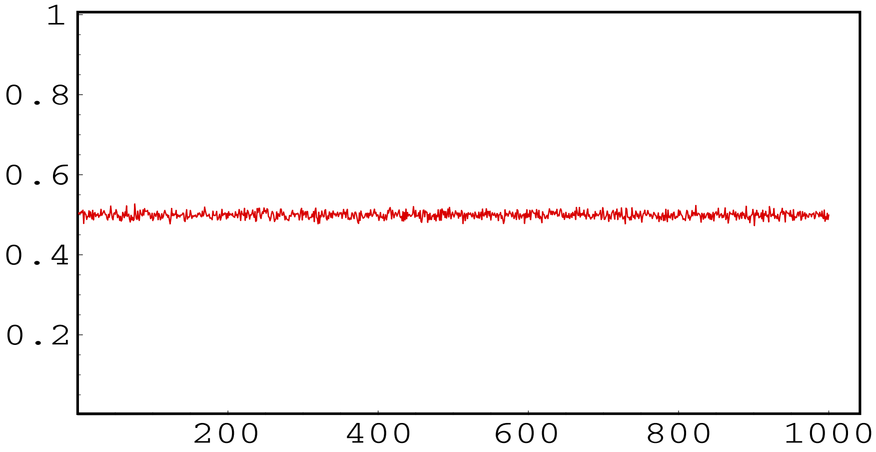
SRD and LRD OUTPUT

N= *BATCH SIZE of AVERAGED DATA*

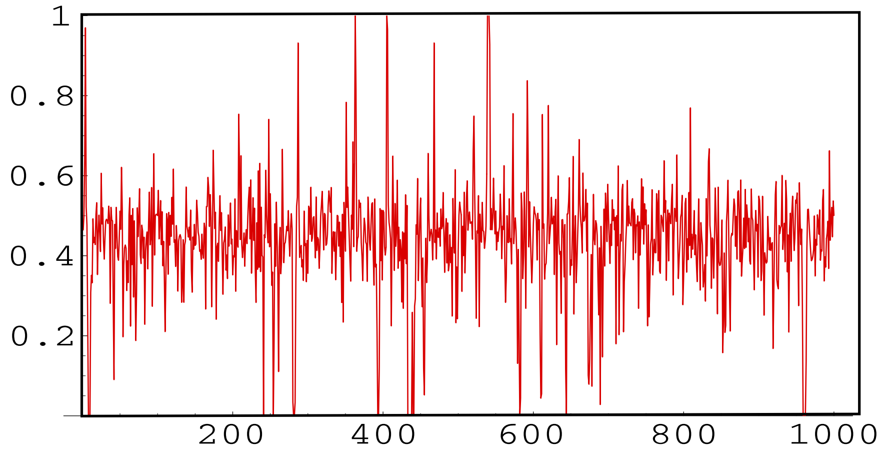


N=100





N=10000

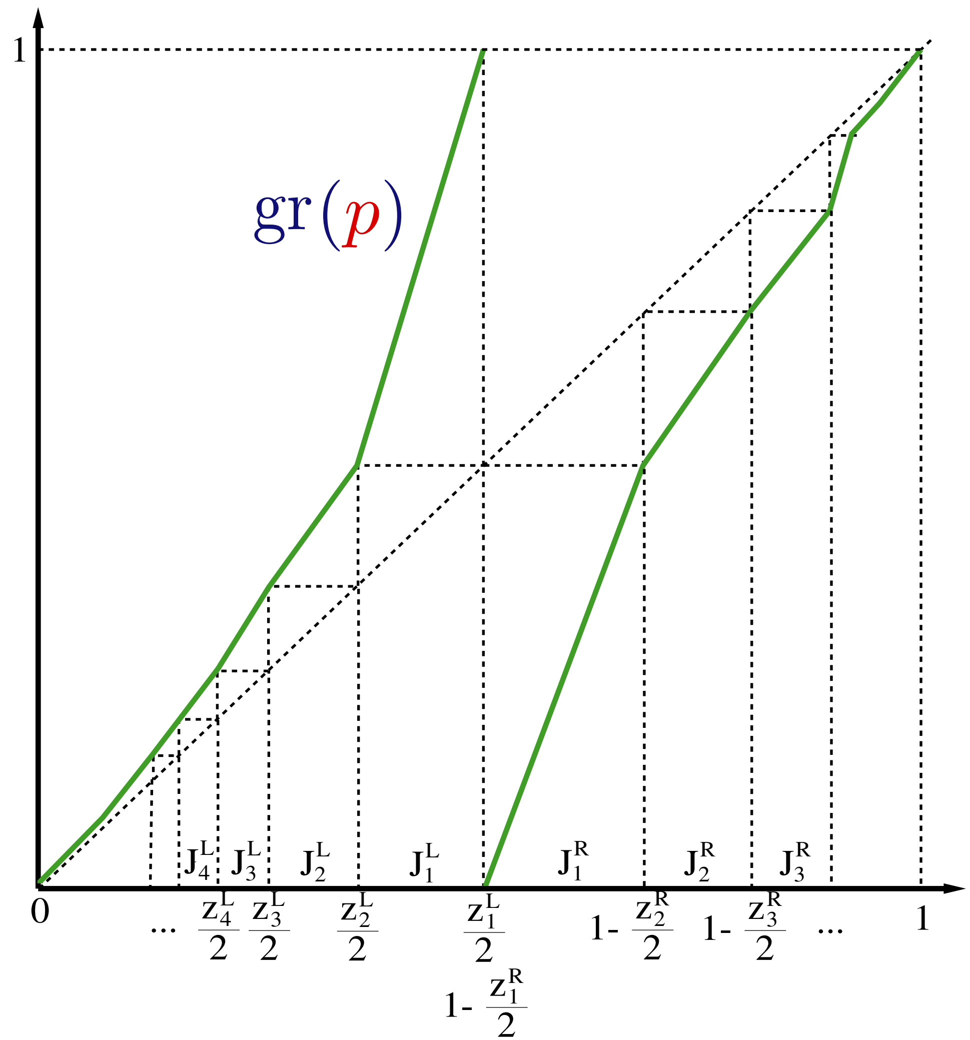


Short range dependent

Long range dependent

PIECEWISE LINEAR MAP p with intermittency

z_i^L and z_i^R - null monotonic decreasing sequences with $z_1^L = z_1^R = 1$



Unit interval is partitioned into subintervals :

$J_i^L = (z_{i+1}^L/2, z_i^L/2]$,

$J_i^R = (1 - z_i^R/2, 1 - z_{i+1}^R/2]$,

Length : $\Delta_i^* = |J_i^*|$

Map $p : I \to I$:

(i) affine on each sub-interval

(ii) $p(J_i^*) = J_{i-1}^*$, $i = 1, 2, \ldots$

(iii) $p(J_i^*) =_{i=1}^{\infty} J_i^{\bar{*}}$

DOUBLE INTERMITTENCY AUTOCORRELATION

Theorem(Cj. Giovanardi *et al* NOLTA '98). Let $p : \mathbf{I} \to \mathbf{I}$ be the double intermittency piece-wise linear map with sequences

$$z_i^L = z^{-\alpha} \text{ and } z_i^R = z^{-\beta} \text{ , } \alpha,\beta > 1.$$

The autocorrelation of the binary output of the PL-map p has the decay,

$$c(k) \sim Ck^{-\gamma}$$

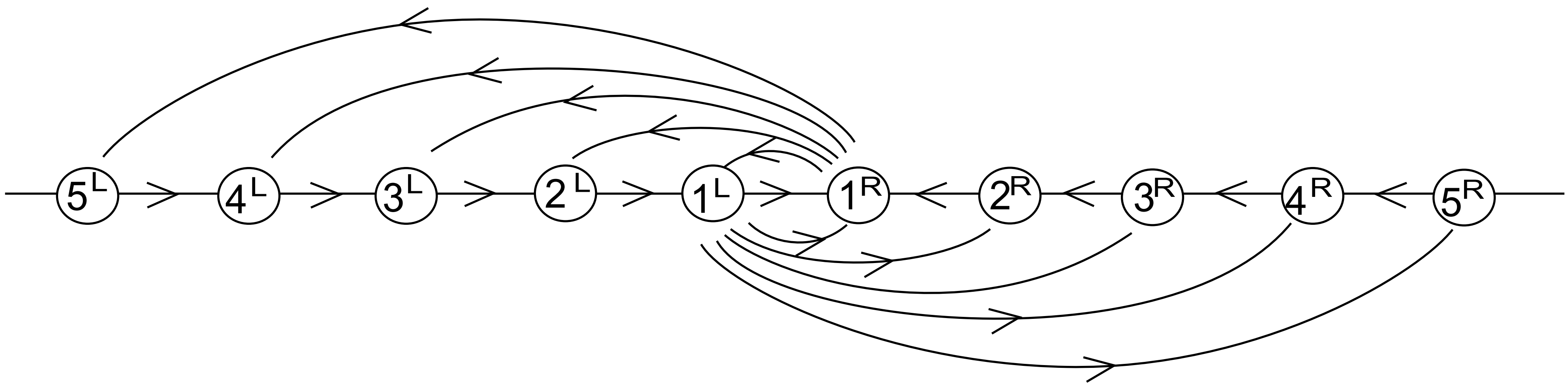
for lag k, where $\gamma = \mathrm{Min}\{\alpha, \beta\} - 1$ and C constant.

$$c(k) = \frac{E(X_t X_{t+k}) - E(X_t)E(X_{t+k})}{\sqrt{(\mathrm{Var}(X_t)\mathrm{Var}(X_{t+k}))}} = \frac{E(X_t X_{t+k}) - \mu^2}{\mu(1-\mu)}$$

MARKOV CHAIN STRUCTURE

p is isomorphic to a *Markov* chain.

interval J_i^* in the map ⟷ State i^* in the Markov chain



Transition probabilities

$$Prob(Y_{t+1} = *_i | Y_t = *_{i+1}) = 1$$

$$Prob(Y_{t+1} = *_i | Y_t = \bar{*}_1) = \Delta_i^*$$

SOME CRITICAL INPUT FOR PROOF

A. *Convolution theorems* Let $\{a_i\}$ and $\{b_i\}$ be two sequences with polynomial asymptotics

$$a_i \sim K_a i^{-a} \qquad b_i \sim K_b i^{-b}$$

where $K_a, K_b > 0$, $\mathrm{Max}\{a, b\} > 1$, $\mathrm{Min}\{a, b\} > 0$.

Define

$$C_m = \sum_{k=1}^{m-1} a_k b_{m-k}.$$

Then

$$C_m \sim \begin{cases} K_a S_b m^{-a} & \text{if } a < b; \\ K_b S_a m^{-b} & \text{if } a > b; \\ (K_b S_a + K_a S_b) m^{-a} & \text{if } a = b. \end{cases}$$

where $S_a = \sum_{i=1}^{\infty} a_i$, and $S_b = \sum_{i=1}^{\infty} b_i$.

B. Recursion formulae

$$T_{01}(n,1) = \sum_{i=1}^{n-1} \Delta_i^L \Delta_{n-i}^R$$

$$T_{01}(n) = \sum_{i=2}^{n-2} T_{01}(i,1) T_{01}(n-i)$$

$$T_{00}(n) = \Delta_n^L + \sum_{i=2}^{n-1} T_{01}(i) \Delta_{n-i}^L$$

$$Z(n) = P_1 \sum_{i=1}^{n-1} z_i^R z_{n-i}^R$$

$$P_{11}(n) = P_1 \sum_{i=n}^{\infty} z_i^R$$

$$c(n) = P_{11}(n) + \sum_{i=1}^{n-2} T_{00}(i) Z(n-i)$$

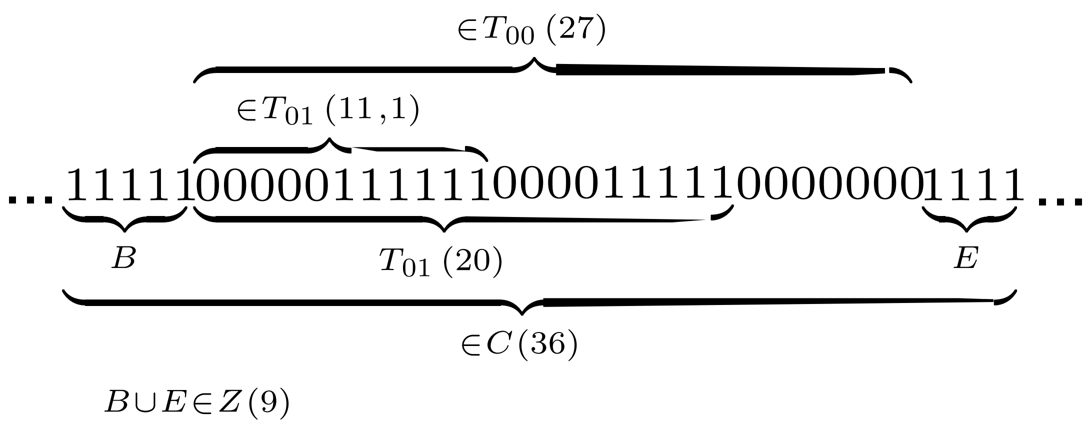
$T_{01}(n,1)$ is the probability associated with all transition sequences of length n, starting with a '0' and finishing with a '1' with a single uninterrupted spell of either symbol;

$T_{01}(n)(T_{00}(n))$ corresponds to *all* transition sequences of length n from a '0' to a '1' ('0' to a '0');

$Z(n)$ probability of occurence of those sequences with two uninterrupted spells of '1's of total length n;

$P_{11}(n)$ probability of a sequence of ones of length n

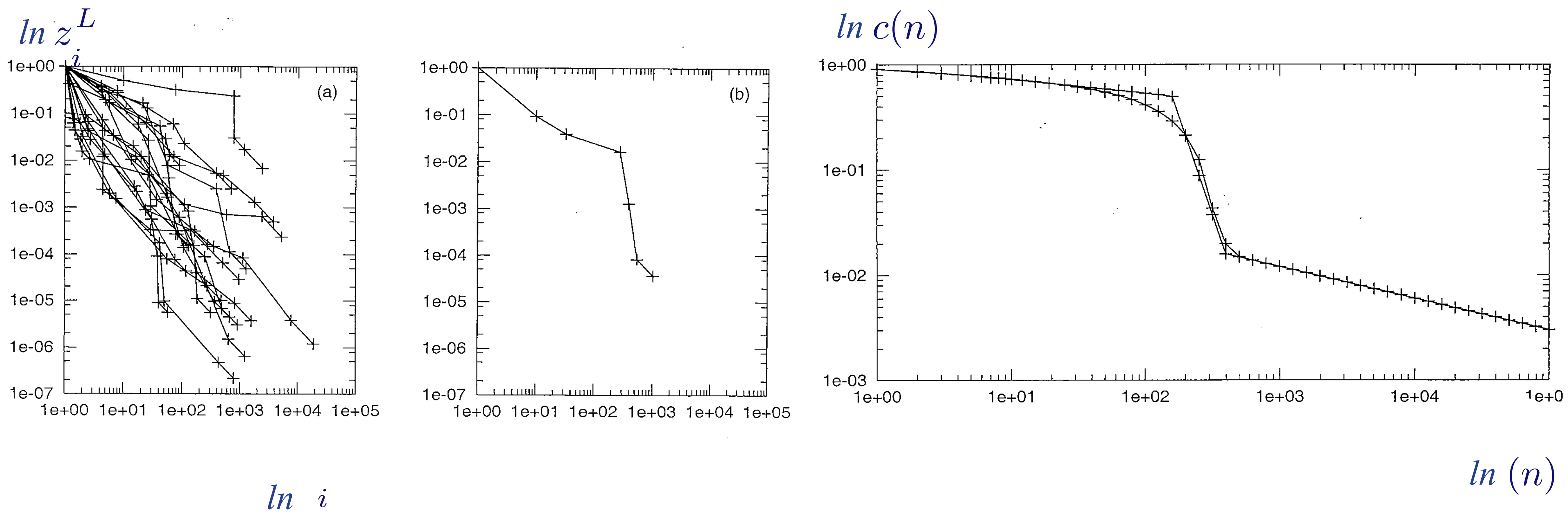
$c(n)$ is the autocorrelation function with time lag n



Realization of autocorrelation sequences using PL maps

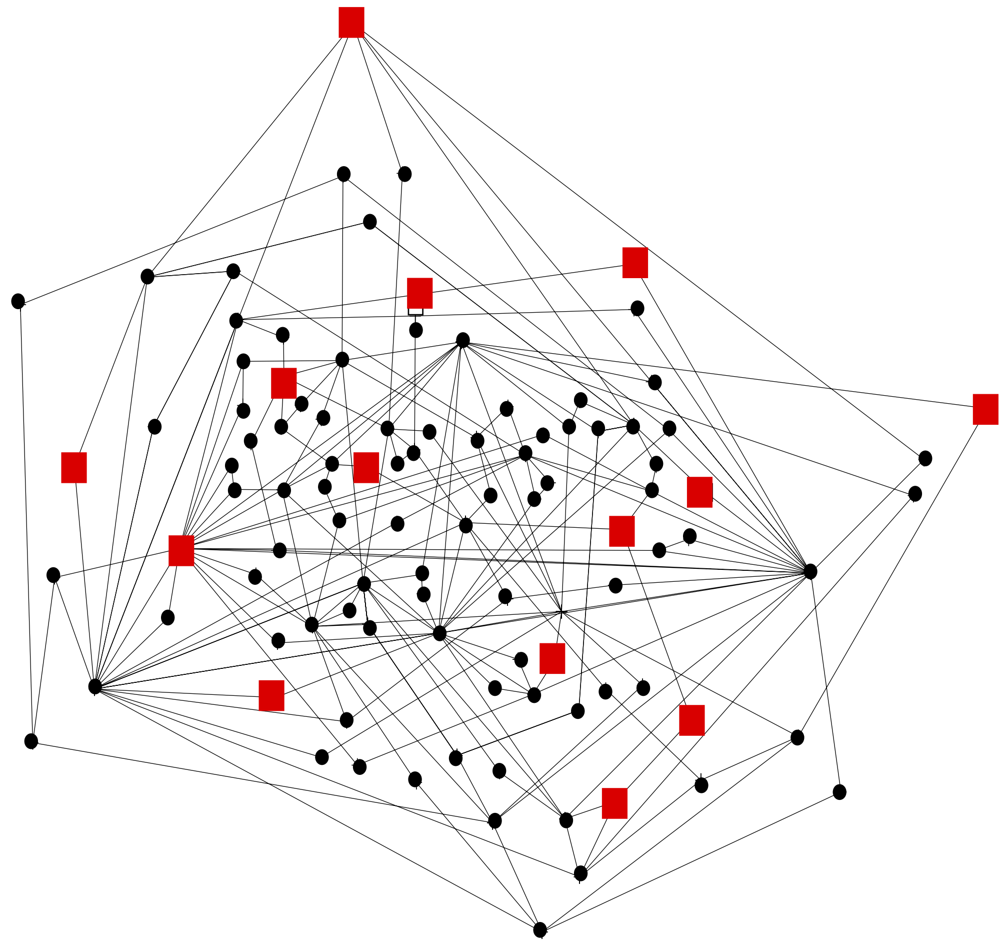
Problem: Choose sequences z_i^L, z_i^R to produce a PL map with a prescribed correlation vector $c(n)$, n=1,2,3,...

Use of genetic algorithms to search out optimal PL map

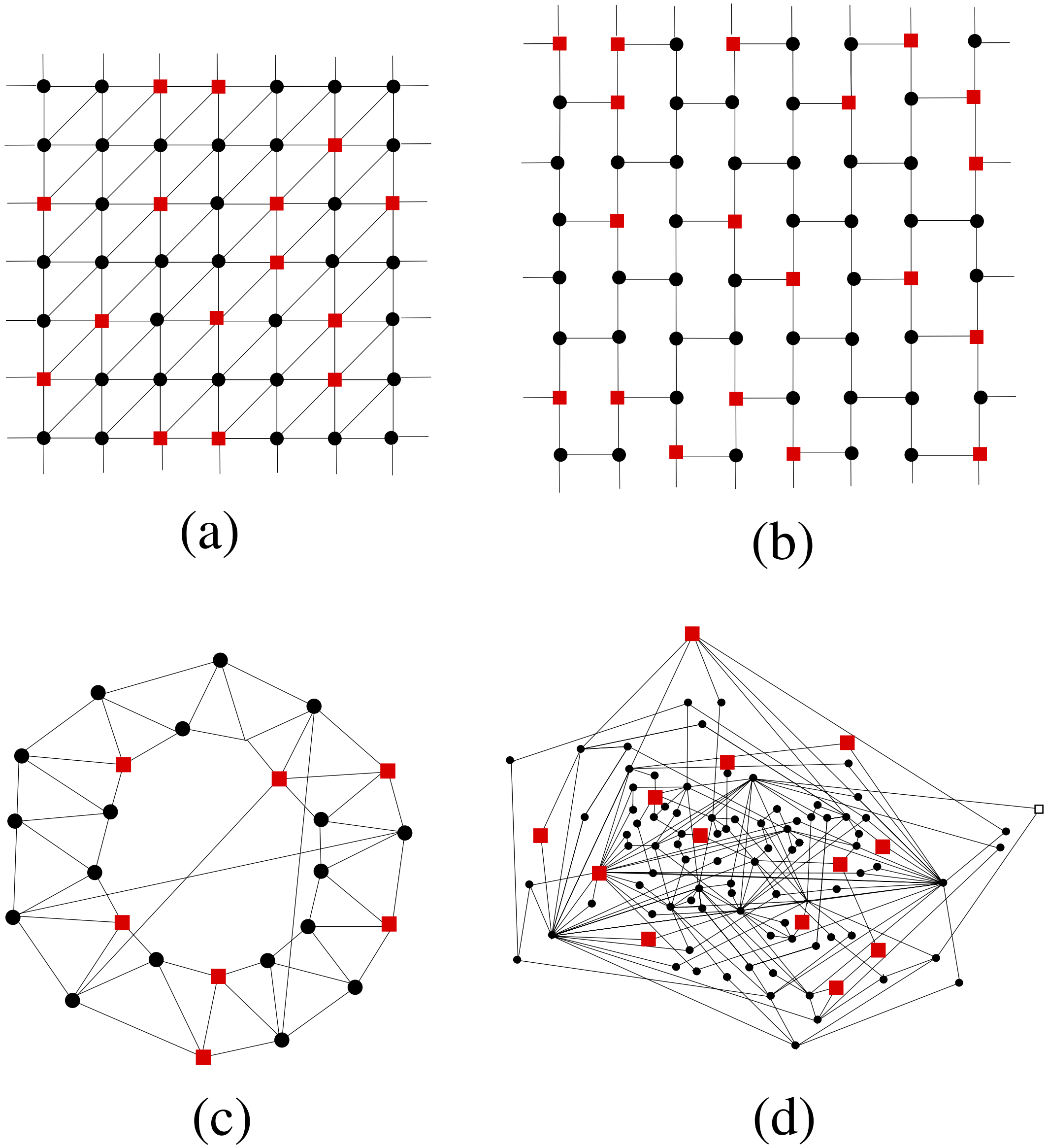


NETWORK PACKET TRAFFIC MODEL

- hosts ■ can source, transfer and receive packets
- random host destination
- routers ● can transfer packets
- every node has a buffer for queueing packets
- packets at head of queue move one step closer to destination for each time step



NETWORK MODELS



different types of graph:

REGULAR:

(a) triangular

(b) hexagonal,

and

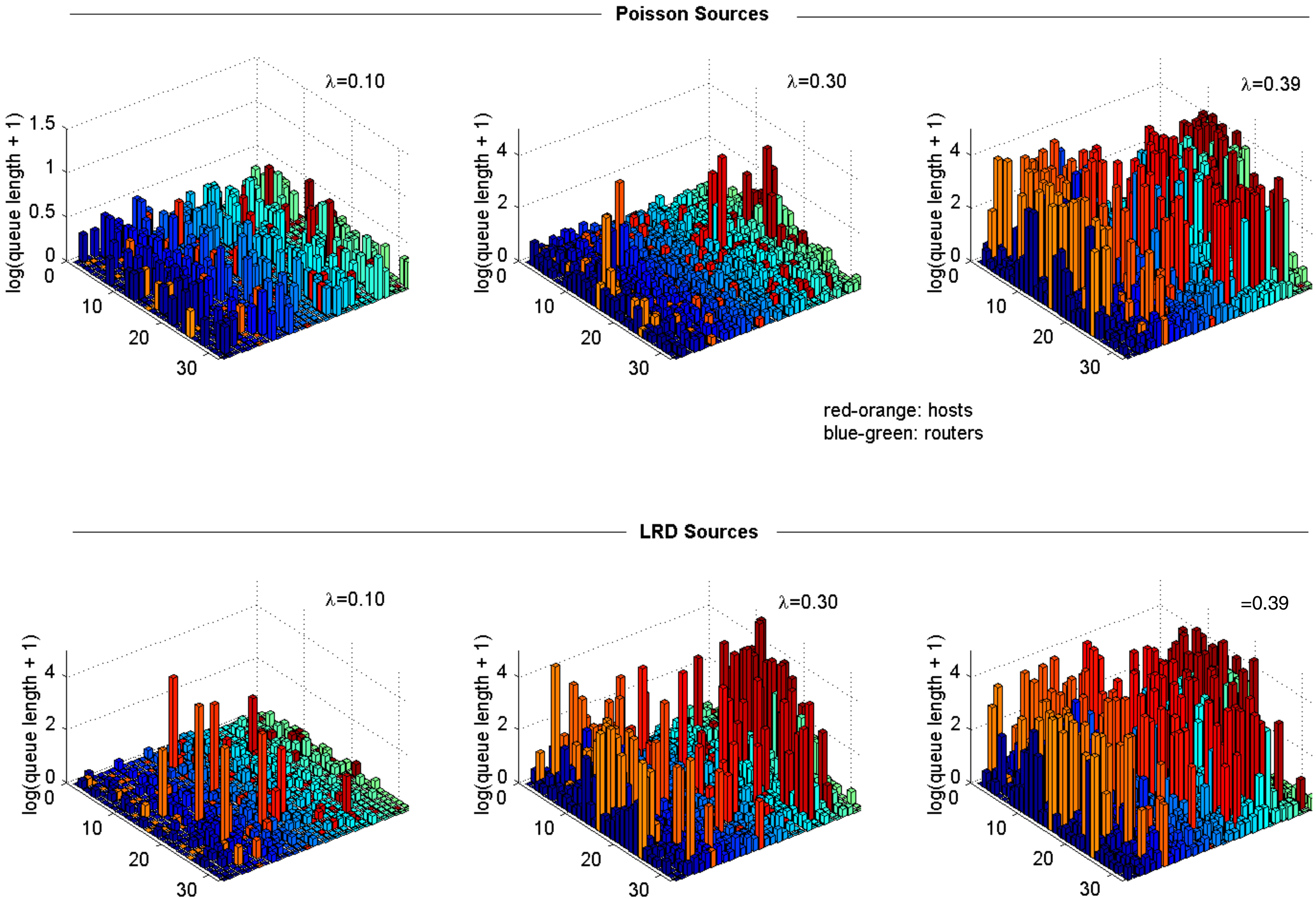
(c) SMALL-WORLD

random connections added
to increase connectivity

(d) SCALE-FREE

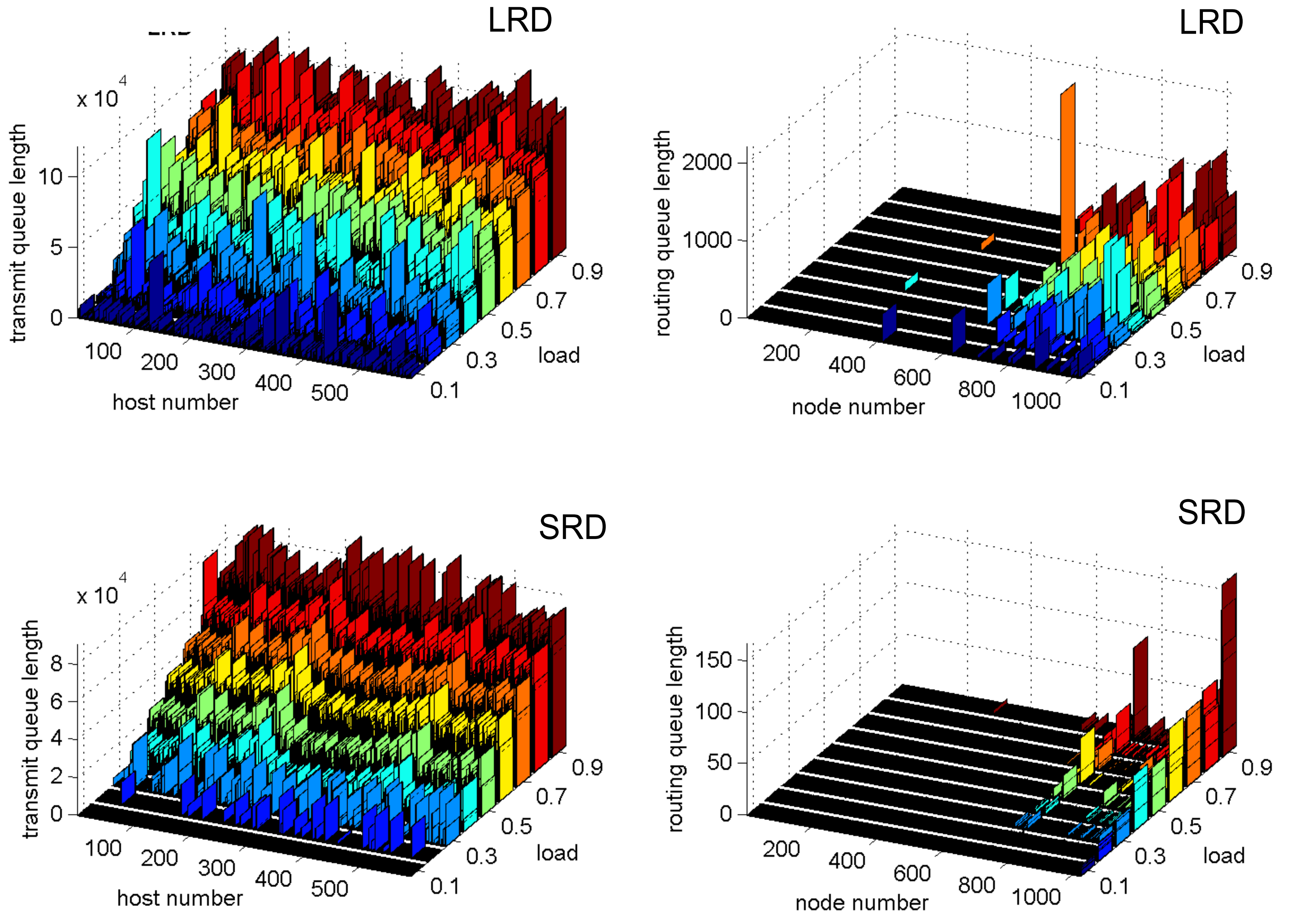
Prob(vertex valency =n) = $n^{-\gamma}$
with exponent $\gamma \in [2,3]$.

Manhattan network queues as load increases : Poisson vs. LRD



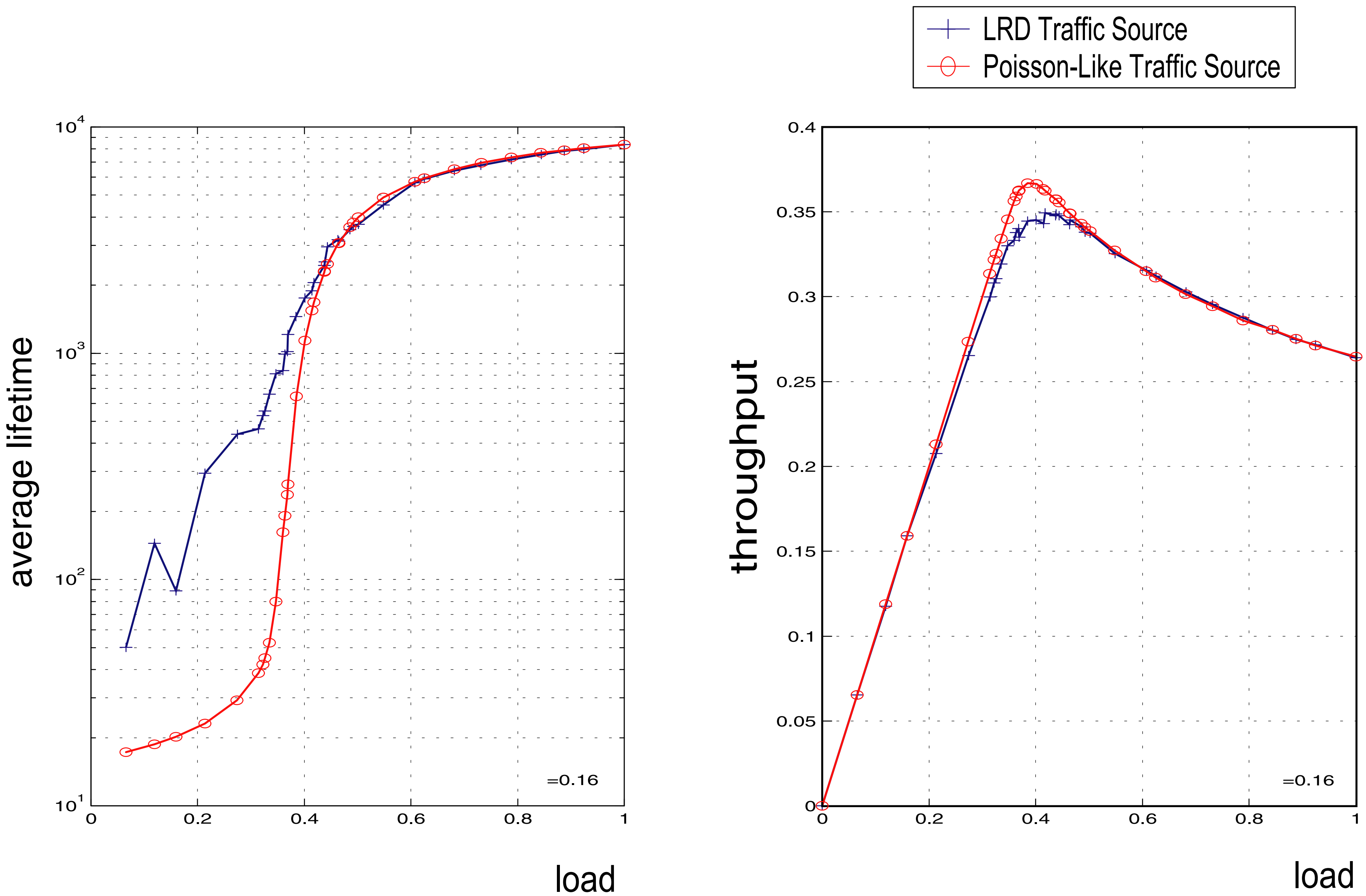
Comparing Queue Lengths for a Manhattan Network with Poisson and LRD sources

Scale-free queues as load increases : Poisson vs. LRD

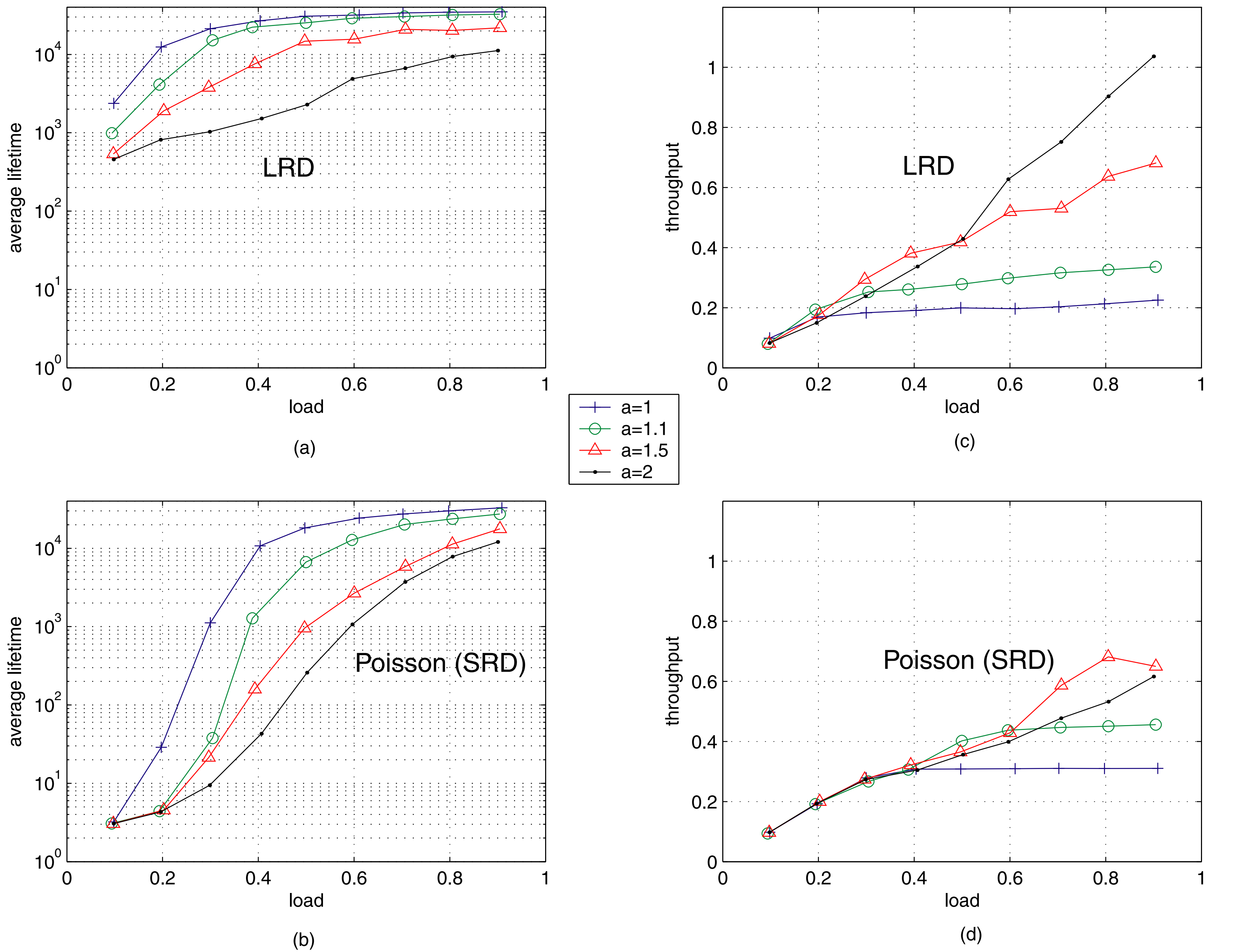


ONSET of CONGESTION

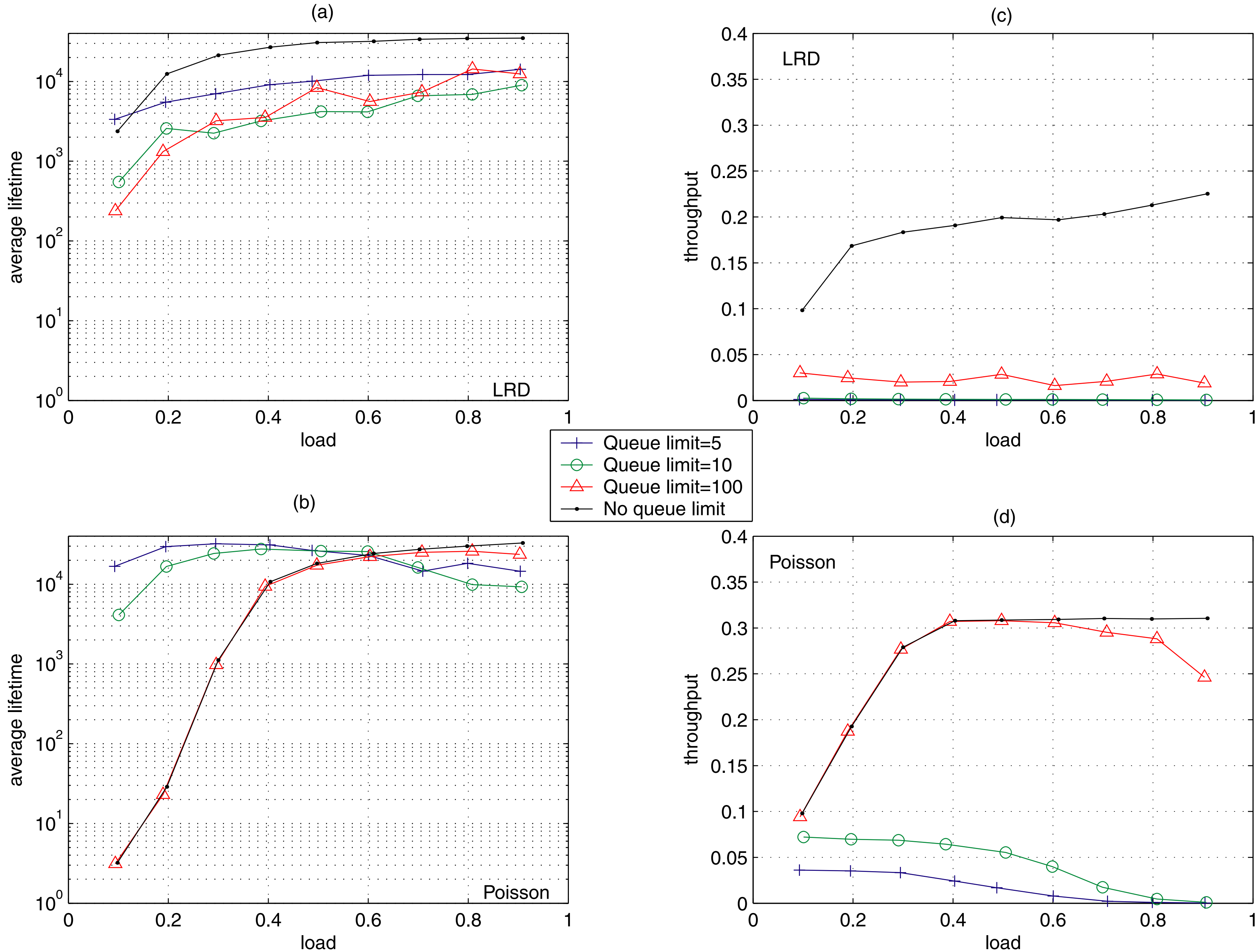
`Phase transition' for Poisson-like and LRD traffic



AVERAGE LIFETIME/THROUGHPUT WITH INCREASING SERVER STRENGTH



QUEUE LIMITING RESPONSES



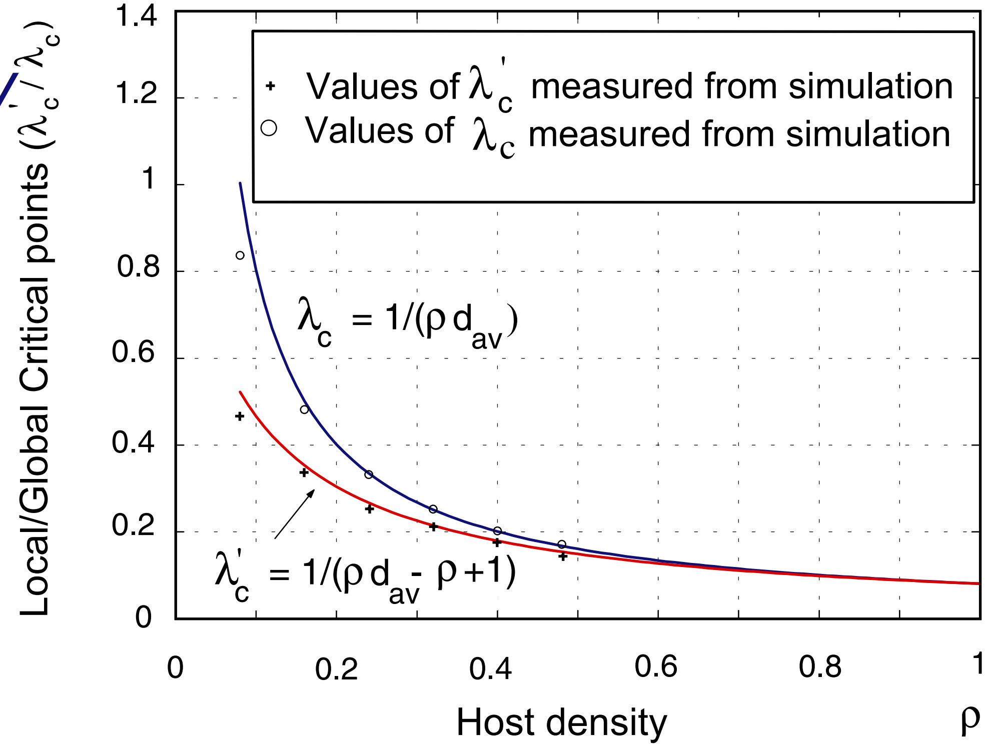
MEAN FIELD THEORY- *criticality*

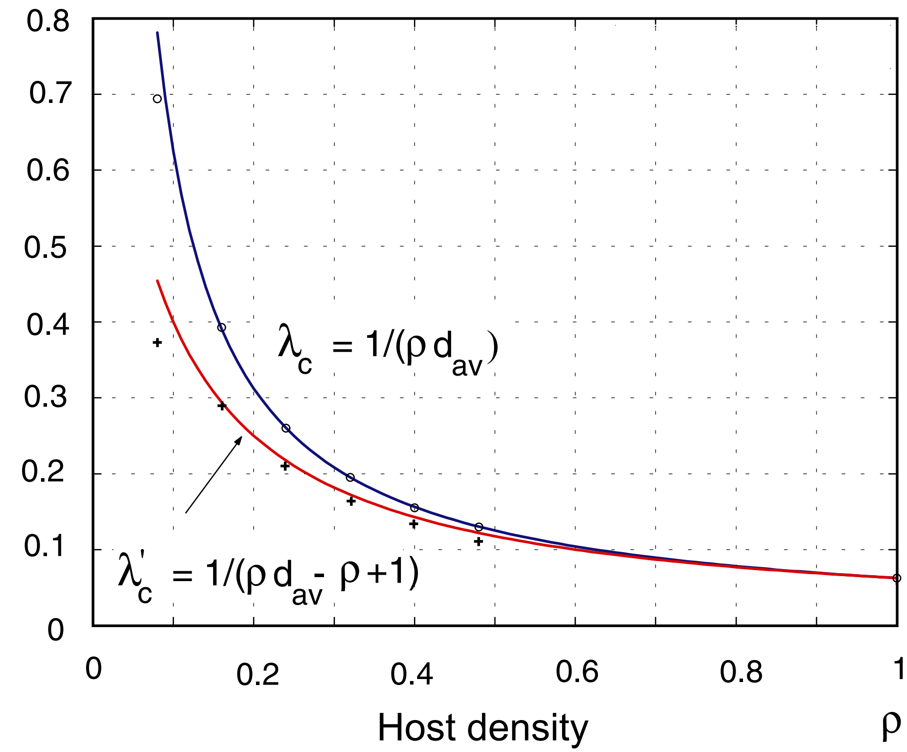
D_t = total distance of packets to destination at time t $L x L$ grid

$$D_{t+1} = D_t + \rho\lambda L^2 d_{av} - L^2$$

pre-congestion <=> non-increasing D_t

$$\lambda_c = 1/\rho\, d_{av}$$





Remarks

Exact autocorrelation results are available for the piecewise linear (PL) double intermittency map

Realization techniques of PL maps with specific piece-wise power law decay characteristics using genetic algorithms

Dynamical modelling allows for good comparison of LRD/SRD effects in different types of regular/scale-free network

Robust results: (a) queue length increases for LRD vs. Poisson
(b) average lifetime/throughput comparisons

TCP dynamics (Erramilli) and limited queue lengths have been introduced within this framework

Automatic server strength adjustments have been considered

Intended work

Neural network growth algorithms to be considered for dynamic network modelling