Measurability of vacuum fluctuations and dark energy

Christian Beck
School of Mathematical Sciences
Queen Mary, University of London
*Mile End Road, London E1 4NS, UK**

Michael C. Mackey
Centre for Nonlinear Dynamics in Physiology and Medicine
Departments of Physiology, Physics and Mathematics
McGill University, Montreal, Quebec, Canada†

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Vacuum fluctuations of the electromagnetic field induce current fluctuations in resistively shunted Josephson junctions that are measurable in terms of a physically relevant power spectrum. In this paper we investigate under which conditions vacuum fluctuations can be gravitationally active, thus contributing to the dark energy density of the universe. Our central hypothesis is that vacuum fluctuations are *gravitationally active* if and only if they are *measurable* in terms of a physical power spectrum in a suitable macroscopic or mesoscopic detector. This hypothesis is consistent with the observed dark energy density in the universe and offers a resolution of the cosmological constant problem. Using this hypothesis we show that the observable vacuum energy density ρ_{vac} in the universe is related to the largest possible critical temperature T_c of superconductors through $\rho_{vac} = \sigma \cdot \frac{(kT_c)^4}{\hbar^3 c^3}$, where σ is a small constant of the order 10^{-3}. Our hypothesis is testable in Josephson junctions where we predict there should be a cutoff in the measured spectrum at 1.7 THz if the hypothesis is true.

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I. INTRODUCTION

A fundamental problem relevant to both quantum field theories and cosmology is whether vacuum fluctuations are 'real' in the sense that the corresponding vacuum energy has a gravitational effect [1, 2, 3, 4]. Quantum electrodynamic (QED) explanations of phenomena such as the Casimir effect, the Lamb shift, van der Waals forces, spontaneous emission from atoms, etc. provide indirect evidence for the existence of vacuum fluctuations. However, Jaffe [1] emphasizes that the Casimir effect, which is generally believed to provide stringent evidence for the existence of QED vacuum fluctuations, can be explained without any reference to vacuum fluctuations. Indeed it seems that for most QED phenomena vacuum fluctuations are a useful mathematical tool to theoretically describe these effects, but there are formulations such as Schwinger's source theory [3] that completely avoid any reference to vacuum fluctuations.

It is well known that the amount of vacuum energy formally predicted by quantum field theories is too large by many orders of magnitude if gravitational coupling is taken into account. While the currently observed dark energy density in the universe is of the order m_ν^4, where m_ν is a typical neutrino mass scale, quantum field theories predict that there is an infinite vacuum energy density, since the corresponding integrals diverge. Assuming a cutoff on the order of the Planck scale, one still has a vacuum energy density too large by a factor 10^{120}. This is the famous cosmological constant problem [5, 6, 7, 8, 9, 10].

The relevant question would thus seem to be not whether vacuum fluctuations exist (they certainly exist as a useful theoretical tool), but under which conditions they have a physical reality in the sense that they produce a directly measurable spectrum of fluctuations in macroscopic or mesoscopic detectors which could have a gravitational effect [11, 12, 13, 14]. With respect to this question, a very interesting experiment was performed by Koch, van Harlingen and Clarke [15] in 1982. Koch et al. experimentally measured the spectral density of the noise current in a resistively shunted Josephson junction and showed that the data were described by the theoretically derived spectrum [16, 17, 18, 19, 20, 21]

$$S_I(\omega) = \frac{2\hbar\omega}{R}\coth\left(\frac{\hbar\omega}{2kT}\right) = \frac{4}{R}\left[\frac{1}{2}\hbar\omega + \frac{\hbar\omega}{\exp(\hbar\omega/kT)-1}\right], \qquad (1)$$

where R is the shunting resistance, $\omega = 2\pi\nu$ is the frequency, h is Planck's constant, k is Boltzmann's constant, and T is the temperature. The first term in this equation is independent of temperature and increases linearly with the frequency ν. This term is induced

*Electronic address: c.beck@qmul.ac.uk; URL: http://www.maths.qmul.ac.uk/~beck
†Electronic address: mackey@cnd.mcgill.ca; URL: http://www.cnd.mcgill.ca/people_mackey.html; also: Mathematical Institute, University of Oxford, 24-29 St Giles', Oxford OX1 3LB, UK

by zero-point fluctuations of the electromagnetic field, which produces measurable noise currents in the Josephson junction [19]. The second term is due to ordinary Bose-Einstein statistics and describes thermal noise. The experimental data of [15], reproduced in Figure 1, confirm the form (1) of the spectrum up to a frequency of approximately $\nu \simeq 6 \times 10^{11}$ Hz.

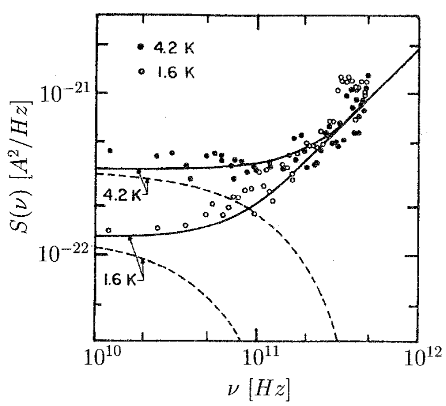

FIG. 1: Spectral density of current noise as measured in [15] for two different temperatures. The solid line is the prediction of eq. (1), while the dashed line neglects the zero-point term.

The data in Fig. 1 represent a *physically measured* spectrum induced by vacuum fluctuations. The spectrum is measurable due to subtle nonlinear mixing effects in Josephson junctions. These mixing effects, which have nothing to do with either the Casimir effect or van der Waals forces, are a consequence of the AC Josephson effect [22, 23, 24].

In [11] we suggested an extension of the Koch experiment to higher frequencies. We based this suggestion on the observation that if the vacuum energy associated with the measured zero-point fluctuations in Fig. 1 is gravitationally active (in the sense that the vacuum energy is the source of dark energy), then there must be a cutoff in the measured spectrum at a critical frequency ν_c. Otherwise the corresponding vacuum energy density would exceed the currently measured dark energy density [25, 26, 27] of the universe. The relevant cutoff frequency was predicted in [11] to be given by

$$\nu_c \simeq (1.69 \pm 0.05) \times 10^{12} \quad \text{Hz}, \tag{2}$$

only 3 times larger than the largest frequency reached in the 1982 Koch et al. experiment [15].

In this paper we discuss the measurability of vacuum fluctuations as inspired by the Koch et al. experiment. We are motivated by the fact that this experiment will now be repeated with new types of Josephson junctions capable of reaching the cosmologically interesting frequency ν_c [28].

The central hypothesis that we explore in this paper is that vacuum fluctuations are *gravitationally active* (and hence contribute to the dark energy density of the universe) if and only if they are *measurable* (in the form of a spectral density) in a suitable macroscopic or mesoscopic detector. We will show that this basic hypothesis: a) provides a possible solution to the cosmological constant problem; b) predicts the correct order of magnitude of dark energy currently observed in the universe; and c) is testable in future laboratory experiments based on Josephson junctions. We also argue that the optimal detector for measurable quantum noise spectra will typically exploit macroscopic quantum effects in superconductors. From our measurability assumption we obtain a formula for the observable dark energy density in the universe that is a kind of analogue of the Stefan-Boltzmann law for vacuum energy, but in which the temperature T is not a free parameter but rather given by the largest possible critical temperature T_c of high-T_c superconductors.

This paper is organized as follows. In Section II we explain the central hypothesis of this paper and discuss how it can help to avoid the cosmological constant problem. In Section III we show that assuming the central hypothesis is true one obtains the correct order of magnitude of dark energy density in the universe. Section IV shows how the near-equilibrium condition of the fluctuation dissipation theorem can be used to obtain further constraints on the dark energy density. Finally, in Section V we provide some theoretical background for our measurability approach. We discuss the fluctuation dissipation theorem and its potential relation to dark energy, as well as the AC Josephson effect and its relation to measurability of vacuum fluctuation spectra.

II. MEASURABILITY AND GRAVITATIONAL ACTIVITY OF VACUUM FLUCTUATIONS

Let us once again state the following **central hypothesis**:

> Vacuum fluctuations are *gravitationally active* if and only if they are *measurable* in terms of a physically relevant power spectrum in a macroscopic or mesoscopic detector

It is important to make our usage of terms precise.

1. By 'vacuum fluctuations that are gravitationally active' we mean those that contribute to the currently measured dark energy density of the universe.

2. By 'measurable vacuum fluctuations' we mean those that induce a measurable quantum noise power spectrum in a suitable macroscopic or mesoscopic detector, for example in a resistively shunted Josephson junction.

The central hypothesis is testable experimentally, e.g. in the experiment by Warburton, Barber and Blamire

[28] currently under way. If the central hypothesis is true, then the vacuum fluctuations producing the measured spectra in the Josephson junction are gravitationally active (i.e. contributing to dark energy density), and hence there will be a cutoff near ν_c in the measured spectrum. Otherwise the dark energy density of the universe is exceeded. The converse is also true: If the cutoff is not observed, the observed vacuum fluctuations in the Josephson junction cannot be gravitationally active. In this case the central hypothesis is false.

We now demonstrate how, under the assumption that the central hypothesis is true, the cosmological constant problem is solved and at the same time the correct order of magnitude of dark energy density in the universe is predicted.

Recall that quantum field theories predict a divergent (infinite) amount of vacuum energy given by

$$\hat{\rho}_{vac} = \frac{1}{2}(-1)^{2j}(2j+1)\int_{-\infty}^{\infty}\frac{d^3k}{(2\pi)^3}\sqrt{\vec{k}^2+m^2} \qquad (3)$$

in units where $\hbar = c = 1$. Here $\vec{k}$ is the momentum associated with a vacuum fluctuation, m is the mass of the particle under consideration, and j is the spin. The central hypothesis now immediately explains why most of the vacuum energy in eq. (3) is not gravitationally active: In most cases there will be no suitable detector to measure the vacuum fluctuations under consideration. And what is not measurable is also not observable. Only in very rare cases will there be such a detector, and only in these cases the corresponding vacuum energy is measurable and thus physically relevant. According to our hypothesis, the detectable part of vacuum energy is gravitationally active and responsible for the current accelerated expansion of the universe.

For strong and electro-weak interactions it is unlikely that a suitable macroscopic detector exists that can measure the corresponding vacuum spectra. The only candidate where we know that a suitable macroscopic detector exists is the electromagnetic interaction. Photonic vacuum fluctuations induce measurable spectra in superconducting devices, as experimentally confirmed by Koch et al. [15] up to frequencies of 0.6 THz. For photons, $m = 0$ and the integration over all $\vec{k}$ in eq. (3) is just an integration over all frequencies ν since $E = \sqrt{\vec{k}^2+m^2} = |\vec{k}| = h\nu = \hbar\omega$.

These detectors of photonic vacuum fluctuations will no longer function if the frequency ν becomes too large. To see this, consider the spectrum

$$S(\omega) = \frac{1}{2}\hbar\omega + \frac{\hbar\omega}{\exp(\hbar\omega/kT)-1} \qquad (4)$$

occurring in the square brackets of eq. (1). This spectrum is at the root of the problem considered here and it is of relevance for other mesoscopic systems as well [29]. The spectrum in eq. (4) is identical (up to a factor $4/R$) to the measured spectrum in Josephson junctions. It arises out of the fluctuation dissipation theorem [16, 30, 31] in a universal way and it formally describes a quantum mechanical oscillator of frequency ω. The linear term in ω describes the zeropoint energy of this quantum mechanical oscillator, whereas the second term describes thermal states of this oscillator. There is vacuum energy associated with the zeropoint term and we may identify it with the source of dark energy (for more details on the theoretical background, see Section 5). For the vacuum fluctuation noise term $\frac{1}{2}\hbar\omega$ to be measurable, it must be not too small relative to the thermal noise term $\hbar\omega/(e^{\hbar\omega/kT}-1)$. For low frequencies the thermal noise dominates, for large frequencies the quantum noise. The frequency ω_0 where both terms have the same size is given by

$$\omega_0 = \frac{kT}{\hbar}\ln 3. \qquad (5)$$

Now suppose we want to measure vacuum fluctuation spectra at very large frequencies ω. From eq. (5) we can achieve higher frequencies by choosing to do our measurements at higher temperatures since increasing the temperature increases the frequency ω_0. As ω_0 increases, the vacuum fluctuation term dominates relative to thermal noise for all frequencies $\omega > \omega_0$.

There is however a practical limit to this procedure. Namely, if the temperature T becomes too high, then a superconducting state will no longer exist. Since superconducting devices such as Josephson junctions appear to be the only experimentally feasible devices to measure high frequency quantum noise spectra (see the arguments in section 5), this means that there is a maximum frequency ω_c above which a superconducting detector is no longer functional and the quantum fluctuation spectrum becomes unmeasurable. This critical frequency is given by

$$\hbar\omega_c \sim kT_c, \qquad (6)$$

where T_c is the largest possible critical temperature of any superconductor. Currently, the largest T_c known for high-T_c superconductors is approximately $T_c = 139$ K [32].

In fact, technically feasible solutions of superconducting materials that are used in practice to build Josephson junctions have lower T_c. For example, the well-known YBCO materials have a maximum critical temperature T_c of 93 K [23]. To optimize a Josephson quantum noise detector one needs to avoid quasiparticle currents, which means that the junction should operate at a temperature T_c' well below T_c. Let us choose as a rough estimate $T_c' \approx 80$ K. One obtains $\nu_c' \sim kT_c'/h \approx 1.7$ THz. It is encouraging that this value is so close to that of eq. (2), thus providing us with the correct order of magnitude of measurable dark energy density if the central hypothesis is true.

III. ESTIMATE OF OBSERVABLE DARK ENERGY DENSITY

To get a better estimate of the proportionality constant in eq. (6), let us more carefully analyze when an experiment based on Josephson junctions will be able to resolve quantum noise. In the data in Fig. 1, the overall precision by which the power spectrum can be measured at a given frequency in the Koch experiment is of the order 10-40%. Other experiments based on SQUIDs yield fluctuations of similar order of magnitude in the measured power spectrum [33]. The quantum noise term $\frac{1}{2}\hbar\omega$ is measurable as soon as it is larger than the standard deviation of the fluctuations of the measured spectrum. That is to say,

$$\frac{1}{2}\hbar\omega > \eta \cdot \frac{\hbar\omega}{e^{\hbar\omega/kT}-1} \tag{7}$$

where we estimate $\eta \simeq 0.1 \rightarrow 0.4$ from the fluctuations of the Koch data in Fig. 1. Condition (7) can be written as

$$\frac{\hbar\omega}{kT} > \ln(1+2\eta). \tag{8}$$

We thus obtain from eq. (6) the critical frequency ν_c beyond which measurements of the spectrum become impossible as

$$\nu_c \approx \frac{\ln(1+2\eta)}{2\pi}\frac{kT_c}{\hbar}. \tag{9}$$

The largest critical temperature of a high-T superconductors achieved so far is approximately $T_c = 139$K [32]. Our result yields, with $T_c = 139$K and $\eta \simeq 0.1 \rightarrow 0.4$,

$$\nu_c \approx \ln(1+2\eta) \times 2.89 \times 10^{12}\text{Hz} \simeq 0.5 \rightarrow 1.7\text{THz}. \tag{10}$$

Recall that the dark energy density measured in astronomical observations is correctly reproduced for $\nu_c = 1.69 \times 10^{12}$ Hz [11]. It is interesting that our argument based on measurability of vacuum fluctuations predicts the correct order of magnitude of dark energy density in the universe. This is especially hearting since the cosmological constant problem is usually plagued by estimates of vacuum energy too large by a factor 10^{120} [5, 6]. Our proposition for the resolution of the cosmological constant problem is simply that for frequencies higher than ν_c a macroscopic detector no longer exists to measure the spectrum of vacuum fluctuations. As soon as the vacuum fluctuations are no longer measurable, by our central hypothesis they also can no longer have a large-scale gravitational effect.

From this perspective, the dark energy density of the universe is given by integrating the energy density over all *measurable* vacuum fluctuations, to obtain

$$\rho_{dark} = \int_0^{\nu_c} \frac{8\pi\nu^2}{c^3}\frac{1}{2}h\nu d\nu \tag{11}$$

$$= \frac{\pi h}{c^3}\nu_c^4 \tag{12}$$

$$= \frac{\ln^4(1+2\eta)}{8\pi^2}\frac{(kT_c)^4}{\hbar^3c^3}. \tag{13}$$

This result can be considered an analogue of the Stefan-Boltzmann law

$$\rho_{rad} = \frac{\pi^2}{15}\frac{(kT)^4}{\hbar^3c^3} \tag{14}$$

for radiation energy density. The difference is that in eq. (13) the temperature is the largest possible critical temperature T_c of superconductors, and the proportionality constant $\frac{\ln^4(1+2\eta)}{8\pi^2} \approx 1.4 \times 10^{-5} \rightarrow 1.5 \times 10^{-3}$ is much smaller than for the Stefan-Boltzmann law (i.e. compared with $\frac{\pi^2}{15} \simeq 0.66$). Using $T_c = 139$ K and the known dark energy density $\rho_{dark} = (3.9 \pm 0.4)$ GeV/m^3 [25, 26, 27] we may write eq. (13) as

$$\rho_{dark} = \sigma\frac{(kT_c)^4}{\hbar^3c^3} \tag{15}$$

where the dimensionless constant σ is given by

$$\sigma = \frac{\ln^4(1+2\eta)}{8\pi^2} = \hbar^3c^3\frac{\rho_{dark}}{(kT_c)^4} = (1.46 \pm 0.15) \times 10^{-3}. \tag{16}$$

IV. FURTHER CONSTRAINTS ON DARK ENERGY DENSITY

The explanation of why vacuum fluctuations produce a measurable spectrum of noise in resistively shunted Josephson junctions is based on two fundamental effects, the fluctuation dissipation theorem [16, 17, 20, 30, 31] and the AC Josephson effect [22, 23, 24]. The Josephson effect requires the existence of a superconducting state, and in this way we were lead to an estimate on observable (measurable) dark energy density in the previous section. The fluctuation dissipation theorem requires that the system under consideration is close to thermal equilibrium. We now show that the latter condition also provides an upper bound on observable (measurable) dark energy density. In other words, both effects imply that the cosmological constant is small.

The fluctuation dissipation theorem predicts a current (I) spectrum (A^2/Hz) in the shunt resistor given by

$$S_I(\omega) = 2\hbar\omega \coth\left(\frac{\hbar\omega}{2kT}\right) Re\ Z^{-1}(\omega), \tag{17}$$

and a voltage (V) spectrum (V^2/Hz) given by

$$S_V(\omega) = 2\hbar\omega \coth\left(\frac{\hbar\omega}{2kT}\right) Re\ Z(\omega), \tag{18}$$

where $Z(\omega)$ is the impedance. As said before, the derivation of the fluctuation dissipation theorem is based on the assumption of thermal equilibrium in the resistor, or that the system is at least near thermal equilibrium.

The noise spectra measured with Josephson junctions correspond to real currents of real electrons, rather than virtual fluctuations, and hence these currents will generate heat in the shunting resistor. The dissipated power is $P = I \times V$, which again provides a reason why there must be an upper cutoff in the measurable spectrum. Namely, if the frequency becomes too high, then the associated heat production through the power dissipation will become so large that it takes the system away from thermal equilibrium. Consequently, the fluctuation dissipation theorem would no longer be valid.

To illustrate this, assume there is a physical device that could measure the vacuum noise spectra to frequencies much higher than ν_c, say of the same order of magnitude as relevant for the Casimir effect.

Consider two Casimir plates separated by a distance L. The wavelengths of vacuum fluctuations that will fit into the cavity formed by the plates must satisfy $\lambda \leq L$ but since $\lambda = c/\nu$ this means that the minimum frequency of vacuum fluctuations associated with the Casimir effect must satisfy

$$\nu_{min} \geq \frac{c}{L}.$$

Experimental measurements of the Casimir effect are made with $L \simeq O(0.1\,\mu) \to O(1\,\mu)$ [34], so assume $L = 1\,\mu = 10^{-6}$ m to give $\nu_{min} \simeq 3 \times 10^{14}$ Hz, some 200 times greater than the cutoff frequency ν_c in eq. (2). If the predicted cutoff frequency in the Josephson junction were increased to ν_{min}, this would imply an energy density of about

$$\rho_{dark}\left(\frac{\nu_{min}}{\nu_c}\right)^4 \simeq 3.9 \times 2^4 \times 10^8 \quad \mathrm{GeV/m^3} \tag{19}$$

$$= 6.2 \times 10^9 \quad \mathrm{GeV/m^3} \tag{20}$$

which is dissipated in the shunt resistor. Suppose that this energy density came from a black body described by the Stefan-Boltzmann law, i.e.

$$\frac{\pi^2 k^4}{15\hbar^3 c^3} T^4 = 6.2 \times 10^9 \quad \mathrm{GeV/m^3}.$$

What would the corresponding temperature be? Quite high, namely $T \simeq 8 \times 10^3$ K. It is clear that the Josephson experiment could not operate with a heat source at that temperature, and thermal equilibrium would be destroyed long before. This simple argument once again shows that there must be a cutoff in the measurable spectrum at frequencies much smaller than ν_{min}.

If we make the same estimate for the dark energy density, the corresponding temperature of a black body with the same energy density as dark energy density is $T \simeq 8 \times 10^3/(2 \times 10^2)^4) \simeq 0.5 \times 10^{-5}$ K, so this neither disturbs thermal equilibrium, nor disturbs a junction operating in a superconducting state.

Note that the experiments that successfully confirm the QED predictions of the Casimir effect (see, e.g., [34]) just provide measurements of the Casimir force, they do not provide us with a measured spectrum of vacuum fluctuations, as required by the central hypothesis. If the central hypothesis is correct, it is clear that the vacuum fluctuations that formally (i.e. by entering into the QED calculations) influence the Casimir effect are gravitationally inactive since their frequency is much larger than ν_c (see also [35]).

V. THEORETICAL BACKGROUND

A. The fluctuation dissipation theorem and dark energy

Consider an arbitrary observable $x(t)$ of a quantum mechanical system and consider the quantum mechanical expectation of the time derivative of x, denoted by $\langle \dot{x}(t) \rangle$. Let $F(t)$ be an external force. If linear response theory is applicable we may write

$$\langle \dot{x}(t) \rangle = \int_{-\infty}^{t} G(t-t')F(t')dt', \tag{21}$$

where the function G is the 'generalized conductance'. Its Fourier transform is given by

$$G(\omega) = \int_0^\infty dt e^{i\omega t} G(t). \tag{22}$$

The fluctuation dissipation theorem [16, 17, 20, 30, 31] yields a very general relation between the power spectrum $S_{\dot{x}}$ of the stochastic process $\dot{x}(t)$ and the real part of $G(\omega)$:

$$S_{\dot{x}}(\omega) = 2\hbar\omega \coth\left(\frac{\hbar\omega}{2kT}\right) Re\, G(\omega) \tag{23}$$

$$= \left[\frac{1}{2}\hbar\omega + \frac{\hbar\omega}{\exp(\hbar\omega/kT) - 1}\right] \cdot 4 Re\, G(\omega) \tag{24}$$

The function in square brackets is *universal*, it does not depend on details of the quantum system considered, in particular on its Hamiltonian H. However, $G(\omega)$ is system dependent. The universal function in the square brackets can be physically associated with the mean energy of a quantum mechanical oscillator of frequency ω at temperature T. Its ground state energy is given by $\frac{1}{2}\hbar\omega$. Note that this oscillator has nothing to do with the original Hamiltonian H of the quantum system under consideration. Also note that the fluctuation dissipation theorem is valid for arbitrary Hamiltonians H, H need not describe a harmonic oscillator at all but can be a much more complicated Hamiltonian function. Nevertheless, the universal function that occurs in the square brackets of eq. (24) can always be formally interpreted as the mean energy of a harmonic oscillator.

Our physical interpretation is to identify the zeropoint energy of this formal harmonic oscillator as the source of dark energy. This oscillator is universally present everywhere, but typically manifests measurable effects only in dissipative media. Note that the term $\frac{1}{2}\hbar\omega$, which describes the zeropoint energy of this oscillator, is invariant under transformations $H \to H + const$ of the original

Hamiltonian H. This is obvious, since the fluctuation dissipation theorem is valid for *any* Hamiltonian. In other words, the zeropoint energy of our dark energy oscillator is an invariant, it is invariant under renormalization of the original Hamiltonian. This is a strong hint that the corresponding vacuum energy $\frac{1}{2}\hbar\omega$ is indeed a physically observable quantity. It is likely to have gravitational relevance since it is invariant under arbitrary re-definitions of the original Hamiltonian.

A further interpretation (as suggested in the classical papers and textbooks on the subject [16, 17, 19]) is to associate the term $\frac{1}{2}\hbar\omega$ with the zeropoint fluctuations of the electromagnetic field. We thus arrive at the following, in our opinion, most plausible interpretation of the fluctuation dissipation theorem: The term $\frac{1}{2}\hbar\omega$ describes the *gravitationally active* part of the vacuum fluctuations of the electromagnetic field.

B. The AC Josephson effect and measurability of vacuum fluctuations

The fluctuation dissipation theorem quite generally explains the existence of quantum noise in resistors, but on its own it does not suffice to make the power spectrum of quantum fluctuations measurable in an experiment. Putting a voltmeter directly into the dissipative medium would not prove to be a feasible method to measure the zeropoint spectrum at high frequencies. Rather, we need a more sophisticated method, and the AC Josephson effect [22, 23, 24] satisfies this requirement.

To briefly explain this effect, remember that a Josephson junction consists of two superconductors with an insulator sandwiched in between. In the Ginzburg-Landau theory, each superconductor is described by a complex wave function, whose absolute value squared yields the density of superconducting electrons. Denote the phase difference between the two wave functions of the two superconductors by $\varphi(t)$. Josephson [22] made the remarkable prediction that at zero external voltage a superconductive current given by

$$I_s = I_c \sin\varphi \tag{25}$$

flows between the two superconducting electrodes. Here I_c is the maximum superconducting current the junction can support. Moreover, he predicted that if a voltage difference V is maintained across the junction, then the phase difference φ evolves according to

$$\dot{\varphi} = \frac{2eV}{\hbar}, \tag{26}$$

i.e. the current in eq. (25) thus becomes an oscillating current with amplitude I_c and frequency

$$\nu = \frac{2eV}{h}. \tag{27}$$

This frequency is the well-known Josephson frequency, and the corresponding effect is called the AC Josephson effect. The quantum energy $h\nu$ given by eq. (27) can be interpreted as the energy change of a Cooper pair that is transferred across the junction. The AC Josephson effect is a very general and universal effect that always occurs whenever two superconducting electrodes are connected by a weak link.

The AC Josephson effect connects quantum mechanics (i.e. differences of phases of macroscopic wave functions) with measurable classical quantities (currents or voltages). A Josephson junction can be regarded as a perfect voltage-to-frequency converter, satisfying the relation $2eV = h\nu$. For distinct DC voltages, it is also a perfect frequency-to-voltage converter. This inverse AC Josephson effect is, for example, used to maintain the SI unit Volt.

In the experiments of Koch et al. [15] the quantum noise in the shunt resistor is mixed down at the Josephson frequency $2eV/h$ to produce measurable voltage fluctuations. The measurement frequency in these experiments is usually much smaller than the Josephson frequency. However, due to the specific nonlinear properties of Josephson junctions, the measured voltage fluctuations are influenced by quantum fluctuations at the Josephson frequency and its harmonics [18, 21]. In this way quantum fluctuations in the THz regime become experimentally accessible. The frequency variable ν in Fig. 1 is experimentally varied by applying different DC voltages across the junction, thus making direct use of formula (27). Josephson oscillations are clearly necessary to do these types of measurements.

The AC Josephson effect has been experimentally observed up to Josephson frequencies in the low-THz region. The energy gap in cuprates limits the maximum value of the Josephson frequency to $\nu \sim 15$ THz, but in practice one seems to be able to only reach the 2 THz region [36]. If the central hypothesis is true, then the largest attainable Josephson frequency also constrains the dark energy density of the universe.

VI. CONCLUSIONS AND OUTLOOK

In this paper we have proposed a possible solution to the cosmological constant problem, based on the central hypothesis that vacuum fluctuations are only *gravitationally active* if they are *measurable* in the form of a noise spectrum in suitable macroscopic or mesoscopic detectors. From this assumption a universal low-energy cutoff frequency ν_c consistent with the currently observed dark energy density in the universe is predicted in a rather natural way. One obtains $\nu_c \sim kT_c/h$, where T_c is the largest possible critical temperature of high-T_c superconductors. This means ν_c is in the THz region. Moreover, the dark energy density of the universe is most naturally identified as ordinary electromagnetic vacuum energy of virtual photons with frequency $\nu < \nu_c$, and given by a kind of analogue of the Stefan-Boltzmann law for dark energy, $\rho_{dark} \sim (kT_c)^4/(\hbar^3 c^3)$.

Suppose the universal cutoff $\nu_c = 1.7$ THz corresponding to the dark energy density is observed in the experiment planned by Warburton et al. [28]. Would this invalidate successful QED predictions for other effects such as the Casimir effect [34, 35] or Lamb shift?

We think the answer is 'no'. Once again we emphasize that the cutoff is predicted for the *measurable* spectrum. Virtual photons as a useful tool for the theoretician are still allowed to persist at higher frequencies. These can then still be used as a theoretical tool to do calculations for the Casimir effect, Lamb shift, spontaneous emissions of atoms etc. in just the same way as before, keeping in mind that in many cases, e.g. for the Casimir effect, they are not needed at all to explain the effect [1].

The central hypothesis merely implies that for photonic vacuum fluctuations the *measurability* in the form of a physically relevant spectrum ceases to exist for $\nu > \nu_c = 1.7$ THz, for the reasons we have given above. This is connected with a kind of *phase transition* for the gravitational activity of the virtual photons at $\nu = \nu_c$. It is indeed plausible that vacuum fluctuations are only gravitationally active if they are measurable in the form of a frequency spectrum in a macroscopic or mesoscopic detector, as stated by our central hypothesis. How else should or could these vacuum fluctuations push galaxies apart and accelerate the expansion of the universe if the effect is not measurable with a macroscopic detector? As shown above the central hypothesis leads to the correct dark energy density in the universe, and the cosmological constant problem is avoided. The validity or non-validity of the central hypothesis will soon be tested [28].

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